Physics 571

Fall 2021 Semester

Mathematical Methods for Physics

The instructor reserves the right to revise, alter, or amend this Syllabus as necessary. Students will be notified in writing of any such changes. Updated August 11, 2021.

Logistics

Instructor	Dr. Norman Mannella
Office	Nielsen 210 / JIAM 313
Telephone (Both UT and JIAM)	(865) 974 - 6123
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Lecture Time and Location	Tuesday and Thursday, 9:50 – 11:05, Nielsen 304
Office Hours	After class, or by appointment

General Course Description

The course will treat in a rigorous fashion some of the mathematical principles and ideas that are used in Classical and Quantum Physics, Quantum Chemistry, and Engineering. The study of some of the most common mathematical methods is of fundamental importance for the professional formation of physicists, chemists, and engineers.

Course Objectives

- **Advance skills and ability in formulating and solving problems**. Expand and exercise the students' intuition and thinking process through the understanding of the theory and application of mathematics to the solution of problems.
- **Increase mathematical sophistication**. Learn and apply advanced mathematical techniques and methods in solving problems.

Prerequisites

The course and text presume a familiarity with calculus and calculus concepts (vectors, differential and integral calculus), linear algebra (matrices, determinants, eigenvalues/eigenvectors etc.), and ordinary differential equations. A background in these topics is highly recommended and is necessary for success in the course. You should have either taken these classes and retain some basic understanding, or you should get textbooks and teach yourselves as needed.

Course content

Sets, elementary topology, and algebraic structures – Notion of sets, Ordered sets, Inf. and Sup., Finite, Countable, and Uncountable sets. Concepts of transformations, injections, surjections, one-to-one mapping. Maps composition, Invertible maps. Metric and metric spaces. Limit points, open and closed sets, dense sets. Sequences and their convergence in metric spaces, Cauchy sequences, Complete metric spaces. Concepts of limit and continuity of functions in metric spaces. Algebraic structures, Groups, Fields, Vector Spaces. Linear independence of vectors, Bases. Direct sum of vector spaces. Normed vector spaces, Banach spaces. Euclidian spaces R^n and Complex spaces C^n. Schwartz inequality.

Basic Theory of Linear Transformations – Linear transformations, Multilinear functions, Linear and multilinear forms (Functionals). Matrices as representation of linear transformations. Algebraic matrix operations. Matrix composition, transposition, cofactors, trace, inversion. Determinants, Laplace's expansions. Change of basis, similarity transformations. Eigenvalues and eigenvectors. Systems of linear equations. Caley-Hamilton theorem. Matrix diagonalization, modal matrix. Normal transformations. Functions of matrices.

Hilbert spaces: basic theory – Inner product, Hilbertian norm. Hilbert spaces. Schwartz inequality. Orthogonality. Orthogonal sum. Projection theorem. Orthonormal systems. Bessel inequality. Fischer-Riesz theorem. Gram-Schmidt process. Complete orthonormal systems (C.O.N.S).

Linear transformations in inner product spaces and Hilbert spaces – Orthogonal and non-orthogonal bases, metric tensor. Dirac's notation. Linear transformations and Dirac notation. Adjoint operators, Riesz representation theorem. Self-adjoint (Symmetric and Hermitian), and isometric (orthogonal and unitary) transformations. Change of basis for orthonormal systems. Projection operators. Diagonalization of self-adjoint transformations. Bi-linear and quadratic forms, and their diagonalization.

The Spectral Theorem – The Spectral Theorem in finite dimensional spaces: C.O.N.S., Diagonal representation, Orthogonal projectors, Autospaces. Distinct and degenerate eigenvalues, uniqueness of C.O.N.S. Matrix representations on the C.O.N.S. basis, block matrices. Commutation of two self-adjoint operators, construction of common C.O.N.S., Spectral Theorem, Simultaneous Diagonalization of operators and quadratic forms. Complete system of operators. Functions of operators.

Sequences, series and improper integrals – Numerical sequences, convergence and Cauchy sequences in normed spaces, convergence tests. Numerical series and their convergence. Improper integrals of first and second kind. Existence criteria. Taylor's approximation formula, remainder. Cauchy principal value.

Elements of Complex Analysis – Complex numbers, sequences and series in C. Power series. Differentiation of functions and power series. Multi-valued functions, branch points and branch cuts. Integration in C: basic notions, Fundamental Theorem of contour integration, Cauchy-Goursat Theorem. Analytic functions and their expansion in Taylor's series. Cauchy Integral formula. Singularities and Laurent series. Poles, residues, Residue Theorem. Application to evaluation of integrals. Jordan lemma. Principle value integrals. Plemelj's formulas. Dispersion relations.

Functional Hilbert Spaces – Sequences and series of functions, point-wise and uniform convergence, convergence criteria. L^2 spaces. C.O.N.S. of eigenfunctions. Lebesgue integrals. Dirac's delta function. Convergence in the mean. Stone-Weierstrass Theorem. Continuous bases. Linear Operators in functional spaces. Integration measure.

Fourier's Integral Transform – Definition and properties. Convolution Theorem. Plancherel's and Parseval's Theorems. Correlations, Wiener-Khintchine Theorem. The Uncertainty Principle Theorem.

Ordinary Differential Equations (ODE) – First and second order linear ODE. Wronski determinant. Homogenous solutions and particular integrals, variation of constants. Series solution of linear ODE. Fuchs singularities, Frobenius method. Systems of linear ODEs.

Linear Second Order Ordinary Differential Operators (ODO) – Homogeneous and non-homogeneous boundary conditions. Adjoint of an ODO and adjoint Boundary Conditions. Generalized Green Identity. Formally self-adjoint, and self-adjoint ODO. Sturm Liouville and normal forms of an ODO. Eigenvalues and Eigenfunctions of an ODO. The equations of Mathematical Physics. Eigenvalues and eigenfucntions of common ODO (Laplace, Helmholtz, Legendre, Associated Legendre, Spherical harmonics, Bessel, Spherical Bessel, Laguerre, Hermitte).

Green functions and their application to Ordinary Differential Equations – Particular integrals expressed as Green functions. Initial value problems. Boundary value problems. Construction of the Green function.

Partial Differential Equations (PDE) – PDE of the elliptic, parabolic, and hyperbolic type. Separation of variables. Dirichlet and Neumann Boundary Value Problems. Application of Green function techniques to PDE.

Reference Material

There is no "official" textbook. I will follow a set of **Lecture Notes** that will be provided to you. A couple of books that cover the majority of the topics developed in the course are

- **"Mathematics of Classical and Quantum Physics", by Byron and Fuller (Dover)**
- **"Mathematics for Physicists", Dennery and Krzywicki (Dover)**

The following textbooks are recommended as supplements to the course, either for review, or to get a different perspective on parts of the material.

Undergraduate Texts:

for linear algebra and ordinary differential equations,

- **"Introduction to Linear Algebra and Differential Equations", by John W. Dettman, (Dover)**
- **Linear Algebra, S. Lang**
- **"Ordinary Differential Equations", M. Tenenbaum and H. Pollack (Dover)**

for differential and integral calculus

- **"Vector and Tensor Analysis", by A. I. Borisenko and I. E. Tarapov, (Dover)**
- **"Introduction to Vector and Tensor Analysis", by Robert C. Wrede, (Dover)**
- **"Introduction to Vector Analysis", by Harry F. Davis and Arthur D. Snider**
- **"Div Grad Curl and all that", by H. M. Schey**
- **"Advanced Calculus of Several Variables", by C. H. Edwards (Dover)**

For Mathematical Methods

- **"Applied Analysis by the Hilbert Space Method", S. Holland (Dover)**

- **"Mathematical Methods for Physics and Engineering", Riley, Hobson and Bence**

Graduate Texts

- **"Mathematical Methods in Physics and Engineering", by J. W. Dettman (Dover)**
- **"Physical Mathematics", by K. Cahill (Cambridge)**
- **"Mathematical Methods of Physics", Mathews and Walker**
- **"Mathematical Methods for Physicists", Arfken and Weber**

It is also possible to learn lots of math from texts in Quantum Mechanics and Electromagnetism

- **"Classical Electrodynamics", by J. D. Jackson**
- **"Classical Electrodynamics", by W. Greiner**
- **"Classical Electromagnetism", by Jerrold Franklin**
- **"Modern Quantum Mechanics", J. Sakurai**

Announcements, Lecture Notes, Course Material, and Course Updates

Lectures and Canvas (Online@UT) are my primary modes of communication with the class. Announcements, Lecture Notes, additional Course Material, Homework, solutions to Homework, and Course Updates including definite dates for exams etc. will be posted on Canvas. Please note that it will be your responsibility to be aware of the content of any communication taking place in class, be it an announcement or anything related to the course material, in case you are not present. You are required to have an official UT email address and read announcements on Canvas and your email on a daily basis. Information that cannot be transmitted to you during the lectures or on Canvas, or any personal communication, will be given to you via email.

Contacting the Instructor

I prefer personal contact to e-mail contact, and therefore encourage you to come to office hours, or schedule an appointment. As another alternative, I am going to be in my office most of the time, if you show up chances are that I might see you, unless I am really in the middle of something. Call me in the office to check. **Anyhow, I strongly encourage you to see me.** As a general rule, I do NOT address homework problems by e-mail. On the other hand, if you have a personal emergency, e-mail is fine.

Attendance Policy

Attendance is highly recommended, and most likely essential for a successful completion of the course. If you are not present in class, it will be your responsibility to be aware of the content of any communication taking place in class, be it an announcement or anything related to the course material.

Homework

There will be approximately 10 problem sets. You will be notified in Canvas when the HW is available. **Homework will always be collected at the beginning of the class session**, with due date specified on the HW itself and communicated in Canvas when the HW is made available. Please note the following policies, which are quite strict:

No extensions or make-up problem sets will be given. If there are extremely serious circumstances supported by proper documentation, exception to this policy may be considered at my discretion.

Due dates and time for HW are firm. I post all assignments at least one week in advance, so please plan ahead. Problem sets turned in 1 day late will receive 90% of the maximum score. Your work will receive 0 points if turned in later than 1 day.

Each problem will be graded on a 0 - 5 scale (0 = no work, 1 = poor, 2 = fair, 3 = good, 4 = very good, 5 = excellent), or similarly if worth more points (e.g. 0 = no work, 2 = poor, etc.). Solutions to the HW will be posted on Canvas.

NB.: *A point is a point*: this means that different HW sets might have different total scores, and your total score will be given by the sum of the points that you will collect.

Questions regarding the HW problems may be asked in lecture or during office hours, NOT by email. For each HW set, I will make available for you a **Forum in Canvas** where you can discuss among yourself.

Midterm and Final Exam

There will be one Midterm Exams and one comprehensive, Final Exam.

Grading

The semester Grade will be based on the following Weighted Average:

Homework	=	**55%**
Midterm	=	**15%**
Final Exam	=	**30%**

Grades in the C range represent performance that is unsatisfactory, and does not meet expectations; Grades in the B range represent performance that meets expectations; Grades in the A range represent work that is very good/excellent.

Please note: **Ordinarily make-up exams will NOT be given. Missing the Exams is very serious and may well result in failure of the course.** However, if there are extremely serious circumstances supported by proper documentation, a make-up for Midterms and/or Final exam may be considered at my discretion.

Appeals

You are welcome to discuss the grading of a given assignment, be it homework, Midterm or Final Exam. Appeals must be dropped in my mailbox, with a brief explanation of the issue. Any appeal will be entertained if it is raised no later than one week after the date on which the graded Exams/HW are made available for return to the class. After this "appeal period" of one week, exam grades will be considered final and will not be altered.

Cheating and Plagiarism will not be tolerated

Cheating will not be tolerated. Everyone must have an equal chance to do well. The penalty for cheating on any aspect of this course will be an "**F**" for the course.

Plagiarism of any kind will not be tolerated. Working together on homework does not count as plagiarism. A line by line copy of another student's homework does. If you use a source (book, articles, internet material etc.), you must quote it. Use of a source without citation is plagiarism.

Cheating and/or plagiarism cases found to be in Violation of the Academic Honesty policies will result in disciplinary actions according to the University rules, without exception.

Department of Physics and Astronomy Statement on Civility & Community

The Department of Physics & Astronomy at the University of Tennessee is committed to creating an environment that welcomes all people, regardless of their identities. We value the diversity that enriches our department. We understand the importance of free and open dialogue that includes the free exchange of ideas. We do not tolerate uncivil speech or any form of discourse that infringes on others' rights to express themselves, or has a negative impact on their education, or work environment. We actively promote an environment of collegiality and an atmosphere of mutual respect and civility. We understand that respect includes being considerate of others' feelings, circumstances, and their individuality. We recognize the necessity of a civil community in realizing the potential of individuals in teaching, learning, research, and service. We believe these values extend beyond the department into our work within physics regionally, nationally, and internationally, as well as work and studies in the university, and the broader community. We encourage all members of the department to intervene and report any incidents involving bigotry, or that violate the university code of conduct. http://www.phys.utk.edu/about/civility-community.html 3

Reporting

Anyone who experiences or observes any such incident is encouraged to report it to the Department Head (Prof. H. Weitering) or one of the Associate Heads (Prof. M. Breinig and Prof. K. Jones). Students can also speak to any faculty or staff member with whom they feel comfortable. Incidents that involve sexual harassment or stalking will be reported to the office of Title IX under mandatory reporting requirements.

http://www.phys.utk.edu/about/civility-community.html 3

University Civility Statement

"Civility is genuine respect and regard for others: politeness, consideration, tact, good manners, graciousness, cordiality, affability, amiability and courteousness. Civility enhances academic freedom and integrity and is a prerequisite to the free exchange of ideas and knowledge in the learning community. Our community consists of students, faculty, staff, alumni, and campus visitors. Community members affect each other's well-being and have a shared interest in creating and sustaining an environment where all community members and their points of view are valued and respected. Affirming the value of each member of the university community, the campus asks that all its members adhere to the principles of civility and community adopted by the campus" https://civility.utk.edu/

Academic Integrity

Each student is responsible for his/her personal integrity in academic life and for adhering to UT's Honor Statement. The Honor Statement reads: "An essential feature of the University of Tennessee, Knoxville is a commitment to maintaining an atmosphere of intellectual integrity and academic honesty. As a student of the university, I pledge that I will neither knowingly give nor receive any inappropriate assistance in academic work, thus affirming my own personal commitment to honor and integrity."

For students with disabilities

Students needing an accommodation based on the impact of a disability should contact me privately to discuss specific needs. Students are also responsible to contact the Office of Disability Services at 865-974-6087 in Hoskins Library to coordinate reasonable accommodations for students with documented disabilities. https://sds.utk.edu/

Wellness

The Student Counseling Center is the university's primary facility for personal counseling, psychotherapy, and psychological outreach and consultation services. The Center for Health Education and Wellness manages 974-HELP, the distressed student protocol, case management, the Sexual Assault Response Team, and the Threat Assessment Task Force. https://counselingcenter.utk.edu/ and https://wellness.utk.edu/

COVID-19 Guidelines

With the spread of the Delta variant of COVID-19, students, faculty, and staff will be required to wear masks in classrooms, labs, and for indoor academic. This requirement will remain in place until conditions improve and the university communicates new instructions.

The university strongly recommends that all members of the campus community be vaccinated for their own protection, to prevent disruption to the semester, and to prevent the spread of COVID-19. Vaccination information and appointment signups are available at *tiny.utk.edu/vaccine*. The Student Health Center medical staff is available to students to

answer questions or discuss concerns about vaccines, and the center provides vaccines free of charge for anyone 18 years or older who would like one.

If you think you are sick or have been exposed to COVID-19, you should contact the Student Health Center or your preferred health care provider. You can also contact the university's COVID-19 support team for guidance by filling out the COVID-19 self-isolation form at *covidform.utk.edu*.
You must not attend class if you have tested positive for COVID-19 and are in the isolation period, if you have COVID-19 symptoms and have not been cleared by a medical provider, or if you are an unvaccinated close contact in the quarantine period.

If you need to miss class for illness, please email me as soon as possible.

You can find more information and updates at *utk.edu/coronavirus*.