1. The state space of a certain physical system is three dimensional. Let \( \{ |u_1\rangle, |u_2\rangle, |u_3\rangle \} \) be an orthonormal basis of this space. Consider the following two kets which belong in the same space:

\[
|\psi_0\rangle = \frac{1}{\sqrt{2}} |u_1\rangle + \frac{i}{2} |u_2\rangle + \frac{1}{2} |u_3\rangle \\
|\psi_1\rangle = \frac{1}{\sqrt{3}} |u_1\rangle + \frac{i}{\sqrt{3}} |u_3\rangle
\]

(a) Are these kets normalized?

(b) Calculate the matrices \( \rho_0 \) and \( \rho_1 \) representing, in the \( \{ |u_1\rangle, |u_2\rangle, |u_3\rangle \} \) basis, the projection operators onto the state \( |\psi_0\rangle \) and onto the state \( |\psi_1\rangle \). Verify that these matrices are Hermitian.

2. Consider a physical system whose three dimensional state space is spanned by the three kets \( |u_1\rangle, |u_2\rangle, |u_3\rangle \). In the basis of these three vectors, taken in this order, the two operators \( \hat{H} \) and \( \hat{B} \) are represented by the matrices

\[
\begin{bmatrix}
1 & 0 & 0 \\
0 & -2 & 0 \\
0 & 0 & -2
\end{bmatrix}
\text{ and }
\begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 2 \\
0 & 2 & 0
\end{bmatrix}
\]

where \( \omega_0 \) and \( b \) are real constants.

(a) Are the matrices \( H \) and \( B \) Hermitian?

(b) Show that \( H \) and \( B \) commute. Obtain a basis of eigenvectors common to \( H \) and \( B \).

3. Consider the matrix \( \sigma_x \) defined by:

\[
\sigma_x = \begin{pmatrix}
0 & 1 \\
1 & 0
\end{pmatrix}
\]

Prove the relation:
\[ e^{i\alpha x} = I \cos \alpha + i\sigma_x \sin \alpha , \]

where \( I \) is the 2x2 unit matrix.

4. Le Bellac, page 58, problem 2.4.11.