Problems for chapter 12, due April 23, 2012

1. ***Magnon dispersion relation.*** Derive the magnon dispersion relation (24) for a spin S on a simple cubic lattice, $z = 6$. Hint: Show first that (18a) is replaced by

$$dS_\rho^x/dt = (2JS/\hbar)(6S_\rho^y - \sum_\delta S_{\rho+\delta}^y) \ ,$$

where the central atom is at $\boldsymbol{\rho}$ and the six nearest neighbors are connected to it by six vectors $\boldsymbol{\delta}$. Look for solutions of the equations for dS_ρ^x/dt and dS_ρ^y/dt of the form $\exp(i\mathbf{k}\cdot\boldsymbol{\rho} - i\omega t)$.

2. ***Heat capacity of magnons.*** Use the approximate magnon dispersion relation $\omega = Ak^2$ to find the leading term in the heat capacity of a three-dimensional ferromagnet at low temperatures $k_BT \ll J$. The result is $0.113\ k_B(k_BT/\hbar A)^{3/2}$, per unit volume. The zeta function that enters the result may be estimated numerically; it is tabulated in Jahnke-Emde.

3. ***Néel temperature.*** Taking the effective fields on the two-sublattice model of an antiferromagnetic as

$$B_A = B_a - \mu M_B - \epsilon M_A \ ; \quad B_B = B_a - \mu M_A - \epsilon M_B \ ,$$

show that

$$\frac{\theta}{T_N} = \frac{\mu + \epsilon}{\mu - \epsilon} \ .$$

4.

Discuss why spin waves are more favorable as modes of excitation than local spin modes, particularly at low temperatures.