Magnetism

- How does magnetism come about?
- What is magnetic susceptibility measuring?
- How does the neutron interact with electrons?
- Is the magnetic scattering length a constant?
- Diffraction from magnetic crystals.
In a metal, degeneracy between electrons of opposite spin sharing the same orbital state is resolved by applying a magnetic field. This causes a redistribution of electrons between the two spin orientations.

The difference in the population of up and down spins give rise to a moment

\[ M = \mu_0 (n_+ - n_-) \]

\[ \approx \mu_0 H N(\varepsilon_F) \]

This gives rise to Pauli paramagnetism. It does not depend on temperature and measures the density of states at the Fermi level.
Suppose each atom behaves like a small magnet such as in d- or f-electrons

when a magnetic field is applied, magnetic moments aligned in the direction other field will have energy \( \vec{\mu} \cdot \vec{H} \)

The magnetic susceptibility is given by

\[
\chi \approx \frac{1}{3} \frac{N \langle \mu^2 \rangle}{k_B T}
\]

This is the Curie law for paramagnetic susceptibility. It works for solids with magnetic atoms that are well separated from one another.

In the case of interactions b/n spins, then

\[
\chi = \frac{N \langle \mu^2 \rangle}{3k_B (T - \Theta)}
\]

which is known as the Curie-Weiss law.

\[ T_c = \Theta_{cw} \]

Spontaneous magnetization in a ferromagnet.
Q: In the interaction of a neutron with an atom, how does magnetic scattering come about?

Neutron has a magnetic dipole moment.

$$\mu_n = -\gamma |\mu_n\rangle$$

$$\gamma$$ - gyromagnetic ratio

$$\hat{\sigma}$$ - Pauli spin matrices

When $$s = \frac{1}{2}$$,

$$\sigma_x = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \frac{1}{2} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mu_n = \frac{e\hbar}{2M_p}$$ (nuclear Bohr magneton)

$$\gamma = 1.913$$

Electron has a magnetic dipole moment.

$$\mu_e = -g \mu_B \hat{s}$$

$$\hat{s}$$ - spin angular momentum ($$\pm \frac{1}{2}$$)

$$g$$ - about 2 for free electrons

Spin magnetic moment - roughly speaking, comes from the rotation of the electron

At the same time, electron has orbital magnetic moment

$$\mu_o = \frac{\hbar}{2c} (\hat{\nabla} \times \vec{\nabla})$$

arising from the orbital motion of the electron.
First consider the neutron interaction with a single bound atom. The interaction potential is given by

\[ V(r) = -\hat{\mu} \cdot B(\vec{r}) \quad \text{(dipole tends to orient itself parallel to field)} \]

\( B(\vec{r}) \) is the magnetic-flux density (or magnetic induction) arising from the atom.

The scattering length

\[ b = +\frac{\mu^2 m}{\hbar^2} \left| \langle \vec{k}' | (-\hat{\mu} \cdot B(\vec{r})) | \vec{k} \rangle \right| \]

\[ = -\frac{m}{\pi\hbar^2} \hat{\mu} \cdot B(\hat{q}) \]

where \( B(\hat{q}) = \int d^3\vec{r} \ e^{-i\vec{k}' \cdot \vec{r}} B(\vec{r}) e^{i\vec{k} \cdot \vec{r}} \)

Using Maxwell's equations for time-independent fields:

\[ \hat{J} = c \hat{\nabla} \times \hat{M} \]

\[ \hat{B}_s = \hat{\nabla} \times \hat{A} \]

\[ \hat{A} = -\frac{\mu_0}{c} \hat{J} \]

using \( \hat{\nabla} \rightarrow -i\hat{q} \) so

\[ B(\hat{q}) = -i\hat{q} \times \frac{\mu_0}{c\hat{q}^2} J(\hat{q}) \]

\[ = -i\hat{q} \times \frac{\mu_0}{c\hat{q}^2} \left( \hat{q} \times M_s(\hat{q}) \right) \]

\[ = -\mu_0 \hat{q} \times \hat{q} \hat{M}_\perp(\hat{q}) \]

where \( M_\perp(\hat{q}) \) is the component of magnetization perpendicular to \( \hat{q} \).
Spin magnetization operator is defined as over all electrons in atom

\[ \hat{M}(\vec{q}) = - g \mu_B \left< \sum_j \frac{\mathbf{S}_j e^{i \vec{q} \cdot \vec{r}_j}}{E} \right> \]

\[ = - g \mu_B \hat{S} \cdot \vec{f}(\vec{q}) \]

(from spin only part)

where \( \hat{S} \) is the total spin quantum number.

\( \vec{f}(\vec{q}) \) is the magnetic form factor which obeys \( \vec{f}(\vec{q}=0) = 1 \)

and in the limit \( \vec{f}(\vec{q}) = 0 \) and decreases over a distance given by the spatial extent of the spin density of the atom.

Collecting terms, we have

\[ \vec{B}(\vec{q}) = - 4\pi g \mu_B \hat{S}_\perp \vec{f}(\vec{q}) \]


and so, scattering length

\[ b = - \frac{m}{4\pi \hbar^2} \frac{1}{\sqrt{1 - \gamma^{2/3}}} \cdot (-4\pi g \mu_B \hat{S}_\perp \vec{f}(\vec{q})) \cdot \vec{q} \]

\[ = - 2\pi \alpha g \vec{f}(\vec{q}) \hat{\sigma} \cdot \hat{S}_\perp \]

which is the magnetic scattering length.

\( g \) - Landé splitting factor

\[ g = 1 \pm \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \]

\( J \) - total angular momentum

\( \sigma \) - depends on spin coordinates

\( b \) - depends on spin coordinates

\( \alpha \) - electron charge

\( \gamma \) - depends on spin coordinates

\( \vec{q} \) - wave vector

\( \vec{v} \) - velocity

\( \hbar \) - reduced Planck constant

\( m \) - mass

\( E \) - energy

\( \mu_B \) - Bohr magneton

\( \alpha \) - fine structure constant

\( \gamma \) - gyromagnetic ratio

\( \vec{r} \) - position vector

\( \vec{S} \) - spin vector

\( \vec{q} \) - scattering vector

\( \vec{f} \) - form factor

\( \hat{S}_\perp \) - perpendicular component of spin vector

\( \hat{\sigma} \) - Pauli spin matrix

\( \hat{\sigma} \cdot \hat{S} \) - matrix element

\( \hat{S}_\perp \cdot \hat{S}_\perp \) - matrix element

\( \vec{B} \) - magnetic field

\( \vec{B}(\vec{q}) \) - magnetic field at wave vector \( \vec{q} \)

\( e \) - electron charge

\( \vec{e} \) - electron velocity

\( \vec{E} \) - electric field

\( \vec{E}(\vec{q}) \) - electric field at wave vector \( \vec{q} \)
Q: How does the magnetic form factor vary with the sin θ?

The form factor and the electronic distribution are related as Fourier transforms

\[ U(r) = \frac{2\pi}{\lambda} \int_0^\infty q f(q) \sin(qr) \, dq \quad q = \frac{4\pi \sin\theta}{\lambda} \]

Example of MnF2 - measurements of paramagnetic scattering.
B(q) consists of 2 terms. 

What are they?

Orbital and spin angular momentum

The total magnetic field due to an electron of momentum \( p \) is

\[
\vec{B} = \vec{B}_s + \vec{B}_L
\]

Let's compare with nuclear scattering

For nuclear scattering, the matrix element is

\[
\begin{align*}
\langle j | e^{i\vec{Q}\cdot\vec{r}_j} | k \rangle \\
\text{where } b_j \text{ is constant b/c nuclear potential has short range}
\end{align*}
\]

For magnetic scattering, the matrix element is

\[
-\gamma e_q \gamma \int (\hat{q} \cdot \vec{S} : \vec{S}_L)
\]

The magnetic interactions have long-range and the interactions are not very simple.

For atoms that possess orbital and spin angular momentum interactions b/n the two occur.

\[
\Rightarrow \text{ Russell - Saunders coupling}
\]

Spin magnetic moment, \( \mu_s = \sqrt{S(S+1)} \frac{1}{m_B} \)

Orbital magnetic moment, \( \mu_L = \sqrt{L(L+1)} \frac{1}{m_B} \)

Paramagnetic moment, \( \mu_J = \gamma \sqrt{J(J+1)} \frac{1}{m_B} \)
Consider the case of unpolarized neutrons

**Q: Does magnetic scattering vanish?**

Unpolarized beams consists of particles with spin in an incoherent superposition. The interference effects involve the coherent wave function of a single neutron in whatever spin state it happens to be in.

\[
\frac{d\sigma}{d\Omega} = |\langle b_q \rangle|^2 = (2\pi q)^2 \int f(q)^2 \sum_{\lambda \lambda'} \rho_{\lambda \lambda'} \langle \lambda \sigma | \bar{\sigma} \cdot \vec{S}_{\lambda \lambda'} | \lambda' \sigma' \rangle \langle \lambda' \sigma' | \bar{\sigma} \cdot \vec{S}_{\lambda \lambda'} | \lambda \sigma' \rangle e^{iq(L_{\lambda \lambda'} - k_L)}
\]

\[
= (\cdots) \sum_{\ell \ell'} \rho_{\ell \ell'} \langle \vec{S}_{\ell \ell'} \rangle \cdot \langle \sigma | \sigma' \rangle \langle \sigma' | \sigma \rangle \langle \bar{\sigma} \cdot \vec{S}_{\ell \ell'} \rangle e^{i(q(L_{\ell \ell'} - k_L))}
\]

In the case of a magnetic structure with translational symmetry,

\[
\frac{d\sigma}{d\Omega} = (\cdots) \frac{|f(q)|^2 (2\pi)^3}{V_m} \sum_{\ell m} |F_{m \ell} (q)|^2 \delta(q - \ell_{2m}) \{ \text{for periodic magnetic structures} \}
\]

where \( F_{m \ell} (q) = \sum_{\nu} \langle S_{\nu \ell} \rangle e^{iq \cdot d\nu} \)

which is the magnetic vector structure factor.

\( \ell_m \)- reciprocal lattice vector of magnetic unit cell.

For AFM

the magnetic unit cell is doubled with real space period 2a so \( \ell_m = \frac{2\pi}{2a} = \frac{n}{a} \)
It is also the case that the nuclear-magnetic interference terms average to zero. The structure factor is given by the sum of 2 terms representing the nuclear and magnetic intensities.

From a diffraction pattern, need to identify $F_{\text{magn}}$ for a magnetic material and then use it to determine the magnetic structure.

**Example: Ferromagnetic material**

The magnetic peaks will occur at the same angular position as the nuclear peaks.

**Example: Antiferromagnetic material**

The magnetic unit cell is twice as long as the chemical unit cell in one direction. Extra reflections are observed.
In general, magnetic ions in a crystal have both spin and orbital angular momentum. Both contributions need to be taken into account.

The magnetic field due to the orbital motion of the electron is given by

\[ H = \frac{\mathbf{e} \times \mathbf{v}}{c} \]

As we saw in the previous lecture

Spin part \( \implies B_s = \nabla \times \hat{A} \)

Now, Orbital part \( \implies B_L = -\frac{2\hbar e}{m c^2} \mathbf{p} \times \mathbf{r} \)

The interaction potential in this case is given by

\[ V_L = \frac{1}{\hbar} \frac{\mathbf{p} \times \mathbf{r}}{r^2} \]

Need to evaluate the matrix element

\[ \langle q' \mid V_L \mid q \rangle = \frac{1}{\hbar} \int \frac{e^{-i\mathbf{Q} \cdot \mathbf{r}}}{\hbar} \sum_j \frac{1}{\hbar} \frac{i \mathbf{p}_j \times \mathbf{r}}{r^2} e^{i\mathbf{Q} \cdot \mathbf{r}} d^3r \]

\[ \text{if } \mathbf{r} = \mathbf{r}_j + \mathbf{R} \]

\[ = \sum_j \frac{1}{\hbar} e^{i \mathbf{Q} \cdot \mathbf{r}_j} \int e^{i\mathbf{Q} \cdot \mathbf{r}} \frac{\mathbf{p}_j \times \mathbf{r}}{r^2} d^3r \]

\[ = \sum_j \frac{4\pi i j}{\hbar \mathbf{Q}} e^{i\mathbf{Q} \cdot \mathbf{r}_j} \mathbf{p}_j \times \mathbf{Q} \]

The combined effect for the magnetization

\[ \mathbf{M}_L = \sum_j e^{i \mathbf{Q} \cdot \mathbf{r}_j} \left\{ \mathbf{Q} \times (\mathbf{s}_j \times \mathbf{Q}) + \frac{j}{\hbar \mathbf{Q}} \mathbf{p}_j \times \mathbf{Q} \right\} \]

(which is related to the magnetization)
We defined the spin magnetization operator as

\[ \hat{\mathbf{M}}(\mathbf{q}) = -\frac{q}{\hbar} \mu_B \langle q' | \sum_j \mathbf{e}^{i \mathbf{q} \cdot \mathbf{r}_j} \hat{\mathbf{s}}_j | q \rangle \]

The corresponding orbital term is given by (see Appendix H1 of Squires for details)

\[ M_{LL} = \frac{i}{\hbar \mathbf{Q}} \sum_j \mathbf{e}^{i \mathbf{q} \cdot \mathbf{r}_j} \hat{\mathbf{p}}_j \times \hat{\mathbf{Q}} = \frac{i}{\hbar} \hat{\mathbf{Q}} \times \hat{\mathbf{M}}_L(\mathbf{Q}) \times \mathbf{Q} \]

with \[ \hat{\mathbf{M}}_L(\mathbf{Q}) = \int e^{i \mathbf{Q} \cdot \mathbf{r}} \hat{\mathbf{M}}_L(\mathbf{r}) \, d^3 \mathbf{r} \]

Thus, the operator for the total magnetization is given by

\[ \hat{\mathbf{M}}(\mathbf{q}) = \hat{\mathbf{M}}_S(\mathbf{Q}) + \hat{\mathbf{M}}_L(\mathbf{Q}) \]

and \[ \mathbf{M} = \mathbf{M}_S + \mathbf{M}_L \]

Neutron magnetic scattering is due to the interaction of the magnetic dipole moment of the neutron with the magnetic field produced by the unpaired electrons.

The field is determined by the magnetic moments due to spin and orbital motion.
Rare earth ions have spin and orbital angular momentum
The calculation of the matrix elements of $\mathbf{M}$ is not so simple

LS coupling is considered and the angular momentum state is specified by quantum numbers $L$, $S$ and $J$

In this case the form factor, $f(Q)$, is modified to

$$f(Q) = \frac{g_s}{g} f_0 + \frac{g_L}{g} (f_0 + f_2)$$

$g = g_s + g_L$

$$g_s = 1 + \frac{S(S+1) - L(L+1)}{J(J+1)}$$

$$g_L = \frac{1}{2} + \frac{L(L+1) - S(S+1)}{2J(J+1)}$$

$$f_n = \frac{4\pi}{Q} \int_0^\infty f_n(Qr) S(r) r^2 dr$$

with $f_n(Qr)$ - Bessel function of order $n$

$S(r)$ - density of unpaired electrons averaged over all directions in space.
Let us first consider the case of paramagnetic systems

- the spins of the ions are randomly oriented; no internal field.
- no external field is applied
- spin flips don't change the energy of the system

Spin matrix element

\[ \langle S_i^x (0) S_i^x (t) \rangle = \langle S_i^x S_i^x \rangle \quad \text{time-independent} \]

For elastic scattering, \( t = \infty \), the cross-section is as follows

\[
\frac{d\sigma}{dQ} = r_0^2 \left\{ \frac{1}{2} g f(Q)^2 e^{-2W} \sum_{\alpha \beta} \frac{1}{2} \delta_{\alpha \beta} \langle S_i^\alpha \delta_{x\xi} \rangle N \sum_\xi e^{i \delta_{\xi \zeta}} \langle S_i^\xi S_i^\zeta \rangle \right\}
\]

\( \alpha, \beta \) correspond to \( x, y, z \)

For a paramagnet, there is no spatial correlation between spins

for \( l \neq l' \) \( \langle S_i^x S_i^y \rangle = 0 \)

for \( l = l' \) \( \langle S_i^x S_i^y \rangle = \frac{1}{3} \delta_{\alpha \beta} S(S+1) \)

\( \sigma \) is only non-zero for \( l = l' \) and \( \alpha = \beta \)

In this case

\[
\frac{d\sigma}{dQ} = \frac{2}{3} r_0^2 N \left\{ \frac{1}{2} g f(Q)^2 e^{-2W} S(S+1) \right\}
\]

For large spin values, paramagnetic scattering goes up.
There is no coherent scattering b/c paramagnetic ions are randomly oriented in space.
The only dependence to \( Q \) comes from the form factor

Magnetic scattering is diffuse

Q: what happens in the presence of a magnetic field?

An the presence of an \( \mathbf{H} \) field and at low enough temperatures, a paramagnet will be polarized
Part of the scattering that is proportional to the component of spin pointing in the same direction as the field will contribute to Bragg peaks; the rest will be part of the diffuse scattering.
Consider scattering from magnetic crystals where spins are oriented.

**Time independent spin correlations \( \Rightarrow \) elastic scattering**

\[
\frac{d\sigma}{d\Omega} = (\gamma r_0)^2 \left| \frac{g}{2} F(k) \right|^2 e^{-2W(k)} \sum_{\alpha\beta}(\delta_{\alpha\beta} - \hat{k}_\alpha \hat{k}_\beta) \sum_{l} e^{i\xi \tau} \langle S_\alpha^l \rangle \langle S_\beta \rangle
\]

1. **Ferromagnetic crystals**

Even with no field, spins are aligned in one direction within domains

- **Z-axis, direction of the spins.**
  
  then
  
  \[ \langle S_i^z \rangle = \langle S_i^\beta \rangle = 0 \]

\[ \langle S_i^z \rangle = \langle S_i \rangle = \text{magnetization of the domain proportional to this} \]

The cross-section becomes

\[
\frac{d\sigma}{d\Omega} = \frac{g_0}{2} N(2\pi)^3 \langle S^z \rangle \sum_x \left\{ \frac{1}{2} g f(x) \right\} \frac{1}{2} e^{-2W} \cdot \left\{ 1 - (\hat{\eta} \cdot \hat{\tau}) \right\} \langle S \rangle \langle \hat{S} \rangle
\]

\[ \hat{\eta} - \text{unit vector in the average direction of spins} \]

- a. Since \( d\sigma \) depends on \( \langle S^z \rangle \), it becomes very temperature dependent.
- b. Form factor falls off rapidly with \( |\tau| \)

**Q: what happens with external field?**

If a magnetic field is applied in the direction \( \tau \), the spin directions within domains will align so that \( \eta \) is along \(-\tau\). Then \( \tau \cdot \eta = -1 \).

In this case, magnetic scattering will disappear.
Simple cubic antiferromagnet

\[ \mathbf{F}^-(\mathbf{k}) = \frac{m_s}{2\mu_B} \mathbf{F}(\mathbf{k}) e^{-2W(\mathbf{k})} 8\sin\pi h \sin\pi k \sin\pi l \]

\[ \frac{d\sigma}{d\Omega} = N \left( \gamma r_0 \frac{m_s}{2\mu_B} \right)^2 e^{-2W(\mathbf{k})} |\mathbf{F}(\mathbf{k})|^2 \left( 1 - \kappa_z^2 \right) \frac{(2\pi)^3}{V} \sum_m \delta(\mathbf{k} - \mathbf{\bar{r}}_m) \]

No magnetic diffraction for \( \mathbf{k} \parallel \langle S \rangle \)

Remember:

\[ \mathbf{F}^-(\mathbf{k}) = \sum_d \frac{g_d}{2} F_d(\mathbf{k}) e^{-2W_d(\mathbf{k})} \langle S_d \rangle e^{i\mathbf{k} \cdot \mathbf{d}} \]

What happens at \( T_N \), the Neel transition temperature?

If the average \( \langle S^n \rangle \) is taken over all ions in a domain, it would be \( \phi \).

\( \langle S^n \rangle \) varies with temperature and falls to zero at \( T_N \)

\[ \text{At one magnetic Bragg peak} \]

\[ I(\Omega) \sim (\Omega - \Omega_0)/\kappa \]

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Other types of magnetic ordering exist besides FM and AFM.

Not so simple Heli-magnet: 
**MnO₂**

\[
\langle s_i^x \rangle = \langle S \rangle \cos (\vec{q} \cdot \vec{r}) \\
\langle s_i^y \rangle = \langle S \rangle \sin (\vec{q} \cdot \vec{r}) \\
\langle s_i^z \rangle = 0
\]
Spin waves are quantized with energy relative to the ground state,

\[ \hbar \omega \]

- **\( n \):** integer
- **\( \omega \):** angular frequency of the wave
- **\( \hbar \):** magnon

At finite temperatures, spins are not all aligned. The spin deviations are represented by traveling sinusoidal waves

\[ \Rightarrow \text{SPIN waves} \]

Neutron can be scattered by a process where \( n \rightarrow n \pm 1 \)

\[
S^\pm (\bar{k}, \omega) = \frac{S}{2} \{ \delta(\epsilon(\bar{k}) - \hbar \omega)(n(\hbar \omega) + 1) + \delta(\epsilon(\bar{k}) + \hbar \omega)n(\hbar \omega) \}
\]

**Magnon creation**

**Magnon destruction**

Conditions to satisfy:

\[
\frac{\hbar^2}{2m} (k^2 - k'^2) = \hbar \omega \quad \text{(spin wave energy)}
\]

\[
\bar{k} - \bar{k}' = \bar{\epsilon} + \bar{q}
\]

**Dispersion relation**

\[
\epsilon(\bar{k}) = 2S \left( J(0) - J(\bar{k}) \right)
\]

**Magnon occupation prob.**

\[
n(E) = \frac{1}{\exp\left(\frac{E}{k_B T}\right) - 1}
\]

**Magnon frequencies**

The spin Hamiltonian that can describe this system is

Gadolinium

\[
\begin{align*}
\begin{array}{c}
\text{Gadolinium} \\
\text{dispersion curve}
\end{array}
\end{align*}
\]
\[ H = -\sum_{\ell \ell'} J(\ell - \ell') \hat{s}_\ell \cdot \hat{s}_{\ell'} \]
Spin waves in an antiferromagnet

\[
S^\perp(\vec{k}, \omega) = \frac{S}{2} J \left( 1 - \frac{1}{z} \sum_d e^{i\vec{k}\cdot\vec{d}} \right) \frac{\varepsilon(\vec{k})}{\varepsilon(\vec{k})} \\
\times \left\{ \delta(\varepsilon(\vec{k}) - \hbar \omega) \left( n(\hbar \omega) + 1 \right) + \delta(\varepsilon(\vec{k}) + \hbar \omega) n(\hbar \omega) \right\}
\]

Dispersion relation

\[
\varepsilon(\vec{k}) = 2S \sqrt{J(0)^2 - J(\vec{k})^2}
\]
$S^{\alpha\beta}(\kappa, \omega)$ and the magnetic susceptibility

$$S^{\alpha\beta}(\kappa, \omega) = \int dt e^{-i\omega t} \frac{1}{N} \sum_{ll'} e^{i\kappa(n_{l'}-n_{l})} \langle S^\alpha_l(0)S^\beta_{l'}(t) \rangle$$

Compare to the generalized susceptibility

$$\chi^{\alpha\beta}_q(\omega) = \left(\frac{g \mu_B}{N}\right)^2 \int dt e^{-i\omega t} \sum_{ll'} e^{i\kappa(n_{l'}-n_{l})} \langle [S^\alpha_l(t), S^\beta_{l'}(0)] \rangle$$

They are related by the fluctuation dissipation theorem

$$S^{\alpha\beta}(q, \omega) = \frac{\text{Im}\{\chi^{\alpha\beta}_q(\omega)\}}{\pi(g \mu_B)^2} \frac{1}{1 - e^{-\beta\hbar\omega}}$$

We convert inelastic scattering data to $\chi^{\alpha\beta}_q(\omega)$ to

- Compare with bulk susceptibility data
- Isolate non-trivial temperature dependence
- Compare with theories
Magnetic Properties of the Neutron

- The neutron has a magnetic moment of $-9.649 \times 10^{-27}$ JT$^{-1}$
  \[ \mu_n = -\gamma \mu_N \vec{\sigma} \]

  where $\mu_N = \frac{e\hbar}{2m_p}$ is the nuclear magneton,
  
  $m_p = $ proton mass, $e = $ proton charge and $\gamma = 1.913$
  
  $\vec{\sigma}$ is the Pauli spin operator for the neutron. Its eigenvalues are $\pm 1$

- Note that the neutron’s spin and magnetic moment are antiparallel
- Because of its magnetic moment, the neutron feels a potential given by:
  \[ V_m(\vec{r}) = -\vec{\mu}_n \cdot \vec{B}(\vec{r}) \text{ where } \vec{B}(\vec{r}) = \mu_0 \mu H(\vec{r}) = \mu_0 [\vec{H}(\vec{r}) + \vec{M}(\vec{r})] \]

- Thus the neutron senses the distribution of magnetization in a material
- Homework problems: What is the Zeeman energy in meV of a neutron in a 1 Tesla field? At what temperature is the Boltzmann energy equal to this Zeeman energy? What is the effective scattering length of a “point” magnetic moment of one Bohr magneton?
Magnetic Scattering of the Neutron

- For nuclear scattering, the matrix element that appears in the expression for the scattering cross section (see last lecture) is: \( \sum_j b_j e^{i \vec{Q} \cdot \vec{r}_j} \)

- The equivalent matrix element for magnetic scattering is:
  \[ n_0 \frac{1}{2 \mu_B} \vec{\sigma} \cdot \vec{M}_\perp(\vec{Q}) \]
  where \( \mu_B = \frac{e\hbar}{2m_e} \) is the Bohr magneton \( (9.27 \times 10^{-24} \text{ JT}^{-1}) \)

  and \( n_0 = \frac{\mu_0 e^2}{4\pi m_e} \) is classical radius of the electron \( (2.818 \times 10^{-5} \text{ nm}) \)

- Here \( \vec{M}_\perp(\vec{Q}) \) is the component of the Fourier transform of the magnetization that is perpendicular to the scattering vector \( \vec{Q} \). This form arises directly from the dipolar nature of the magnetic interaction.

- Unlike the neutron-nucleus interaction, the magnetic interaction of the neutron with a scattering system specifically depends on neutron spin
Derivation

The \( \mathbf{B} \) field at distance \( \mathbf{R} \) from a magnetic moment \( \mathbf{M} \) is

\[
\frac{\mu_0}{4\pi} \nabla \Lambda \left( \frac{\mathbf{M} \cdot \mathbf{R}}{R^3} \right) = -\frac{\mu_0}{4\pi} \nabla \Lambda \left( \mathbf{M} \nabla \left( \frac{1}{R} \right) \right)
\]

Since

\[
\int_0^1 \frac{1}{q^2} \exp(iq \cdot \mathbf{R}) dq = 2\pi \int_0^1 dq \int_0^\infty dq \exp(iqR\cos\theta)d\cos\theta = 4\pi \int_0^\infty dq \frac{\sin(qR)}{qR} = \frac{2\pi^2}{R}
\]

\[
\nabla \Lambda \left( \frac{\mathbf{M} \cdot \mathbf{R}}{R^3} \right) = -\frac{1}{2\pi^2} \int \frac{1}{q^2} \nabla \Lambda \left( \mathbf{M} \nabla \left( \exp i\mathbf{q} \cdot \mathbf{R} \right) \right) dq
\]

But \( \mathbf{M} \nabla \left( \exp i\mathbf{q} \cdot \mathbf{R} \right) = i\mathbf{M} \mathbf{q} \exp i\mathbf{q} \cdot \mathbf{R} \) and \( \nabla \Lambda \mathbf{M} \mathbf{q} \exp i\mathbf{q} \cdot \mathbf{R} = i\mathbf{q} \Lambda \mathbf{M} \mathbf{q} \exp i\mathbf{q} \cdot \mathbf{R} \)

so

\[
\nabla \Lambda \left( \frac{\mathbf{M} \cdot \mathbf{R}}{R^3} \right) = \frac{1}{2\pi^2} \int \frac{1}{q^2} \nabla \Lambda \left( \mathbf{M} \mathbf{q} \right) \left( \exp i\mathbf{q} \cdot \mathbf{R} \right) dq = \frac{1}{2\pi^2} \int M_{\perp}(\mathbf{q}) \left( \exp i\mathbf{q} \cdot \mathbf{R} \right) dq
\]