9.1. Consider a volume $V$ bounded by a surface $S$ filled with a magnetization $\mathbf{M}(\mathbf{r}')$ that depends on the position $\mathbf{r}'$. The vector potential $\mathbf{A}$ produced by a magnetization $\mathbf{M}(\mathbf{r})$ is given by

$$\mathbf{A}(\mathbf{r}) = \int d^3\mathbf{r}' \frac{\mathbf{M}(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}.$$ 

(a) Show that $\nabla' \left( \frac{1}{|\mathbf{r} - \mathbf{r}'|} \right) = \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3 \mathbf{r}}$.

(b) Use this result together with the divergence theorem to show that $\mathbf{A}(\mathbf{r})$ can be written as

$$\mathbf{A}(\mathbf{r}) = \int_V d^3\mathbf{r}' \frac{\nabla' \times \mathbf{M}(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + \oint_S dS' \frac{\mathbf{M}(\mathbf{r}') \times \mathbf{n}'}{|\mathbf{r} - \mathbf{r}'|},$$

where $\mathbf{n}$ is a unit vector outward normal to the surface $S$. The volume integration is carried out over the volume $V$ of the magnetized material. The surface integral is carried out over the surface bounding the magnetized object.

9.2. Demonstrate for yourself that Table 9.1 is correct by placing $\uparrow$ or $\downarrow$ arrows according to Hund’s rules as shown below for Cr of atomic configuration $(3d)^5 (4s)^1$.

<table>
<thead>
<tr>
<th>$l_z$</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>-1</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>3d-shell</td>
<td>$\uparrow\uparrow\uparrow\uparrow\uparrow$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4s-shell</td>
<td></td>
<td>$\uparrow$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Clearly $S = \frac{1}{2} \times 6 = 3$, $L = 0$, $J = L + S = 3$, and

$$g = \frac{3}{2} + \frac{1}{2} \frac{3(3 + 1) - 0(0 + 1)}{3(3 + 1)} = 2.$$ 

Therefore, the spectroscopic notation of Cr is $^7S_3$.

Use Hund’s rules (even though they might not be appropriate for every case) to make a similar table for $Y^{39}$, $Nb^{41}$, $Tc^{43}$, $La^{57}$, $Dy^{66}$, $W^{74}$, and $Am^{95}$. 