I.a. The conductor in the figure below is a wire that carries a current \( I \) in the direction indicated by the arrows. The conductor consists of two circular arcs and two straight sections. Using the Law of Biot & Savart, determine the magnitude of the net magnetic field at point \( P \) (the center of the circles). Clearly **EXPLAIN YOUR REASONING**.

\[
\begin{align*}
\text{Law of Biot and Savart:} & \\
\frac{dB}{dr} = \frac{\mu_0 I \, d\varphi}{4\pi} \quad \frac{d^2 B}{dr^2} = \frac{\mu_0 I \, \pi a^2}{4\pi a^2} = \frac{\mu_0 I}{4a} \\
\end{align*}
\]

\[
\vec{B} = \frac{\mu_0 I}{4a} \left[ \left( \frac{1}{b} + \frac{1}{a} \right) \right] T
\]

1b. What is the direction of the net magnetic at point \( P \) (the center of the circle)? Clearly **EXPLAIN YOUR REASONING**.

Using the Right Hand Rule, \( P \) goes into the page.

\[
\hat{d} \perp \hat{t} \perp \hat{B}
\]

\[
\hat{d} \parallel \hat{x}, \quad \text{so} \quad B \text{ must be} \parallel \hat{z} \text{ into or out}
\]
2. The triangular loop of wire in the figure carries a current of I in the direction shown. The loop is in a uniform magnetic field of magnitude B in the same direction as the current in side PQ of the loop. Be sure to indicate both magnitude and direction if the answer is a vector.

2a. What is the force exerted by the magnetic field on each side of the triangle?

\[
F_{PB} = I \times B = I \times B \sin \theta = 0 \text{ N}
\]

\[
F_{PR} = I \times B = I \times B \sin (90^\circ - \theta) = IB \cos \theta = 0.6IB \text{ into the page}
\]

\[
F_{QR} = I \times B = I \times B \sin (90^\circ - \theta) = IB \cos \theta = 0.6IB \text{ out of the page}
\]

2b. What is the net force on the triangle?

\[0.6IB - 0.6IB = 0\] using superposition

2c. What is the magnitude torque on the triangle? What is the axis about which this torque would tend to rotate the triangle?

The triangle would rotate about the X axis because the dipole moment (direction into page) is at the lowest potential when \(I\) is \(B\) (direction +\(\hat{j}\))

\[
J = I \times B = IAB \sin \theta
\]

\[
= IB \times 0.6 \times 0.8 \times 0.6 = 0.24IB
\]
3. The figure below is cross sectional view of two very long parallel conducting wires that are perpendicular to the xy plane (the plane of the paper). Each wire carries current \( I \) but in opposite directions (the current in the upper wire in out of the plane of the paper).

3a. On the figure, sketch the directions of magnetic fields from the upper and lower wires at point P which at a distance \( x \) from the midpoint between the wires.
   To receive full credit, you must explain your reasoning, and your sketch must clearly indicate the direction with respect to the positions in the figure (i.e. is it parallel to something or is it a particular angle with respect to the axis, etc.).

   Magnetic field direction is \( \perp \) to the direction of the current and the position vector \( \vec{r} \) and do to RHR, the direction of \( \vec{B} \) from current \( I_1 \) is up and to the right \( \perp \) to \( \vec{r} \) and \( \vec{I}_1 \). The direction of \( \vec{B} \) from \( I_2 \) is down and to the right \( \perp \) to \( \vec{r} \) and \( \vec{I}_2 \). Both are in the xy plane.

3b. What is the magnitude and direction of the net magnetic field at point P?

   The magnitude of the magnetic field due to a long conducting wire is \( B = \frac{\mu_0 I}{2\pi r} \) where \( r \) is the distance between the wire.

   So \( B_{I_1} = \frac{\mu_0 I_1}{2\pi r_1} \) in components
   \[
   B_{I_1x} = \frac{\mu_0 I_1}{2\pi (a+x)^2} \hat{x}, \quad B_{I_1y} = -\frac{\mu_0 I_1}{2\pi (a+x)^2} \hat{y}
   \]

   \( B_{I_2} = \frac{\mu_0 I_2}{2\pi r_2} \) in components
   \[
   B_{I_2x} = \frac{\mu_0 I_2}{2\pi (a-x)^2} \hat{x}, \quad B_{I_2y} = \frac{\mu_0 I_2}{2\pi (a-x)^2} \hat{y}
   \]

   \[
   \vec{B}_P = B_{I_1x} - B_{I_1y} + B_{I_2x} + B_{I_2y} - \frac{\mu_0 I_1 a}{\pi(a+x)^2} \hat{x} - \frac{\mu_0 I_2 a}{\pi(a-x)^2} \hat{x}
   \]

   Directions in xy cancel

   \( \vec{B}_P \) in the \( -x \) direction which is right.
4. A long wire carrying current $I$ lies in the plane of the paper (x-y plane) as shown. A uniform magnetic field of magnitude $B$ is applied perpendicular to the plane of the paper and INTO the paper.

What is the magnitude and direction of the force on a length $L$ of the conductor?

$$F = ILB\sin\theta$$

$\theta$ is the angle between $L$ and $B = 90^\circ$

$\sin\theta = 1$

Force is $\perp$ to the wire

$$F = ILB$$

$$F = ILB\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$$