One note about the test, do not worry about numbers too much, if you have concept method and formula correct, wrong numerical answer will at most cost you 1 point out of 10.

Chapter 5 Homework

Prob 1

The number of moles per cm^3 is $81 \times 10^{-3}/3 = 27 \times 10^{-3}$, so that the concentration is 16×10^{21} atoms cm^{-3}. The mass of an atom of He^3 is $(3.017)\ (1.661) \times 10^{-24} = 5.01 \times 10^{-24}$ g. Thus $\varepsilon_F \simeq [(1.1 \times 10^{-54})/10^{-23}][(30)(16) \times 10^{21}]^{2/3} \approx 7 \times 10^{-16}$ erg, or $T_F \approx 5K$.

Prob 2

The energy eigenvalues are $\varepsilon_k = \frac{\hbar^2}{2m}k^2$. The mean value over the volume of a sphere in k space is

$$< \varepsilon > = \frac{\hbar^2}{2m}\frac{\int k^2 dk \cdot k^2}{\int k^2 dk} = \frac{3}{5} \cdot \frac{\hbar^2}{2m} k_F{}^2 = \frac{3}{5}\varepsilon_F.$$

The total energy of N electrons is

$$U_0 = N \cdot \frac{3}{5}\varepsilon_F.$$

Prob 3

(a) $n = N/V = (m/M)/V = \rho_m/(63.546*10^{-3}/6.02*10^{-23}) = 8.464 \times 10^{28}$ Assuming 1 Copper atom contributes 1 electron. This problem is mostly plugging in numbers once you have the correct equation. Also the mass in the equations refer to effective mass of eletrons.

(b) $\tau = m/(ne^2\rho)$

(c) $E_F = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3}$

(d) $v_F = \frac{\hbar}{m}(3\pi^2 n)$

(e) $l_F = v_F\tau$ $l_F = v_F\tau$

Chapter 7 Homework

Prob 1

(a) The dispersion relation is $\omega = \omega_m |\sin \frac{1}{2} Ka|$. We solve this for K to obtain $K = (2/a)\sin^{-1}(\omega/\omega_m)$, whence $dK/d\omega = (2/a)(\omega_m{}^2 - \omega^2)^{-1/2}$ and, from (15), $D(\omega)$ $= (2L/\pi a)(\omega_m{}^2 - \omega^2)^{-1/2}$. This is singular at $\omega = \omega_m$. (b) The volume of a sphere of radius K in Fourier space is $\Omega = 4\pi K^3/3 = (4\pi/3)[(\omega_0 - \omega)/A]^{3/2}$, and the density of orbitals near ω_0 is $D(\omega) = (L/2\pi)^3 |d\Omega/d\omega| = (L/2\pi)^3 (2\pi/A^{3/2})(\omega_0 - \omega)^{1/2}$, provided $\omega < \omega_0$. It is apparent that $D(\omega)$ vanishes for ω above the minimum ω_0.

Prob 2

(a) The motion is constrained to each layer and is therefore essentially two-dimensional. Consider one plane of area A. There is one allowed value of K per area $(2\pi/L)^2$ in K space, or $(L/2\pi)^2 = A/4\pi^2$ allowed values of K per unit area of K space. The total number of modes with wavevector less than K is, with $\omega = vK$,

$$N = (A/4\pi^2)(\pi K^2) = A\omega^2/4\pi v^2 .$$

The density of modes of each polarization type is $D(\omega) = dN/d\omega = A\omega/2\pi v^2$. The thermal average phonon energy for the two polarization types is, for each layer,

$$U = 2\int_0^{\omega_D} D(\omega)\ n(\omega,\tau)\ \hbar\omega\ d\omega = 2\int_0^{\omega_D} \frac{A\omega}{2\pi v^2}\ \frac{\hbar\omega}{\exp(h\omega/\tau) - 1} d\omega,$$

where ω_D is defined by $N = \int_D^{\omega_D} D(\omega)\ d\omega$. In the regime $\hbar\omega_D >> \tau$, we have

$$U \cong \frac{2A\tau^3}{2\pi v^2 \hbar^2} \int_0^\infty \frac{x^2}{e^x - 1}\ dx.$$

Thus the heat capacity $C = k_B\ \partial U/\partial\tau \propto T^2$.

(b) If the layers are weakly bound together, the system behaves as a linear structure with each plane as a vibrating unit. By induction from the results for 2 and 3 dimensions, we expect $C \propto T$. But this only holds at extremely low temperatures such that $\tau << \hbar\omega_D \approx \hbar v N_{layer}/L$, where N_{layer}/L is the number of layers per unit length.

Prob 3

$$D(\omega) = \frac{dN}{d\omega} = \frac{dN}{dk}\frac{dk}{d\omega}$$

$$1D - N = \frac{L}{2\pi} 2k$$

$$2D - N = (\frac{L}{2\pi})^2 \pi k^2$$

$$3D - N = (\frac{L}{2\pi})^3 \frac{4}{3}\pi k^3$$

$$U = \int_0^{\omega_D} D(\omega) < n(\omega) > \hbar\omega d\omega$$

$$N = \int_0^{\omega_D} D(\omega) < n(\omega) > d\omega$$

ω_D can be found from last equation. It will not depend on temperature.

In Kittel the case for 3D is done. What is different in 1D and 2D is that $D(\omega)$ is different.

1D $D(\omega) \propto 1$

2D $D(\omega) \propto \omega$

3D $D(\omega) \propto \omega^2$

Following what was done in Kittel you will see, following a simple pattern that

1D $U \propto T^2 \int_0^{x_D} dx \frac{x}{e^x - 1}$

2D $U \propto T^3 \int_0^{x_D} dx \frac{x^2}{e^x - 1}$

3D $U \propto T^4 \int_0^{x_D} dx \frac{x^3}{e^x - 1}$

Thus in the low temperature limit the integrals are constants and since $C = \frac{dU}{dT}$,

$C \propto T, T^2, T^3$ for 1D, 2D, 3D

In high temperature limit, x_D is small thus use $e^x - 1 \approx x + x^2/2 + x^3/6$ you will see $U \propto T$ in all cases. Thus in high temperature limit specific heat is constant. The exact expressions can be derived following Kittel and what I have here.

Chapter 8 Homework
Prob 1

a. $\mathrm{E_d} = 13.60\ \mathrm{eV} \times \frac{\mathrm{m}^*}{\mathrm{m}} \times \frac{1}{\varepsilon^2} \simeq 6.3 \times 10^{-4}\ \mathrm{eV}$

b. $\mathrm{r} = \mathrm{a_H} \times \varepsilon \times \frac{\mathrm{m}}{\mathrm{m}^*} \simeq 6 \times 10^{-6} \mathrm{cm}$

c. Overlap will be significant at a concentration

$$\mathrm{N} = \frac{1}{\frac{4\pi}{3} \mathrm{r}^3} \approx 10^{15} \mathrm{atoms\ \ cm^{-3}}$$

Prob 2

The velocity components are $\mathrm{v_x} = \hbar \mathrm{k_x} / \mathrm{m_t}$; $\mathrm{v_y} = \hbar \mathrm{k_y} / \mathrm{m_t}$; $\mathrm{v_z} = \hbar \mathrm{k_z} / \mathrm{m_\ell}$. The equation of motion in k space is $\hbar\ \mathrm{dk/dt} = -(\mathrm{e/c})\, \mathrm{v} \times \mathrm{B}$. Let B lie parallel to the k_x axis; then

$dk_x / dt = 0;\ dk_y / dt = -\omega_\ell k_z;\ \omega_\ell \equiv eB/m_\ell c;\ dk_z / dt = \omega_t k_y;\ \omega_t \equiv eB/m_t c$. We differentiate with respect to time to obtain $d^2k_y / dt^2 = -\omega_\ell\ dk_z / dt$; on substitution for dk_z/dt we have $d^2k_y / dt^2 + \omega_\ell \omega_t k_y = 0$, the equation of motion of a simple harmonic oscillator of natural frequency

$$\omega_0 = (\omega_\ell \omega_t)^{1/2} = eB/(m_\ell m_t)^{1/2} c.$$

Prob 3

Simply plug in numbers to Kittel Chapter 8 Eq(45,47) to get intrinsic concentration and chemical potential(also known as Fermi level)

Prob 4

(a) $\tau = \mu m / e$, μ is the mobility

(b) $\sigma = ne\mu_e + pe\mu_h$

(c) $\sigma \propto T^{3/2} e^{-\frac{E_g}{2k_B T}}$

$$Log\sigma = Cons\tan t + \frac{3}{2} LogT - \frac{E_g}{2k_B T}$$