The Brave ν World

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Colloquium – University of Tennessee

October 5, 2015
O, wonder!
How many goodly creatures are there here!
How beauteous mankind is! O brave new world,
That has such people in’t!

W. Shakespeare, “The Tempest,” Act V, Scene 1
Neutrinos are among a handful of known fundamental, point-like particles.

http://www.particlezoo.net

October 5, 2015

Brave \( \nu \) World
Neutrinos are Very, Very Abundant.

\[ p + p \rightarrow ^2\text{H} + e^+ + \nu_e \quad 99.96 \quad < 0.423 \]
\[ p + e^- + p \rightarrow ^2\text{H} + \nu_e \quad 0.044 \quad 1.445 \]
\[ ^2\text{H} + p \rightarrow ^3\text{He} + \gamma \quad 100 \quad - \]
\[ ^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + p + p \quad 85 \quad - \]
\[ ^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma \quad 15 \quad - \]
\[ ^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu_e \quad 15 \quad 0.863(90\%) \quad 0.386(10\%) \]
\[ ^7\text{Li} + p \rightarrow ^4\text{He} + ^4\text{He} \quad - \]
\[ ^7\text{Be} + p \rightarrow ^8\text{B} + \gamma \quad 0.02 \quad - \]
\[ ^8\text{B} \rightarrow ^8\text{Be}^* + e^+ + \nu_e \quad < 15 \]
\[ ^8\text{Be} \rightarrow ^4\text{He} + ^4\text{He} \quad - \]
\[ ^3\text{He} + p \rightarrow ^4\text{He} + e^+ + \nu_e \quad 0.00003 \quad < 18.8 \]

*Note: Adapted from Ref. 12. Please refer to Ref. 12 for a more detailed explanation.*

around 100 billion go through your thumb every second!
Also Closer to Home...
Supernova: 100 times more energy released in the form of neutrinos!
Neutrinos are Relics of the Big Bang:

Neutrinos are Everywhere
However, Neutrinos Are Really Hard To Detect:

Neutrinos have no charge (unlike, say, the electrons) and don’t interact via the strong nuclear forces (unlike, say, a neutron).

They interact only via the WEAK force – which, as it turns out, is really weak.

You need a wall of lead as thick as the solar system in order to stop a neutrino produced in the Sun!

How did we get around this? With lots and lots of neutrinos, and really big detectors!
16 years ago, this is how we pictured neutrinos:

- come in three flavors (see figure);
- interact only via weak interactions ($W^\pm, Z^0$);
- have ZERO mass – helicity good quantum number;
- $\nu_L$ field describes 2 degrees of freedom:
  - left-handed state $\nu$,
  - right-handed state $\bar{\nu}$ (CPT conjugate);
- neutrinos carry lepton number (conserved):
  - $L(\nu) = L(\ell) + 1$,
  - $L(\bar{\nu}) = L(\bar{\ell}) = -1$. 

- short Journey
Something Funny Happened on the Way to the 21st Century

\( \nu \) Flavor Oscillations

Neutrino oscillation experiments have revealed that neutrinos change flavor after propagating a finite distance. The rate of change depends on the neutrino energy \( E_\nu \) and the baseline \( L \). The evidence is overwhelming.

- \( \nu_\mu \rightarrow \nu_\tau \) and \( \bar{\nu}_\mu \rightarrow \bar{\nu}_\tau \) — atmospheric and accelerator experiments;
- \( \nu_e \rightarrow \nu_\mu,\tau \) — solar experiments;
- \( \bar{\nu}_e \rightarrow \bar{\nu}_{\text{other}} \) — reactor experiments;
- \( \nu_\mu \rightarrow \nu_{\text{other}} \) and \( \bar{\nu}_\mu \rightarrow \bar{\nu}_{\text{other}} \) — atmospheric and accelerator expts;
- \( \nu_\mu \rightarrow \nu_e \) — accelerator experiments.

The simplest and only satisfactory explanation of all this data is that neutrinos have distinct masses, and mix.
Mass-Induced Neutrino Flavor Oscillations

Neutrino Flavor change can arise out of several different mechanisms. The simplest one is to appreciate that, once neutrinos have mass, leptons can mix. If neutrinos have mass, there are two different ways to define the different neutrino states.

(1) Neutrinos with a well defined mass:

\[ \nu_1, \nu_2, \nu_3, \ldots \quad \text{with masses} \quad m_1, m_2, m_3, \ldots \]

(2) Neutrinos with a well defined flavor:

\[ \nu_e, \nu_\mu, \nu_\tau \]

These are related by a unitary transformation:

\[ \nu_\alpha = U_{\alpha i} \nu_i \quad \alpha = e, \mu, \tau, \quad i = 1, 2, 3 \]

\( U \) is a unitary mixing matrix.
The Propagation of Massive Neutrinos

Neutrino mass eigenstates are eigenstates of the free-particle Hamiltonian:

$$|\nu_i\rangle = e^{-i(E_i t - \vec{p}_i \cdot \vec{x})} |\nu_i\rangle, \quad E_i^2 - |\vec{p}_i|^2 = m_i^2$$

The neutrino flavor eigenstates are linear combinations of $\nu_i$'s, say:

$$|\nu_e\rangle = \cos \theta |\nu_1\rangle + \sin \theta |\nu_2\rangle.$$  

$$|\nu_\mu\rangle = -\sin \theta |\nu_1\rangle + \cos \theta |\nu_2\rangle.$$  

If this is the case, a state produced as a $\nu_e$ evolves in vacuum into

$$|\nu(t, \vec{x})\rangle = \cos \theta e^{-ip_1 x} |\nu_1\rangle + \sin \theta e^{-ip_2 x} |\nu_2\rangle.$$  

It is trivial to compute $P_{e\mu}(L) \equiv |\langle \nu_\mu | \nu(t, z = L) \rangle|^2$. It is just like a two-level system from basic undergraduate quantum mechanics! In the ultrarelativistic limit (always a good bet), $t \simeq L$, $E_i - p_{z,i} \simeq (m_i^2)/2E_i$, and

$$P_{e\mu}(L) = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E_\nu} \right)$$
oscillation parameters: \[
\left\{
\begin{aligned}
\pi \frac{L}{L_{osc}} &\equiv \frac{\Delta m^2 L}{4E} = 1.267 \left( \frac{L}{\text{km}} \right) \left( \frac{\Delta m^2}{\text{eV}^2} \right) \left( \frac{\text{GeV}}{E} \right) \\
\text{amplitude } \sin^2 2\theta
\end{aligned}
\right.
\]
A Realistic, Reasonable, and Simple Paradigm:

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

Definition of neutrino mass eigenstates (who are \(\nu_1, \nu_2, \nu_3\)?):

- \(m_1^2 < m_2^2\) \quad \Delta m_{13}^2 < 0 – Inverted Mass Hierarchy
- \(m_2^2 - m_1^2 \ll |m_3^2 - m_{1,2}^2|\) \quad \Delta m_{13}^2 > 0 – Normal Mass Hierarchy

\[
\tan^2 \theta_{12} = \frac{|U_{e2}|^2}{|U_{e1}|^2}; \quad \tan^2 \theta_{23} = \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2}; \quad U_{e3} = \sin \theta_{13} e^{-i\delta}
\]

[For a detailed discussion see e.g. AdG, Jenkins, PRD78, 053003 (2008)]
Three Flavor Mixing Hypothesis Fits All* Data Really Well.

<table>
<thead>
<tr>
<th></th>
<th>Normal Ordering ($\Delta \chi^2 = 0.97$)</th>
<th>Inverted Ordering (best fit)</th>
<th>Any Ordering</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>bfp ±1σ</td>
<td>3σ range</td>
<td>bfp ±1σ</td>
</tr>
<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>$0.304^{+0.013}_{-0.012}$</td>
<td>$0.270 \rightarrow 0.344$</td>
<td>$0.304^{+0.013}_{-0.012}$</td>
</tr>
<tr>
<td>$\theta_{12}/^\circ$</td>
<td>$33.48^{+0.78}_{-0.75}$</td>
<td>$31.29 \rightarrow 35.91$</td>
<td>$33.48^{+0.78}_{-0.75}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>$0.452^{+0.052}_{-0.028}$</td>
<td>$0.382 \rightarrow 0.643$</td>
<td>$0.579^{+0.025}_{-0.037}$</td>
</tr>
<tr>
<td>$\theta_{23}/^\circ$</td>
<td>$42.3^{+3.0}_{-1.6}$</td>
<td>$38.2 \rightarrow 53.3$</td>
<td>$49.5^{+1.5}_{-2.2}$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{13}$</td>
<td>$0.0218^{+0.0011}_{-0.0010}$</td>
<td>$0.0186 \rightarrow 0.0250$</td>
<td>$0.0219^{+0.0011}_{-0.0010}$</td>
</tr>
<tr>
<td>$\theta_{13}/^\circ$</td>
<td>$8.50^{+0.20}_{-0.21}$</td>
<td>$7.85 \rightarrow 9.10$</td>
<td>$8.51^{+0.20}_{-0.21}$</td>
</tr>
<tr>
<td>$\delta_{CP}/^\circ$</td>
<td>$306^{+39}_{-70}$</td>
<td>$0 \rightarrow 360$</td>
<td>$254^{+63}_{-62}$</td>
</tr>
<tr>
<td>$\frac{\Delta m^2_{21}}{10^{-5} \text{ eV}^2}$</td>
<td>$7.50^{+0.19}_{-0.17}$</td>
<td>$7.02 \rightarrow 8.09$</td>
<td>$7.50^{+0.19}_{-0.17}$</td>
</tr>
<tr>
<td>$\frac{\Delta m^2_{3\ell}}{10^{-3} \text{ eV}^2}$</td>
<td>$+2.457^{+0.047}_{-0.047}$</td>
<td>$+2.317 \rightarrow +2.607$</td>
<td>$-2.449^{+0.048}_{-0.047}$</td>
</tr>
</tbody>
</table>

*Modulo a handful of 2σ to 3σ anomalies.

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“Atmospheric Oscillations” in the Electron Sector: Daya Bay, RENO, Double Chooz

\[ \text{phase} = 0.64 \left( \frac{\Delta m^2}{2.5 \times 10^{-3} \text{ eV}^2} \right) \left( \frac{5 \text{ MeV}}{E} \right) \left( \frac{L}{1 \text{ km}} \right) \]

Triumph of the 3 flavor paradigm!

\[ P_{ee} = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \]

[Daya Bay Coll., 1203.1669]
What We Know We Don’t Know: Missing Oscillation Parameters

- What is the $\nu_e$ component of $\nu_3$? ($\theta_{13} \neq 0!$)
- Is CP-invariance violated in neutrino oscillations? ($\delta \neq 0, \pi?$)
- Is $\nu_3$ mostly $\nu_\mu$ or $\nu_\tau$? ($\theta_{23} > \pi/4$, $\theta_{23} < \pi/4$, or $\theta_{23} = \pi/4?$)
- What is the neutrino mass hierarchy? ($\Delta m_{13}^2 > 0?$)

$\Rightarrow$ All of the above can “only” be addressed with new neutrino oscillation experiments

Ultimate Goal: Not Measure Parameters but Test the Formalism (Over-Constrain Parameter Space)

October 5, 2015

Brave $\nu$ World
What we ultimately want to achieve:

We need to do this in the lepton sector!
\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

What we have really measured (very roughly):

- Two mass-squared differences, at several percent level – many probes;
- \(|U_{e2}|^2\) – solar data;
- \(|U_{\mu2}|^2 + |U_{\tau2}|^2\) – solar data;
- \(|U_{e2}|^2|U_{e1}|^2\) – KamLAND;
- \(|U_{\mu3}|^2(1 - |U_{\mu3}|^2)\) – atmospheric data, K2K, MINOS;
- \(|U_{e3}|^2(1 - |U_{e3}|^2)\) – Double Chooz, Daya Bay, RENO;
- \(|U_{e3}|^2|U_{\mu3}|^2\) (upper bound \(\rightarrow\) evidence) – MINOS, T2K.

We still have a ways to go!
Where We Are (?) [This is Not a Proper Comparison Yet!]

NuFIT 2.0 (2014)

But it is a start...

CP-invariance Violation in Neutrino Oscillations

The most promising approach to studying CP-violation in the leptonic sector seems to be to compare $P(\nu_\mu \to \nu_e)$ versus $P(\bar{\nu}_\mu \to \bar{\nu}_e)$. 

The amplitude for $\nu_\mu \to \nu_e$ transitions can be written as

$$A_{\mu e} = U_{e2}^* U_{\mu 2} \left( e^{i\Delta_{12}} - 1 \right) + U_{e3}^* U_{\mu 3} \left( e^{i\Delta_{13}} - 1 \right)$$

where $\Delta_{1i} = \frac{\Delta m_{1i}^2 L}{2 E_i}, \; i = 2, 3$.

The amplitude for the CP-conjugate process can be written as

$$\bar{A}_{\mu e} = U_{e2} U_{\mu 2}^* \left( e^{i\Delta_{12}} - 1 \right) + U_{e3} U_{\mu 3}^* \left( e^{i\Delta_{13}} - 1 \right).$$

[I assume the unitarity of $U$, $U_{e1} U_{\mu 1}^* = -U_{e2} U_{\mu 2}^* - U_{e3} U_{\mu 3}^*$]
In general, $|A|^2 \neq |\bar{A}|^2$ (CP-invariance violated) as long as:

- Nontrivial “Weak” Phases: $\arg(U_{ei}^* U_{\mu i}) \to \delta \neq 0, \pi$;
- Nontrivial “Strong” Phases: $\Delta_{12}, \Delta_{13} \to L \neq 0$;
- Because of Unitarity, we need all $|U_{\alpha i}| \neq 0 \to$ three generations.

All of these can be satisfied, with a little luck: we needed $|U_{e3}| \neq 0$. ✓
Golden Opportunity to Understand Matter versus Antimatter?

The SM with massive Majorana neutrinos accommodates five irreducible CP-invariance violating phases.

- One is the phase in the CKM phase. We have measured it, it is large, and we don’t understand its value. At all.
- One is $\theta_{QCD}$ term ($\theta_G\tilde{G}$). We don’t know its value but it is only constrained to be very small. We don’t know why (there are some good ideas, however).
- Three are in the neutrino sector. One can be measured via neutrino oscillations. 50% increase on the amount of information.

We don’t know much about CP-invariance violation. Is it really fair to presume that CP-invariance is generically violated in the neutrino sector solely based on the fact that it is violated in the quark sector? Why? Cautionary tale: “Mixing angles are small”
Long-Baseline Experiments, Present and Future (Not Exhaustive!)

• [NOW] T2K (Japan), NOνA (USA) – $\nu_\mu \rightarrow \nu_e$ appearance, $\nu_\mu$ disappearance – precision measurements of “atmospheric parameters” $(\Delta m_{13}^2, \sin^2 \theta_{23})$. Pursue mass hierarchy via matter effects. Nontrivial tests of paradigm. First step towards CP-invariance violation.

• [$\sim 2020$] JUNO (China) – $\bar{\nu}_e$ disappearance – precision measurements of “solar parameters” $(\Delta m_{12}^2, \sin^2 \theta_{12})$. Pursue the mass hierarchy via precision oscillations.

• [$\sim 2020$] PINGU (South Pole) – atmospheric neutrinos – pursue mass hierarchy via matter effects.

• [$\sim 2025$] HyperK (Japan), DUNE (USA) – Second (real opportunity for discovery!) step towards CP-invariance violation. More nontrivial tests of the paradigm. Ultimate “super-beam” experiments.

• [$> 2030$?] Neutrino Factories (?) – Ultimate neutrino oscillation experiment. Test paradigm, precision measurements, solidify CP-violation discovery or improve sensitivity significantly.
What We Know We Don’t Know: How Light is the Lightest Neutrino?

So far, we’ve only been able to measure neutrino mass-squared differences.

The lightest neutrino mass is only poorly constrained: $m^2_{\text{lightest}} < 1 \text{ eV}^2$

qualitatively different scenarios allowed:

- $m^2_{\text{lightest}} \equiv 0$;
- $m^2_{\text{lightest}} \ll \Delta m^2_{12,13}$;
- $m^2_{\text{lightest}} \gg \Delta m^2_{12,13}$.

Need information outside of neutrino oscillations:

$\rightarrow$ cosmology, $\beta$-decay, $0\nu\beta\beta$
Big Bang Neutrinos are Warm Dark Matter

- Constrained by the Large Scale Structure of the Universe.

Constraints depend on
- Data set analysed;
- “Bias” on other parameters;
- ...

Bounds can be evaded with non-standard cosmology. Will we learn about neutrinos from cosmology or about cosmology from neutrinos?

Fig. 10.— This figure illustrates the robustness of the neutrino mass detection to other parameter extensions. The marginalized one-dimensional posteriors for $\sum m_\nu$ are shown for two-parameter extensions to $\Lambda$CDM for the combined CMB+BAO+$H_0+$SPT$_{\text{CL}}$ data sets (for $w$, SNe are used instead of $H_0$). Allowing significant curvature or running can significantly reduce the preference for nonzero neutrino masses (to 1.7 and 2.4$\sigma$ respectively). Other extensions increase the preference for positive neutrino masses.

Figure 7. Current constraints and forecast sensitivity of cosmology to the sum of neutrino masses. In the case of an “inverted hierarchy,” with an example case marked as a diamond in the upper curve, future combined cosmological constraints would have a very high-significance detection, with 1-σ error shown as a blue band. In the case of a normal neutrino mass hierarchy with an example case marked as diamond on the lower curve, future cosmology would still detect the lowest $\sum m_\nu$ at greater than 3-σ.

What We Know We Don’t Know: Are Neutrinos Majorana Fermions?

A massive charged fermion \( s=1/2 \) is described by 4 degrees of freedom:

\[
\begin{align*}
(e^-_L &\leftrightarrow \text{CPT} \to e^+_R) \\
\uparrow & \text{“Lorentz”}
\end{align*}
\]

\[
\begin{align*}
(e^-_R &\leftrightarrow \text{CPT} \to e^+_L) \\
\uparrow & \text{“Lorentz”}
\end{align*}
\]

A massive neutral fermion \( s=1/2 \) is described by 4 or 2 degrees of freedom:

\[
\begin{align*}
(\nu_L &\leftrightarrow \text{CPT} \to \bar{\nu}_R) \\
\uparrow & \text{“Lorentz”} \quad \text{‘DIRAC’}
\end{align*}
\]

\[
\begin{align*}
(\nu_R &\leftrightarrow \text{CPT} \to \bar{\nu}_L) \\
\uparrow & \text{“Lorentz”} \quad \text{‘MAJORANA’}
\end{align*}
\]

How many degrees of freedom are required to describe massive neutrinos?
Why Don’t We Know the Answer?

If neutrino masses were indeed zero, this is a nonquestion: there is no distinction between a massless Dirac and Majorana fermion.

Processes that are proportional to the Majorana nature of the neutrino vanish in the limit $m_\nu \to 0$. Since neutrinos masses are very small, the probability for these to happen is very, very small: $A \propto m_\nu/E$.

The “smoking gun” signature is the observation of LEPTON NUMBER violation. This is easy to understand: Majorana neutrinos are their own antiparticles and, therefore, cannot carry any quantum numbers — including lepton number.
Weak Interactions are Purely Left-Handed (Chirality):

For example, in the scattering process $e^- + X \rightarrow \nu_e + X$, the electron neutrino is, in a reference frame where $m \ll E$,

$$|\nu_e\rangle \sim |L\rangle + \left(\frac{m}{E}\right) |R\rangle.$$ 

If the neutrino is a Majorana fermion, $|R\rangle$ behaves mostly like a “$\bar{\nu}_e$,” (and $|L\rangle$ mostly like a “$\nu_e$,”) such that the following process could happen:

$$e^- + X \rightarrow \nu_e + X, \quad \text{followed by} \quad \nu_e + X \rightarrow e^+ + X, \quad P \simeq \left(\frac{m}{E}\right)^2$$

Lepton number can be violated by 2 units with small probability. Typical numbers: $P \simeq (0.1 \text{ eV}/100 \text{ MeV})^2 = 10^{-18}$. VERY Challenging!
Search for the Violation of Lepton Number (or $B - L$)

**Best Bet:** search for

Neutrinoless Double-Beta

Decay: $Z \rightarrow (Z + 2)e^- e^-$

<table>
<thead>
<tr>
<th>$m_{ee}$</th>
<th>in eV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-4}$</td>
<td>1</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>1</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>1</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>1</td>
</tr>
</tbody>
</table>

90% CL (1 dof)

Helicity Suppressed Amplitude $\propto \frac{m_{ee}}{E}$

Observable: $m_{ee} \equiv \sum_i U_{ei}^2 m_i$

⇐ no longer lamp-post physics!
What We Are Trying To Understand:

⇐ NEUTRINOS HAVE TINY MASSES

⇓ LEPTON MIXING IS “WEIRD” ⇓

\[
V_{MNS} \sim \begin{pmatrix}
0.8 & 0.5 & 0.2 \\
0.4 & 0.6 & 0.7 \\
0.4 & 0.6 & 0.7
\end{pmatrix}
\]

\[
V_{CKM} \sim \begin{pmatrix}
1 & 0.2 & 0.001 \\
0.2 & 1 & 0.01 \\
0.001 & 0.01 & 1
\end{pmatrix}
\]

What Does It Mean?
Neutrino Masses: Only* “Palpable” Evidence of Physics Beyond the Standard Model

The SM we all learned in school predicts that neutrinos are strictly massless. Hence, massive neutrinos imply that the SM is incomplete and needs to be replaced/modified.

Furthermore, the SM has to be replaced by something qualitatively different.

* There is only a handful of questions our model for fundamental physics cannot explain (these are personal. Feel free to complain).

- What is the physics behind electroweak symmetry breaking? (Higgs ✓).
- What is the dark matter? (not in SM).
- Why is there more matter than antimatter in the Universe? (not in SM).
- Why does the Universe appear to be accelerating? Why does it appear that the Universe underwent rapid acceleration in the past? (not in SM).
What is the New Standard Model? \( [\nu SM] \)

The short answer is – WE DON’T KNOW. Not enough available info!

Equivalently, there are several completely different ways of addressing neutrino masses. The key issue is to understand what else the \( \nu SM \) candidates can do. [are they falsifiable?, are they “simple”?, do they address other outstanding problems in physics?, etc]

We need more experimental input.
This is much more than a pretty picture.

It is a very powerful, predictive model.

http://www.particlezoo.net
• Result of over 60 years of particle physics theoretical and experimental research.

• Theoretical formalism based on the marriage of Quantum Mechanics and Special Relativity – Relativistic Quantum Field Theory.

• Very Powerful – once we specify the model ingredients: field content (matter particles) and the internal symmetries (interactions), the dynamics of the system is uniquely specified by a finite set of free parameters.
Given the known ingredients of the model and the known rules, we can predict that the neutrino masses are exactly zero.

Neutrino masses require new ingredients or new rules. We are still trying to figure out what these new ingredients are.

On the plus side, we probably know what they could be...
Neutrino Masses, Higgs Mechanism, and New Mass Scale of Nature

The LHC has revealed that the minimum SM prescription for electroweak symmetry breaking — the one Higgs double model — is at least approximately correct. What does that have to do with neutrinos?

The tiny neutrino masses point to three different possibilities.

1. Neutrinos talk to the Higgs boson very, very weakly;

2. Neutrinos talk to a different Higgs boson – there is a new source of electroweak symmetry breaking!;

3. Neutrino masses are small because there is another source of mass out there — a new energy scale indirectly responsible for the tiny neutrino masses, a la the seesaw mechanism.

We are going to need a lot of experimental information from all ares of particle physics in order to figure out what is really going on!
One Candidate $\nu$SM

SM as an effective field theory – non-renormalizable operators

$$\mathcal{L}_{\nu SM} \supset -y_{ij} \frac{L^i H L^j H}{2\Lambda} + \mathcal{O}\left(\frac{1}{\Lambda^2}\right) + H.c.$$  

There is only one dimension five operator [Weinberg, 1979]. If $\Lambda \gg 1\text{ TeV}$, it leads to only one observable consequence...

after EWSB: $$\mathcal{L}_{\nu SM} \supset \frac{m_{ij}}{2} \nu^i \nu^j; \quad m_{ij} = y_{ij} \frac{v^2}{\Lambda}.$$  

- Neutrino masses are small: $\Lambda \gg v \rightarrow m_\nu \ll m_f$ ($f = e, \mu, u, d, \text{ etc}$)
- Neutrinos are Majorana fermions – Lepton number is violated!
- $\nu$SM effective theory – not valid for energies above at most $\Lambda/y$.
- Define $y_{\text{max}} \equiv 1 \Rightarrow$ data require $\Lambda \sim 10^{14}\text{ GeV}$.  

What else is this “good for”? Depends on the ultraviolet completion!
The Seesaw Lagrangian

A simple\textsuperscript{a}, renormalizable Lagrangian that allows for neutrino masses is

\[ \mathcal{L}_\nu = \mathcal{L}_{\text{old}} - \lambda_{\alpha i} L^\alpha H N^i - \sum_{i=1}^{3} \frac{M_i}{2} N^i N^i + H.c., \]

where \( N_i \) (\( i = 1, 2, 3 \), for concreteness) are SM gauge singlet fermions. \( \mathcal{L}_\nu \) is the most general, renormalizable Lagrangian consistent with the SM gauge group and particle content, plus the addition of the \( N_i \) fields.

After electroweak symmetry breaking, \( \mathcal{L}_\nu \) describes, besides all other SM degrees of freedom, six Majorana fermions: \textbf{six neutrinos}.

\textsuperscript{a}Only requires the introduction of three fermionic degrees of freedom, no new interactions or symmetries.
To be determined from data: $\lambda$ and $M$.

The data can be summarized as follows: there is evidence for three neutrinos, mostly “active” (linear combinations of $\nu_e$, $\nu_\mu$, and $\nu_\tau$). At least two of them are massive and, if there are other neutrinos, they have to be “sterile.”

This provides very little information concerning the magnitude of $M_i$ (assume $M_1 \sim M_2 \sim M_3$).

Theoretically, there is prejudice in favor of very large $M$: $M \gg v$. Popular examples include $M \sim M_{\text{GUT}}$ (GUT scale), or $M \sim 1$ TeV (EWSB scale).

Furthermore, $\lambda \sim 1$ translates into $M \sim 10^{14}$ GeV, while thermal leptogenesis requires the lightest $M_i$ to be around $10^{10}$ GeV.

**we can impose very, very few experimental constraints on $M$**
What We Know About $M$:

- $M = 0$: the six neutrinos “fuse” into three Dirac states. Neutrino mass matrix given by $\mu_{\alpha i} \equiv \lambda_{\alpha i} v$.

  The symmetry of $\mathcal{L}_\nu$ is enhanced: $U(1)_{B-L}$ is an exact global symmetry of the Lagrangian if all $M_i$ vanish. Small $M_i$ values are ’tHooft natural.

- $M \gg \mu$: the six neutrinos split up into three mostly active, light ones, and three, mostly sterile, heavy ones. The light neutrino mass matrix is given by $m_{\alpha \beta} = \sum_i \mu_{\alpha i} M_i^{-1} \mu_{\beta i}$

  $[m \propto 1/\Lambda \Rightarrow \Lambda = M/\mu^2]$. This the seesaw mechanism. Neutrinos are Majorana fermions. Lepton number is not a good symmetry of $\mathcal{L}_\nu$, even though $L$-violating effects are hard to come by.

- $M \sim \mu$: six states have similar masses. Active–sterile mixing is very large. This scenario is (generically) ruled out by active neutrino data (atmospheric, solar, KamLAND, K2K, etc).

- $M \ll \mu$: neutrinos are quasi-Dirac fermions. Active–sterile mixing is maximal, but new oscillation lengths are very long (cf. 1 A.U.).
Why are Neutrino Masses Small in the \( M \neq 0 \) Case?

If \( \mu \ll M \), below the mass scale \( M \),

\[
\mathcal{L}_5 = \frac{LHLH}{\Lambda}.
\]

Neutrino masses are small if \( \Lambda \gg \langle H \rangle \). Data require \( \Lambda \sim 10^{14} \) GeV.

In the case of the seesaw,

\[
\Lambda \sim \frac{M}{\lambda^2},
\]

so neutrino masses are small if either

- they are generated by physics at a very high energy scale \( M \gg v \) (high-energy seesaw); or
- they arise out of a very weak coupling between the SM and a new, hidden sector (low-energy seesaw); or
- cancellations among different contributions render neutrino masses accidentally small (“fine-tuning”).
High-Energy Seesaw: Brief Comments

- This is everyone’s favorite scenario.

- Upper bound for $M$ (e.g., Maltoni, Niczyporuk, Willenbrock, hep-ph/0006358):

  $$M < 7.6 \times 10^{15} \text{ GeV} \times \left( \frac{0.1 \text{ eV}}{m_{\nu}} \right).$$

- Hierarchy problem hint (e.g., Casas et al, hep-ph/0410298; Farina et al, 1303.7244; AdG et al, 1402.2658):

  $$M < 10^7 \text{ GeV}.$$

- Leptogenesis! “Vanilla” Leptogenesis requires, very roughly, smallest

  $$M > 10^9 \text{ GeV}.$$

- Stability of the Higgs potential (e.g., Elias-Miró et al, 1112.3022):

  $$M < 10^{13} \text{ GeV}.$$

- Physics “too” heavy! No observable consequence other than leptogenesis. Will we ever convince ourselves that this is correct? (Buckley et al, hep-ph/0606088)
Low-Energy Seesaw  [AdG PRD72,033005)]

The other end of the $M$ spectrum ($M < 100$ GeV). What do we get?

- Neutrino masses are small because the Yukawa couplings are very small $\lambda \in [10^{-6}, 10^{-11}]$;
- No obvious connection with other energy scales (EWSB, GUTs, etc);
- Right-handed neutrinos are propagating degrees of freedom. They look like sterile neutrinos $\Rightarrow$ sterile neutrinos associated with the fact that the active neutrinos have mass;
- sterile–active mixing can be predicted – hypothesis is falsifiable!
- Small values of $M$ are natural (in the ‘tHooft sense). In fact, theoretically, no value of $M$ should be discriminated against!
Constraining the Seesaw Lagrangian

\[ \sin^2 \theta_{13} \]

Experimentally Excluded

\[ m_\nu = \ldots \text{eV} \]

\[ M_N (\text{eV}) \]

\[ (\nu_\alpha) \text{ live here} \]

\[ (\nu_\alpha) \text{ live here} \]

\[ 10^{-1} \]

\[ 10^{-2} \]

\[ 10^{-5} \]

[AdG, Huang, Jenkins, arXiv:0906.1611]
Oscillations

Mass (eV)

$\nu_e$
$\nu_\mu$
$\nu_\tau$
$\nu_s^1$
$\nu_s^2$
$\nu_s^3$

Dark Matter(?)
Pulsar Kicks

Also effects in $0\nu\beta\beta$,
tritium beta decay,
supernova neutrino oscillations,
non-standard cosmology.
Understanding Fermion Mixing

One of the puzzling phenomena uncovered by the neutrino data is the fact that Neutrino Mixing is Strange. What does this mean?

It means that lepton mixing is very different from quark mixing:

\[
V_{MNS} \sim \begin{pmatrix}
0.8 & 0.5 & 0.2 \\
0.4 & 0.6 & 0.7 \\
0.4 & 0.6 & 0.7
\end{pmatrix}
\quad V_{CKM} \sim \begin{pmatrix}
1 & 0.2 & 0.001 \\
0.2 & 1 & 0.01 \\
0.001 & 0.01 & 1
\end{pmatrix}
\]

\[| (V_{MNS})_{e3} | < 0.2 \]

They certainly look VERY different, but which one would you label as “strange”?
André de Gouvêa

"Left-Over" Predictions: $\delta$, mass-hierarchy, $\cos 2\theta_{23}$

[Albright and Chen, hep-ph/0608137]
Neutrino Mixing Anarchy: Alive and Kicking!

P$_3$(KS) vs. $\sin^2\theta_{13}$

[AdG, Murayama, 1204.1249]
Anarchy vs. Order — more precision required!

Order: \( \sin^2 \theta_{13} = C \cos^2 2\theta_{23}, \ C \in [0.8, 1.2] \)
Piecing the Neutrino Mass Puzzle

Understanding the origin of neutrino masses and exploring the new physics in the lepton sector will require unique theoretical and experimental efforts . . .

- understanding the fate of lepton-number. Neutrinoless double beta decay!
- A comprehensive long baseline neutrino program. (On-going T2K and NOvA. DUNE and HyperK next steps towards the ultimate “superbeam” experiment.)
- The next-step is to develop a qualitatively better neutrino beam – e.g. muon storage rings (neutrino factories).
- Different baselines and detector technologies a must for both over-constraining the system and looking for new phenomena.
- Probes of neutrino properties, including neutrino scattering experiments.
- Precision measurements of charged-lepton properties ($g - 2$, edm) and searches for rare processes ($\mu \to e$-conversion the best bet at the moment).
- Collider experiments. The LHC and beyond may end up revealing the new physics behind small neutrino masses.
- Neutrino properties affect, in a significant way, the history of the universe (Cosmology). Will we learn about neutrinos from cosmology, or about cosmology from neutrinos?
In Conclusion

The venerable Standard Model sprung a leak in the end of the last century: neutrinos are not massless! (and we are still trying to patch it)

1. We know very little about the new physics uncovered by neutrino oscillations.
   - It could be renormalizable → “boring” (?) Dirac neutrinos.
   - It could be due to Physics at absurdly high energy scales $M \gg 1 \text{ TeV}$ → high energy seesaw. How can we convince ourselves that this is correct?
   - It could be due to very light new physics. Prediction: new light propagating degrees of freedom – sterile neutrinos
   - It could be due to new physics at the TeV scale → either weakly coupled, or via a more subtle lepton number breaking sector.

2. **neutrino masses are very small** – we don’t know why, but we think it means something important.

3. **neutrino mixing is “weird”** – we don’t know why, but we think it means something important.
4. we need a minimal $\nu$SM Lagrangian. In order to decide which one is “correct” we need to uncover the faith of baryon number minus lepton number ($0\nu\beta\beta$ is the best [only?] bet).

5. We need more experimental input These will come from a rich, diverse experimental program which relies heavily on the existence of underground facilities capable of hosting large detectors (double-beta decay, precision neutrino oscillations, supernova neutrinos, nucleon decay). Also “required”

- Powerful neutrino beam;
- Precision studies of charged-lepton lepton properties and processes;
- High energy collider experiments (the LHC will do for now);

6. There is plenty of room for surprises, as neutrinos are potentially very deep probes of all sorts of physical phenomena. Remember that neutrino oscillations are “quantum interference devices” – potentially very sensitive to whatever else may be out there (e.g., $\Lambda \simeq 10^{14}$ GeV).
Backup Slides ...
Not all is well(?): The Short Baseline Anomalies

Different data sets, sensitive to $L/E$ values small enough that the known oscillation frequencies do not have “time” to operate, point to unexpected neutrino behavior. These include

- $\nu_\mu \rightarrow \nu_e$ appearance — LSND, MiniBooNE;
- $\nu_e \rightarrow \nu_{\text{other}}$ disappearance — radioactive sources;
- $\bar{\nu}_e \rightarrow \bar{\nu}_{\text{other}}$ disappearance — reactor experiments.

None are entirely convincing, either individually or combined. However, there may be something very very interesting going on here...
MiniBooNE & LSND

\[ \text{Courtesy of G. Mills} \]

October 5, 2015

[Brave $\nu$ World]
Bugey 40 m

Distance to Reactor (m)

October 5, 2015

Brave $\nu$ World
What is Going on Here?

- Are these “anomalies” related?
- Is this neutrino oscillations, other new physics, or something else?
- Are these related to the origin of neutrino masses and lepton mixing?
- How do clear this up definitively?

Need new clever experiments, of the short-baseline type!

Observable wish list:

- $\nu_\mu$ disappearance (and antineutrino);
- $\nu_e$ disappearance (and antineutrino);
- $\nu_\mu \leftrightarrow \nu_e$ appearance;
- $\nu_{\mu,e} \rightarrow \nu_\tau$ appearance.
High-energy seesaw has no other observable consequences, except, perhaps, . . .

**Baryogenesis via Leptogenesis**

One of the most basic questions we are allowed to ask (with any real hope of getting an answer) is whether the observed baryon asymmetry of the Universe can be obtained from a baryon–antibaryon symmetric initial condition plus well understood dynamics. [Baryogenesis]

This isn’t just for aesthetic reasons. If the early Universe undergoes a period of inflation, baryogenesis is required, as inflation would wipe out any pre-existing baryon asymmetry.

It turns out that massive neutrinos can help solve this puzzle!
In the old SM, (electroweak) baryogenesis does not work – not enough CP-invariance violation, Higgs boson too light.

**Neutrinos help by providing all the necessary ingredients for successful baryogenesis via leptogenesis.**

- **Violation of lepton number,** which later on is transformed into baryon number by nonperturbative, finite temperature electroweak effects (in one version of the $\nu$SM, lepton number is broken at a high energy scale $M$).

- **Violation of C-invariance and CP-invariance** (weak interactions, plus new CP-odd phases).

- **Deviation from thermal equilibrium** (depending on the strength of the relevant interactions).
E.g. – thermal, seesaw leptogenesis,
\[ \mathcal{L} \supset -y_{i\alpha} L^i H N^\alpha - \frac{M_N^{\alpha\beta}}{2} N_\alpha N_\beta + H.c. \]

[Fukugita, Yanagida]

- L-violating processes
- \( y \Rightarrow \) CP-violation
- deviation from thermal eq. constrains combinations of \( M_N \) and \( y \).
- need to yield correct \( m_\nu \)

not trivial!

E.g. – thermal, seesaw leptogenesis,

\[ \mathcal{L} \supset -y_{\ell \alpha} L^i H N^\alpha - \frac{M_N^{\alpha \beta}}{2} N_\alpha N_\beta + H.c. \]

It did not have to work – but it does

MSSM picture does not quite work – gravitino problem

(there are ways around it, of course...)
Relationship to Low Energy Observables?

In general ... no. This is very easy to understand. The baryon asymmetry depends on the (high energy) physics responsible for lepton-number violation. Neutrino masses are a (small) consequence of this physics, albeit the only observable one at the low-energy experiments we can perform nowadays.

see-saw: $y, M_N$ have more physical parameters than $m_\nu = y^t M_N^{-1} y$.

There could be a relationship, but it requires that we know more about the high energy Lagrangian (model dependent). The day will come when we have enough evidence to refute leptogenesis (or strongly suspect that it is correct) - but more information of the kind I mentioned earlier is really necessary (charged-lepton flavor violation, collider data on EWSB, lepton-number violation, etc).
The most direct probe of the lightest neutrino mass – precision measurements of $\beta$-decay

Observation of the effect of non-zero neutrino masses kinematically.

When a neutrino is produced, some of the energy exchanged in the process should be spent by the non-zero neutrino mass.

Typical effects are very, very small – we’ve never seen them! The most sensitive observable is the electron energy spectrum from tritium decay.

$$^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}$$

Why tritium? Small $Q$ value, reasonable abundances. Required sensitivity proportional to $m^2/Q^2$.

In practice, this decay is sensitive to an effective “electron neutrino mass”:

$$m_{\nu e}^2 \equiv \sum_i |U_{ei}|^2 m_i^2$$
Experiments measure the **shape** of the end-point of the spectrum, not the value of the end point. This is done by counting events as a function of a low-energy cut-off.

note: LOTS of Statistics Needed!

![Graph](image)

Figure 2: The electron energy spectrum of tritium $\beta$ decay: (a) complete and (b) narrow region around endpoint $E_0$. The $\beta$ spectrum is shown for neutrino masses of 0 and 1 eV.
NEXT GENERATION: The Karlsruhe Tritium Neutrino (KATRIN) Experiment:

(not your grandmother’s table top experiment!)

sensitivity $m_{\nu_e}^2 > (0.2 \text{ eV})^2$
And that is not all! Neutrinos are unique probes of several different physics phenomena from vastly different scales, including...

- Dark Matter;
- Weak Interactions;
- Nucleons;
- Nuclei;
- the Earth;
- the Sun;
- Supernova explosions;
- The Origin of Ultra-High Energy Cosmic Rays;
- The Universe.
Making Predictions, for an inverted mass hierarchy, \( m_4 = 1 \text{ eV}(\ll m_5) \)

- \( \nu_e \) disappearance with an associated effective mixing angle \( \sin^2 2\vartheta_{ee} > 0.02 \). An interesting new proposal to closely expose the Daya Bay detectors to a strong \( \beta \)-emitting source would be sensitive to \( \sin^2 2\vartheta_{ee} > 0.04 \);

- \( \nu_\mu \) disappearance with an associated effective mixing angle \( \sin^2 2\vartheta_{\mu\mu} > 0.07 \), very close to the most recent MINOS lower bound;

- \( \nu_\mu \leftrightarrow \nu_e \) transitions with an associated effective mixing angle \( \sin^2 \vartheta_{e\mu} > 0.0004 \);

- \( \nu_\mu \leftrightarrow \nu_\tau \) transitions with an associated effective mixing angle \( \sin^2 \vartheta_{\mu\tau} > 0.001 \). A \( \nu_\mu \rightarrow \nu_\tau \) appearance search sensitive to probabilities larger than 0.1\% for a mass-squared difference of 1 eV\(^2\) would definitively rule out \( m_4 = 1 \text{ eV} \) if the neutrino mass hierarchy is inverted.