1. **25 Points** The resistors labeled 1, 2, 3, and 4 have resistances $R$, $2R$, $2R$, and $R$ respectively as shown on the figure below. Provide an expression for the current in each of the resistors. Be sure to provide an indication of the direction of the current. To receive partial credit you must CLEARLY explain your reasoning and show your work. (Continue work on back of page if necessary)

![Resistor Diagram](image)

**Use Kirchhoff's Rules**

\[
\begin{align*}
\sum I &= 0 \text{ Junction} \\
\sum V &= 0 \text{ Loop}
\end{align*}
\]

\[
\begin{align*}
I_1 + I_2 &= I_3 \quad \text{(a) Junction} \\
I_1 &= I_4 \quad \text{(b) Obvious, but also true by Junction}
\end{align*}
\]

**Loop 1**

\[+2I_2R + V - I_4R - 2V - I_1R = 0 \quad (2)\]

**Loop 2**

\[+2I_2R + V + 2I_3R = 0 \quad (4)\]

\[-2I_2R - V - 2I_1R = 0 \quad (b) \quad (c)\]

\[-2R(I_2 + I_1) = V \]

\[I_1 + I_2 = -\frac{V}{2R} \quad (d) \quad (e)\]

\[2I_2R + V + 2(I_1 + I_2)R = 0 \quad (d) \quad (e)\]

\[2I_2R + V - 2(-\frac{V}{2R})R = 0\]

\[2I_2R + V - 2V = 0\]

\[2I_2R = 0 \quad I_2 = 0 \quad \text{No current flows through } I_2\]

\[I_1 = -\frac{V}{2R}\quad \text{sign means } I_1 \text{ goes "up"}\]

\[I_3 = -\frac{V}{2R}\quad \text{sign means } I_3 \text{ goes "down"}\]

\[I_4 = -\frac{V}{2R}\quad \text{sign means } I_4 \text{ is Left - Right.}\]
2. **25 Points** Three charges are arranged on the vertices of an equilateral triangle with side length L as shown. The signs of the charges are as indicated. Point a is located at the center of the horizontal side of the triangle.

1a. What is the electrostatic potential at point a? Clearly explain your reasoning and clearly indicate whether the result is + or -.

\[ V = \frac{1}{4\pi \varepsilon_0} \left( \frac{Q}{L/2} + \frac{Q}{L/2} - \frac{4Q}{\sqrt{3}L/2} \right) = \frac{2Q}{4\pi \varepsilon_0} \left( \frac{2}{L} - \frac{4Q}{\sqrt{3}L} \right) \]

\[ V = \frac{Q}{\pi \varepsilon_0} \left( 4 - \frac{2}{\sqrt{3}} \right) \quad \text{since} \quad 2 > \sqrt{3} \]

\[ V \text{ is negative} \]

1b. What is the potential energy associated with the charge of -4Q due to the other two charges? Clearly explain your reasoning and clearly indicate whether the result is + or -.

\[ U = qV = -4Q \left[ \frac{1}{4\pi \varepsilon_0} \left( \frac{Q}{L} + \frac{Q}{L} \right) \right] \quad \text{Use superposition} \]

\[ U = -\frac{2Q^2}{\pi \varepsilon_0} \quad \text{result is negative} \]

1c. How much work must be done to assemble all the charges at \( \infty \)? Clearly explain your reasoning, indicate whether the result is + or -, and explain why.

\[ E = \frac{1}{4\pi \varepsilon_0} \left( \frac{Q^2}{L} - \frac{4Q^2}{L} - \frac{4Q^2}{L} \right) = -\frac{3Q^2}{4\pi \varepsilon_0} \]

Negative result indicates that it does not take work to assemble these charges. They are at a lower potential energy than if all were at \( \infty \).
3. **20 Points** The switch in the circuit below was in position 1 shown for a very long time before t=0. At time t=0, the switch is moved to position 2.

Switch S in position 1

Because switch has been in position 1 for a "long time"

Q = 0 and V\_cap = 0

The charge Q has a sign given by the +/- in the drawing

Switch S in position 2

3a. Give an expression for the charge on the capacitor as a function of time, Q(t) for t>0.

**Explain your reasoning and clearly indicate the meaning of the sign of Q(t).**

\[
Q(t) = Q\_final (1 - e^{-t/RC}) = CE (1 - e^{-t/RC})
\]

where time constant is RC.

Result can also be obtained from K's laws and solving the differential equation.

3b. On the axis below, sketch the charge on the capacitor as a function of time. Be sure to provide an indication the scales on the horizontal and vertical axes.
3c. Give an expression for the current through the resistor as a function of time, I(t) for t>0. Explain your reasoning and clearly indicate the meaning of the sign of I(t).

Initial EMF across capacitor is zero so K’s loop rule at t=0 gives I(0)R = E. After a long time, I(∞) = 0 when the capacitor is fully charged, this gives a solution for I(t) = \( \frac{E}{\frac{1}{R}C} e^{-t/RC} \).

This can also be obtained by differentiating the equation for change \( \frac{d}{dt} Q(t) = \frac{d}{dt} \left[ \frac{C}{E} (1 - e^{-t/RC}) \right] \)

\[ = \frac{C}{E} \left[ 0 - \frac{1}{RC} e^{-t/RC} \right] \]

\[ = \frac{E}{\frac{1}{R}C} e^{-t/RC} \]

3d. On the axis below, sketch the charge on the capacitor as a function of time. Be sure to provide an indication the scales on the horizontal and vertical axes.

[Diagram showing I(t) and Q(t) vs. t]