The surface of a conductor is always an Equipotential Surface.

Equipotential Surfaces are always perpendicular to Electric Field Lines.
Define $C = \frac{Q}{V}$ as the "capacitance."

If I double the charge $Q$, I will double the voltage difference. There is a single voltage difference between two charged conductors.

\[
\int \mathbf{P} \cdot \mathbf{E} \, d\mathbf{A} = \Delta V
\]
Capacitance has units of "Farads".

\[ C = \frac{p}{V} \]

Recall result from infinite flat plate

Integral is easy because field is uniform

\[ \int \vec{E} \cdot d\vec{l} = p \int_0^L \vec{E} \cdot d\vec{r} = q_n A - \pi A L \]

Capacitance between two parallel plates

\[ \frac{V}{\varepsilon} \frac{\sigma}{I} = \frac{q_n A}{\varepsilon} \]

\[ p \vec{E} = \int_0^L \vec{E} \cdot d\vec{r} = q_n A - \pi A L \]
Energy Storage in a Charged Capacitor

\[ E = \frac{1}{2} CV^2 \]

- If I start from NO net charge I get

\[ \frac{\varepsilon_0 A \frac{\bar{C}}{l}}{L} = \frac{\varepsilon_0 \bar{C}}{l} = \frac{C}{\varepsilon_0} = M \]

- To add a finite charge \( \bar{Q} \), I must integrate

\[ b_p \frac{C}{b} = b_p \Lambda = MP \]

- To add a small amount of charge \( dq \) I must do work

\[ \int \frac{C}{l} dq \]

- To add more charge, I must do work

\[ \frac{C}{\bar{Q}} = \Lambda \]

Energy Storage in a Capacitor
**Capacitors in Parallel**

\[ Q_1 = C_1 V \quad \quad Q_2 = C_2 V \]

\[ Q = Q_1 + Q_2 = (C_1 + C_2) V \]

\[ \frac{Q}{V} = C_1 + C_2 \]

\[ C_{equiv} = C_1 + C_2 \]

\[ C_{equiv} = C_1 + C_2 + C_3 + \ldots \]

**Capacitors in Series**

\[ V_{ac} = V_1 = \frac{Q}{C_1} \quad \quad V_{cb} = V_2 = \frac{Q}{C_2} \]

\[ V_{ac} = V = V_1 + V_2 = Q \left( \frac{1}{C_1} + \frac{1}{C_2} \right) \]

\[ \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} \]

\[ \frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2} \]

\[ \frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots \]
11. Resistance and Ohms Law

Definition:
A Resistor is a device that follows Ohms Law:

\[ V = IR \]

If there is a voltage “across” a resistor then
There MUST be a current “through” it.

If there is a current “through” a resistor then
There must be a voltage across it.
Resistors in Series and Parallel

Consider the following circuit:

\[ \text{What is } I? \]

1) We know \( I_1 \) must equal \( I_2 \) so \( V_1 = IR_1 \)
\[ V_2 = IR_2 \]

2) \( V_1 \neq V_2 \) but we know the total potential \( V_1 + V_2 = V \)
\[ IR_1 + IR_2 = V \quad \text{or} \quad V = I(R_1 + R_2) \]

The current in the circuit is the same as if I had one resistor of \( R = R_1 + R_2 \)

We say \( R \) is "the equivalent resistance" to the series combination of \( R_1 + R_2 \).

For \( R_1, R_2, R_3, R_4, R_5, \ldots \)

Equivalent \( R = R_1 + R_2 + R_3 + R_4 \)
Resistors in Parallel

Consider:

\[ I = E / (R_1 + R_2) \]

What is \( I \)?

\[ V_1 = I_1 R_1 = V \]
\[ V_2 = I_2 R_2 = V \]

Connected by "parallel wire"

\[ I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2} \]

But \( I_1 + I_2 = I \)

Where else can current go?

\[ I = \frac{V}{R_1} + \frac{V}{R_2} = V \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

by \( I = \frac{V}{R_{eq}} \).

So the parallel combination behaves the same as a single resistor of \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \)

For \[ R_1 \quad R_2 \quad R_3 \]

the equivalent resistance is

\[ \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots \]
12. Kirchhoff's Rules

Example

Loop Rule – Valid for any closed loop
0 = \sum \Delta V

Junction Rule – Valid at any junction
0 = \sum I
Example Problem: What is the current in each resistor? What is the equivalent resistance of the 5 resistors?

Step 1 - Lay out a clear diagram of circuit:

Step 2 - Label ALL Currents – Use Kirchhoff’s Junction Rule to simplify

The directions of currents you chose is NOT IMPORTANT If you guess wrong – Mr. Kirchhoff will give you a minus sign for the current.
Step 3- Identify Positive and Negative sense for each component

The sense of + and – depends on your choice of current

Step 4- Draw Loops around circuit – every resistor must lie on at least one loop

The directions of the loops is NOT IMPORTANT. It does NOT have to be in the direction of the currents!
Step 5 – Go around the loops to get the equations-

NOTE – If we go from + to – potential change is NEGATIVE
If we go from - to + potential change is POSITIVE

\[13V - I_1(1\Omega) - (I_1 - I_3)(1\Omega) = 0 \quad \text{Loop 1}\]
\[-I_2(1\Omega) - (I_2 + I_3)(2\Omega) + 13V = 0 \quad \text{Loop 2}\]
\[-I_1(1\Omega) - I_3(1\Omega) + I_2(1\Omega) = 0 \quad \text{Loop 3}\]

Step 6- Solve the Simultaneous Equations

\[13V - I_1(1\Omega) - (I_1 - I_3)(1\Omega) = 0 \quad \text{Loop 1}\]
\[-I_2(1\Omega) - (I_2 + I_3)(2\Omega) + 13V = 0 \quad \text{Loop 2}\]
\[-I_1(1\Omega) - I_3(1\Omega) + I_2(1\Omega) = 0 \quad \text{Loop 3}\]

Solve Equation 3 for \(I_2\):

\[I_2 = I_1 + I_3\]

Plug into Equations 1 and 2 (this gives 2 equations and 2 unknowns):

\[13V = I_1(2\Omega) - I_3(1\Omega)\]
\[13V = I_1(3\Omega) + I_3(5\Omega)\]

Multiply top equation by 5 and add two equations:

\[65V = I_1(10\Omega) - I_3(5\Omega)\]
\[13V = I_1(3\Omega) + I_3(5\Omega)\]

\[78V = I_1(13\Omega)\]

\[I_1 = 6A\]
\[I_3 = -1A\]
\[I_2 = 5A\]