The continuum-discretized coupled-channels method applied to exotic nuclei

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In collaboration with

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Challenges in Nuclear Astrophysics

TeV/nucleon

???

Exotic stellar site
Quark matter in compact stars, Big Bang

keV/nucleon

???

Nuclear many-body problem: 
the hardest problem of all physics!

• Interactions are complicated
• Nucleons = composite particle

Typical stellar site
Stellar evolution
Solar neutrino problem is due to $\nu$-oscillations
But this reaction needs to be known more accurately
- Bahcall

\[ \text{Has to be known better than 20\% - Woosley} \]
**Electron screening (in stars)**

*Salpeter 1959*

More positive ions

Ion under consideration

More electrons

valid for

\[ n R_D^3 >> 1 \]

\[ R_D \]

Debye Radius

\[ < \sigma \nu >_{plasma} = f(E) < \sigma \nu >_{bare} \]

\[ ^7\text{Be}(p,\gamma)^8\text{B} \ (T \sim 10^7\text{K}) : \quad f(E) \approx 1.2 \]

(20 % effect)

but in the Sun

\[ n R_D^3 \sim 3 - 5 \]

Mean-field treatment of screening needs to be replaced by molecular dynamics

charges in motion: dynamical effects, non-spherical symmetry

(Carraro, Schaefer, Koonin, 1988)
Adiabatic model: \[ \Delta E = E' - E \]

\[ \sigma_{\text{lab}} \sim \sigma_{\text{bare}} (E + \Delta E) \]

\[ \sim \exp \left[ \pi \eta(E) \frac{\Delta E}{E} \right] \sigma_{\text{bare}} (E) \]

**Screening (in the laboratory)**

\[ ^3\text{He}(d,p)^4\text{He} \]

**Rolfs, 1995**

<table>
<thead>
<tr>
<th>Reaction</th>
<th>( \Delta E ) [eV] experiment</th>
<th>( \Delta E ) [eV] adiabatic limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d(^3\text{He},p)^4\text{He} )</td>
<td>180 ± 30</td>
<td>119</td>
</tr>
<tr>
<td>( ^6\text{Li}(p,\alpha)^3\text{He} )</td>
<td>470 ± 150</td>
<td>186</td>
</tr>
<tr>
<td>( ^6\text{Li}(d,\alpha)^4\text{He} )</td>
<td>380 ± 250</td>
<td>186</td>
</tr>
<tr>
<td>( ^7\text{Li}(p,\alpha)^4\text{He} )</td>
<td>300 ± 280</td>
<td>186</td>
</tr>
<tr>
<td>( ^{11}\text{B}(p,\alpha)^2^4\text{He} )</td>
<td>620 ± 65</td>
<td>348</td>
</tr>
</tbody>
</table>

**Corrections**

- Vacuum Polarization: \( \sim 1\% \)
- Relativity: \( 10^{-3} \)
- Bremsstrahlung: \( 10^{-3} \)
- Atomic polarization: \( 10^{-5} \)
- Nuclear polarization: \( <10^{-10} \)

**Balantekin, CB, Hussein, NPA 1997**

nothing seems to work!
Threshold effect

\[ E_p \geq \frac{\mu^2}{4M_pm_e} \Delta E \geq 8 \text{ keV} \]

He: \(1s^2 \rightarrow 1s2s: 19.8 \text{ eV}\)

Not even this: stopping power at low energies


Theories for astrophysical reactions (few examples)

Radiative capture reactions

\[ \sigma \sim \left| \langle \Psi_{\text{cont}} | r^L Y_{LM} | \Psi_{\text{bound}} \rangle \right|^2 \]

Potential model (for \( \Psi_{\text{bound}} \) and \( \Psi_{\text{cont}} \))

\[ ^{16}\text{O}(p,\gamma)^{17}\text{F} \]

Hybrid model
- NCSM for \( \Psi_{\text{bound}} \)
- potential model for \( \Psi_{\text{cont}} \)

Navratil, CB, Caurier, PLB 634, 191 (2006)  
PRC 73, 065801 (2006)
**Microscopy x reactions**

**No-Core Shell Model**

$$g(r) = \langle \chi^{(A)} | \hat{A} \Phi^{(A-a)} \Phi^{(a)} \delta(r - r_{A-a,a}) \rangle$$

**Resonating Group Method**

$$\int dr' [H(r,r') - EN(r,r')] g(r') = 0 \quad \text{all } r$$

Hill-Wheeler 1955

$$g_{\text{bound}}(r) \to C_{lj} \frac{W_{-\eta,l+1/2}(r)}{r}$$

$$g_{\text{scat}}(r \to \infty) \sim I_l(r) - S_l O_l(r)$$

Quaglioni, Navratil, PRL 2007, PRL 2009

Ito, Itagaki, PRL 2008

E.g. NCSM
\[ f(x) = f(0) + p \cdot (\nabla f) \bigg|_0 + Q \cdot (\nabla^2 f) \bigg|_0 + \cdots \]

\[ L_{\text{EFT}} = N^+ \left( i \partial_0 + \frac{\nabla^2}{2m_N} \right) N + C_0 N^+ N N^+ N \]
\[ \quad + N^+ \frac{\nabla^4}{8m_N^3} N + C_2 N^+ N N^+ N^\nabla^2 N \]
\[ \quad + C'_2 N^+ \nabla N \cdot N^+ \nabla N + \ldots \]

π-less EFT


Higa, C.B., van Kolck, in progress
Solutions with exotic beams

Future: electron-ion scattering (RIKEN, GSI/Darmstadt)
C.B., PLB 624, 203 (2005) skins, halos, soft multipole vibrations

Present:

(A) Trojan horse
Baur, C.B., Rebel, NPA 458, 188 (1988)

(B) Asymptotic normalization coefficients
Mukhamedzahnov, 1991
Tribble, Trache, Spitaleri

(C) Coulomb dissociation
Baur, C.B., Rebel, NPA 458, 188 (1988)

(D) Charge Exchange
Taddeucci, 1981 $e^- + (Z,A) \rightarrow (Z-1, A) + \nu_e$

(E) Knockout reactions
Coulomb dissociation

Shoemaker-Levy comet

\[
\frac{d\sigma}{dE_\gamma d\Omega} = \frac{1}{E_\gamma} \sum_l \frac{dn_l(E_\gamma, \Omega)}{dE_\gamma d\Omega} \sigma_\gamma + a \rightarrow b + c (E_\gamma)
\]

Theory: Baur, CB, Rebel, NPA 458, 188 1986

First applications:
- Motobayashi, PLB 1991, \(^{13}\)N(p,\(\gamma\))\(^{14}\)O
- PRL 1994 \(^{7}\)Be(p, \(\gamma\))\(^{8}\)B

Recently:
Davids, Motobayashi, Suemmerer, Gai, Bracco, Aumann, ...
**Needs consideration**

1- Usual theoretical inputs: interactions, many-body problem.

2- Nuclear breakup contribution

3- Additional reaction problems: inverse methods only studies ground state transitions, etc.

**THIS occurs in stars**

<table>
<thead>
<tr>
<th>s,d</th>
<th>E1</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>p_{3/2}</td>
</tr>
</tbody>
</table>

**capture**

**THIS is obtained in lab**

<table>
<thead>
<tr>
<th>s,p,d</th>
<th>E1+E2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p_{3/2}</td>
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</tbody>
</table>

**break-up**
EM response of loosely-bound nuclei

\[
\frac{d\sigma_C}{dE_x} \approx n_{EL}(E_\gamma) \times C^2 S \left| \langle \psi_k \| r^L Y_L \| \phi_0 \rangle \right|^2 \frac{d^3 k}{(2\pi)^3}
\]

\[E_{rel} = E_\gamma - S\]

\[S = \frac{\hbar^2 \eta^2}{2\mu}\]

**Estimate**

\[\psi_k \approx e^{ik \cdot r}\]

\[\phi_0 \approx \frac{1}{r} e^{-\eta r}\]

\[
\frac{dB(EL)}{dE_\gamma} \approx \frac{\sqrt{SE_{rel}^{L+1/2}}}{(E_{rel} + S)^{2L+2}}
\]

\[\varphi_0 \sim \exp(-\eta r)/r\]

Separation energy of fragments with reduced mass \(\mu\)
\[ \sigma_L^\gamma \sim E_\gamma^{2L+1} B(EL) \]

\[
\frac{dB(EL)}{dE_\gamma} \approx \frac{\sqrt{S} E_{rel}^{L+1/2}}{(E_{rel} + S)^{2L+2}}
\]
**Ex: Breakup of $^{11}$Be**

$^{11}$Be + Pb $\rightarrow$ $^{10}$Be + n + Pb

70 MeV/nucleon

$B(E1) = 1.05 \pm 0.06 \text{ e}^2\text{fm}^2$

(3.29 ± 0.06 W.u)

$S = 0.54 \text{ MeV}$

peak at $E_x = \frac{8}{5} S = 0.76 \text{ MeV}$

$$\frac{dB(EL)}{dE_\gamma} \approx \frac{\sqrt{S}(E_x - S)^{3/2}}{E_x^4}$$
\[ I_{sp} = \langle \psi_k \| r^1 Y_1 \| \phi_0 \rangle \approx \frac{k^2}{(k^2 + \eta^2)^2} \left[ \cos \delta + \sin \delta \frac{\eta(\eta^2 + 3k^2)}{2k^3} \right] \]

\[ \approx \frac{E_{rel}}{(S + E_{rel})^2} \left[ 1 + \left( \frac{\mu}{2\hbar^2} \right) \frac{\sqrt{S(S + 3E_{rel})}}{-1/a + (\mu E_{rel}/\hbar^2)r_{eff}} \right] \]

**Final state interactions**

\( \delta = \) scattering phase shift  
\( a = \) scattering length  
\( r_{eff} = \) effective range

Strongly dependent on final state interactions (phase-shifts)
**Pigmy resonances**

\[ B(EL) \sim \left| \int r^L \delta \rho_{fi} \, d^3r \right|^2 \]

\[ \delta \rho_P(r) \approx Z_{eff}^{GT} \alpha_{GT} \frac{d\rho_0}{dr} + Z_{eff}^{SJ} \alpha_{SJ} j_1(kr) \rho_0(r) \]
Estimate hydrodynamical model

\[ E_{PR} = \left( \frac{3\hbar^2}{2aRm_NA_N} \right) \]

\[ A_r = A_c\left(A - A_c\right)/A \]

\[ \Gamma \sim \frac{\hbar\bar{v}_N}{R} \]

\[ \bar{v}_N = \frac{3}{4}v_F = \frac{3}{4}\sqrt{\frac{2E_F}{m_N}} \]

\[ E_F \sim 35 \text{ MeV} \]

\[ R \sim 5 \text{ fm} \]

\[ \bar{v} \sim \text{relative velocity of core and halo} \]

\[ E_{F} \rightarrow E_{P} \sim 1 \text{ MeV} \]

\[ \Gamma \sim 1 \text{ MeV} \]

\[ \Gamma \sim 6 \text{ MeV} \]

usual GR

pigmy GR

only accurate microscopic models can resolve pigmy from direct breakup

R = nuclear size

\( a = \text{difuseness} \)
**Higher-order effects**

(a) w.f. evolution on space-time lattice

\[ i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \]

\[ \psi(t + \Delta t) = \exp \left[-\frac{i}{\hbar} \hat{H}\Delta t \right] \psi(t) \]

Discrete space lattice: \( x_j, \ j=1, 2, ..., N \)

\[ H = \frac{\hat{p}^2}{2\mu} + V_N(x) + V_C(x,t) \]

\[ \hat{S}\psi_j(t) = \sum_k V_{Ck}\psi_k(t) \]

\[ \tau = \frac{\hbar\Delta t}{4\mu(\Delta x)^2} \]

→ three-dimensions straightforward, but costly

From \( \psi \) any observable can be calculated

Bertsch, C.B., Esbensen, Yabana, Nakatsukasa, Baye, Capel,...
(b) Continuum discretized coupled-channels (CDCC)

\[ |\varphi_0\rangle = e^{-iE_0t/\hbar} \]

\[ |\varphi_{jJM}\rangle = e^{-iE_jt/\hbar} \int \Gamma_j(E) |E,JM\rangle dE \]

\[ \int \Gamma_i(E) \Gamma_j(E) dE = \delta_{ij} \]

From \( a_k \) calculate \( \psi \) and observables of interest
Simplest QM calcs.

Eikonal CDCC

$$\psi = \sum_j S_j(z,b) \exp(ik_j z) \phi_k(\xi)$$

$$S_0 = \frac{1}{i\hbar} \int_{-\infty}^{z} V(r') dz'$$  (ground state)

$$V = V_C + V_N$$

$$i\hbar \frac{\partial S_j(z,b)}{\partial z} = \sum_m \langle j | V | m \rangle S_m(z,b) \exp[-(k_m - k_j)z]$$  (excited states)

Note similarity with semiclassical t.d. equation with $z = vt$

$$\frac{da_k}{dt} = -\frac{i}{\hbar} \sum_j a_j(t) V_{kj}(t) e^{i(E_k - E_j) t / \hbar}$$

Corrections due to energy conservation ($v \neq$ constant) straightforward

From $S_j$ calculate $\psi$ and observables of interest
The Continuum-Discretized Coupled Channels method (CDCC)

\[ \psi(\vec{r}, \vec{R}) = \phi_0(k_0, \vec{r}) \chi_0(K_0, \vec{R}) + \int_0^\infty \phi(k, \vec{r}) \chi(K, \vec{R}) dk \]

Next: theory movie
(only formulas change)
\[
\psi(\vec{r}, \vec{R}) = \phi_0(k_0, \vec{r}) \chi_0(K_0, \vec{R}) + \int_0^\infty \phi(k, \vec{r}) \chi(K, \vec{R}) dk
\]
\[ \phi(k) = \phi_0(k_0) + \int_0^\infty \phi(k, \vec{r}) \chi(K, \vec{R}) \, dk \]
\[ \psi(\vec{r}, \vec{R}) = \phi_0(k_0, \vec{r}) \chi_0(K_0, \vec{R}) + \sum_{i=1}^{i_{\max}} \int_{k_{i-1}}^{k_i} \phi(k, \vec{r}) \chi(K, \vec{R}) \, dk \]

**Breakup**

**Discretization**

**Truncation and Discretization**
\[
\psi(\vec{r}, \vec{R}) \equiv \phi_0(k_0, \vec{r}) \chi_0(K_0, \vec{R}) + \sum_{i=1}^{i_{\text{max}}} \chi(K_i, \vec{R}) \int_{k_{i-1}}^{k_i} \phi(k, \vec{r}) \, dk
\]

Truncation and Discretization
$\psi (\vec{r}, \vec{R}) = \phi_0 (k_0, \vec{r}) \chi_0 (K_0, \vec{R}) + \sum_{i=1}^{i_{\text{max}}} \chi(K_i, \vec{R}) \int_{k_{i-1}}^{k_i} \phi(k, \vec{r}) dk$

$\psi^{\text{CDCC}} (\vec{r}, \vec{R}) = \sum_{i=0}^{i_{\text{max}}} \hat{\phi}_i (\vec{r}) \hat{\chi}_i (K_i, \vec{R})$
\[
\psi(\vec{r}, \vec{R}) \equiv \phi_0(k_0, \vec{r}) \chi_0(K_0, \vec{R}) + \sum_{i=1}^{i_{\text{max}}} \chi(K_i, \vec{R}) \int_{k_{i-1}}^{k_i} \phi(k, \vec{r}) \, dk
\]

\[
\psi^{\text{CDCC}}(\vec{r}, \vec{R}) = \sum_{i=0}^{i_{\text{max}}} \hat{\phi}_i(\vec{r}) \hat{\chi}_i(K_i, \vec{R})
\]
\[ T_R + U_{xa}(\vec{r}, \vec{R}) + U_{ca}(\vec{r}, \vec{R}) + h_a(\vec{r}) \] \[ \psi^{CDCC}(\vec{r}, \vec{R}) = 0, \]

\[ \psi^{CDCC}(\vec{r}, \vec{R}) = \sum_{i=0}^{i_{\text{max}}} \phi_i(\vec{r}) \hat{\chi}_i(K_i, \vec{R}) \]

\[ \hat{\chi}_i \approx U_i^{(-)} \delta_{i0} - \sqrt{K_0 / K_i} S_{i0} U_i^{(+)} \]
More about CDCC

Review papers

✓ M. Kamimura, Yahiro, Iseri, Sakuragi, Kameyama and Kawai, PTP Suppl. 89, 1 (1986)

Theoretical foundation based on the distorted-wave Faddeev formalism

✓ N. Austern, Yahiro and Kawai, PRL 63, 2649 (1989)
✓ N. Austern, Kawai and Yahiro, PRC 53, 394 (1996)

Nuclear BU

✓ $^{58}\text{Ni(}d,d\text{)}$ at 56, 80, 200, 400, 700 MeV; $^{208}\text{Pb(}d,d\text{)}$ at 56 MeV
✓ $(d,pn)$ on $^{12}\text{C, }^{51}\text{V, }^{118}\text{Sn}$ at 56 MeV; $(^3\text{He,dp})$ on $^{12}\text{C, }^{51}\text{V, }^{90}\text{Zr}$ at 90 MeV
✓ $(^3\text{He,dp})$ on $^{12}\text{C, }^{28}\text{Si, }^{58}\text{Ni}$ at 52 MeV
✓ Elastic, inelastic, and BU processes of $^6\text{Li}$ and $^7\text{Li}$ on various stable nuclei

Coulomb (and nuclear) BU

✓ $^{208}\text{Pb(}^6\text{Li,}\alpha d\text{)}$ at 156 MeV: Y. Hirabayashi and Y. Sakuragi, PRL 69, 1892 (1992)
✓ $^{208}\text{Pb(}^8\text{B,}^7\text{Be)}$ at 44 and 83 A MeV, and $^{58}\text{Ni(}^8\text{B,}^7\text{Be)}$ at 26 MeV
  B. Davids et al., PRC 63, 065806 (2001), J.A. Tostevin et al., PRC 63, 024617 (2001)
The Eikonal-CDCC method (E-CDCC)

Eikonal approximation for scattering wave functions $\psi^{E-CDCC}$

$$\psi^{E-CDCC} = \sum_i \phi_i(\vec{r}) \hat{\xi}_i(b,z) \exp(i \vec{K}_i \cdot \vec{R})$$

$$K_i = \sqrt{2 \mu_R (E - \varepsilon_i) / \hbar},$$

Energy conservation

- Boundary condition for $\hat{\xi}_i$

$$\hat{\xi}_i(b,z) \xrightarrow{z \rightarrow -\infty} \delta_{i,0}$$

Eikonal scattering amplitude can be transformed into QM form

$$f_{i,0}^{E} = \sum_L f_L^{E} = \sum_L \frac{2\pi}{iK_i} \sqrt{\frac{2L+1}{4\pi}} i^m Y_{lm}(\Omega) [S_{i,0}^{b(L;i)} - \delta_{i,0}]$$

Hybrid scattering amplitude is given by

$$f_{i,0}^{H} = \sum_{L=0}^{L_C} f_L^{Q} + \sum_{L=L_C+1}^{L_{\text{max}}} f_L^{E}$$

Relativistic E-CDCC

Form factor of non-rel. E-CDCC

\[ F^{(b)}_{c^\prime c} (Z) = \left\langle \Phi_{c^\prime} \left| U_{1A} + U_{2A} \right| \Phi_c \right\rangle_r e^{-i(m-m')\phi} = \sum_\lambda F^{(b);\lambda}_{c^\prime c} (Z) \]

Rel. corr. to the form factor

\[ F^{(b);\lambda}_{c^\prime c} (Z) \rightarrow f_{\lambda,m'-m} \gamma F^{(b)\lambda}_{c^\prime c} (\gamma Z) \]

\[ f^{\text{Coul}}_{\lambda,m'-m} = \begin{cases} 1/\gamma & (\lambda=1, m'-m = 0) \\ \gamma & (\lambda=2, m'-m = \pm 1) \\ 1 & \text{(otherwise)} \end{cases} \]

\[ f^{\text{nucl}}_{\lambda,m'-m} = 1 \]

Assumptions

✓ Point charges for 1, 2 and A
✓ Neglecting far-field \((r_i > R)\) contribution
✓ Correction to nuclear form factor


Feshbach and Zabek, Ann. of Phys. 107 (1977) 110
Examples

Reaction
\(^{208}\text{Pb}(^{8}\text{B},^{7}\text{Be}+\text{p})\) at 250 A MeV and 100 A MeV
\(^{208}\text{Pb}(^{11}\text{Be},^{10}\text{Be}+\text{n})\) at 250 A MeV and 100 A MeV

Projectile wave function and distorting potential

Models space

\begin{align*}
\text{\(^{8}\text{B}\)}
\begin{align*}
l_{\text{max}} &= 3, \\
N_s &= 20, N_{p-d} = 10, \\
N_f &= 5, \\
\varepsilon_{\text{max}} &= 10 \text{ MeV}, \\
r_{\text{max}} &= 200 \text{ fm}, \\
R_{\text{max}} &= 500 \text{ fm}, \\
N_{\text{ch}} &= 138
\end{align*}
\end{align*}

\begin{align*}
\text{\(^{11}\text{Be}\)}
\begin{align*}
l_{\text{max}} &= 3, \\
N_{s,p} &= 20, N_d = 10, \\
N_f &= 5, \\
\varepsilon_{\text{max}} &= 10 \text{ MeV}, \\
r_{\text{max}} &= 200 \text{ fm}, \\
R_{\text{max}} &= 450 \text{ fm}, \\
N_{\text{ch}} &= 166
\end{align*}
\end{align*}
Quantum-mechanical (partial wave expansion x Eikonal) (both with relativistic potentials)

Ogata, CB, PTP 121 (2009), 1399
Ogata, CB, to be published
$^8\text{B} + ^{208}\text{Pb}$ at 250 A MeV

$|\sigma_{\text{CDCC}} - \sigma_{\text{1st order}}|$
\( ^8\text{B} + ^{208}\text{Pb} \) at 250 MeV/nucleon

**CDCC relativistic**

**CDCC non-relativistic**
8B breakup by $^{208}$Pb at 250 A MeV

Relativistic vs non-relativistic effects in continuum-continuum transitions

Ogata, CB, PTP 121 (2009), 1399
Ogata, CB, to be published
Summary

• **Challenges in nuclear astrophysics**
  • Screening - surprising and experimentally confusing
  • poor statistics due to small cross sections
  • Some needed reactions will never be measured directly
  • …

• **Indirect methods with radioactive nuclear beams necessary**
  • Coulomb excitation
  • Charge-exchange
  • Knockout & transfer reactions
  • …

• **New theoretical methods needed**
  • reactions in general
  • bridge internal (structure) to external (reactions) pieces of wfs.
  • understand Nucl. Phys. from fundamental theory (QCD)
  • …