5.1. At $E(X) = 0.25 \frac{h^2}{2ma^2}$ there are two degenerate bands. At $E(X) = 1.25 \frac{h^2}{2ma^2}$ there are four. Determine the linear combinations of degenerate states at these points belonging to IR’s of $G_X$. Do the same for $E(\Gamma) = 1 \left[ \frac{h^2}{2ma^2} \right]$ and $2 \left[ \frac{h^2}{2ma^2} \right]$

5.2. Make a table, with values of $h_1, h_2, h_3$, of the resulting $l_1, l_2, l_3$ and $E(\Gamma)$, $E(X)$, $E(L)$ for all bands with $E(\Gamma) \leq 4$ for an FCC lattice.
5.7. A two dimensional rectangular lattice has a reciprocal lattice whose primitive translations, including the $2\pi$, are $b_1 = \frac{2\pi}{a} \hat{x}$ and $b_2 = \frac{2\pi}{a} \frac{1}{\sqrt{2}} \hat{y}$.

(a) List the operations belonging to $G_\Gamma$.
(b) Do the same for $G_X$ and $G_\Delta$.
(c) For the empty lattice the wave functions and energies can be written

$$\psi_t(k, r) = e^{i(k+K_t) \cdot r}$$

and

$$E_t(k) = \frac{\hbar^2}{2m} (k + K_t)^2.$$ Here, $K_t = l_1 b_1 + l_2 b_2$, and $l_1$ and $l_2$ are integers. Tabulate the energies at $\Gamma$ and at $X$ for $(l_1, l_2) = (0,0), (0, \pm 1), (-1,0), (1,0)$, and $(-1, \pm 1)$.

(d) Sketch (straight lines are OK) $E$ vs. $k$ along the line $\Delta$ (going from $\Gamma$ to $X$) for these bands.

(e) Two degenerate bands at the point $E(\Gamma) = 0.5$ connect to $E(X) = 0.75$. Write down the wave functions for an arbitrary value of $k_x$ for these two bands.

(f) From these wave functions, construct the linear combinations belonging to irreducible representations of $G_\Delta$. 