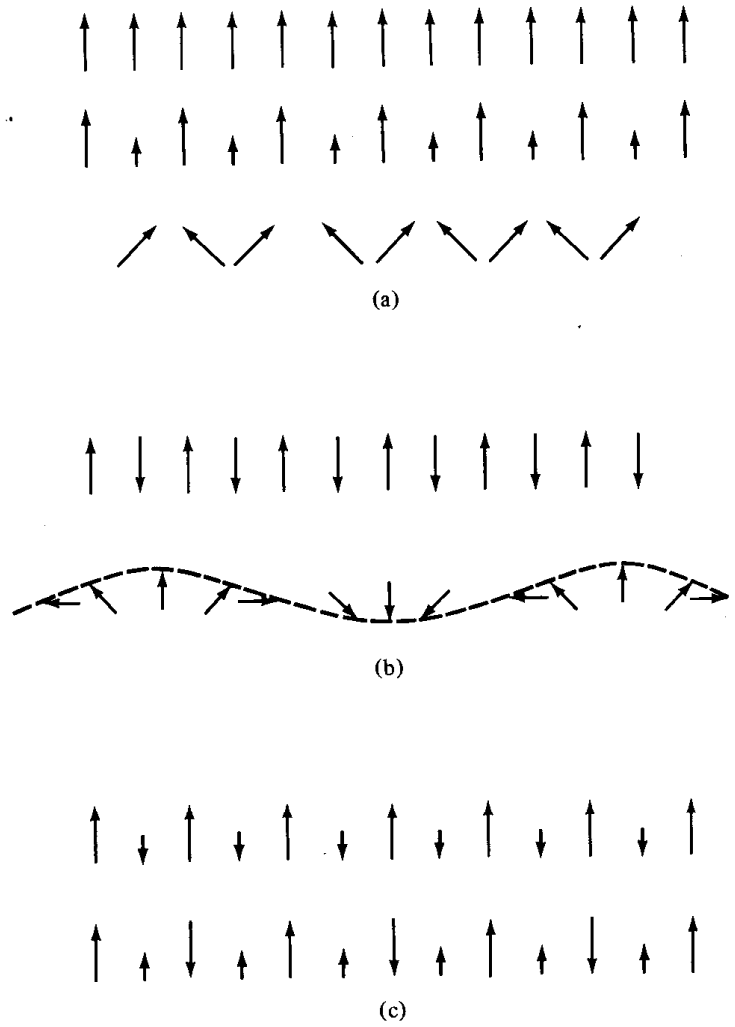
Chapter 12 Ferromagnetism and Antiferromagnetism



A ferromagnet has a spontaneous magnetic moment in zero applied magnetic field.

Antiferromagnet

Ferrimagnetic ordering

Assume each magnetic atom experience a field proportional to the magnetization: $\vec{B}_E = \lambda \vec{M}$, where λ is a constant, independent of temperature. The Curie temperature T_C is the temperature above which the spontaneous magnetization vanishes. When a field B_a is applied, one has $M = \chi_p (B_a + B_E)$. Note that this is only true if the fractional alignment is small. The Curie law states $\chi_p = C/T$, where C is the Curie constant.

$$\chi = \frac{M}{B_a} = \frac{C}{(T - C\lambda)} = \frac{C}{(T - T_C)}; T_C = C\lambda$$ is the Curie-Weiss law.

Detailed calculations predict

$$\chi \propto \frac{1}{(T - T_C)^{1.33}}$$

Assume the exchange coupling between spin $\vec{S}_i$ and $\vec{S}_j$ is J, Heisenberg model has the energy of the system,

$$U = -2J\vec{S}_i \cdot \vec{S}_j$$

Since U is the scalar product of the vector spin operators $\vec{S}_i$ and $\vec{S}_j$, it will favor parallel spins if J is positive and antiparallel if J is negative.

For a system with many spins, the total spin Hamiltonian is simply that for the two-spin case, summed over all pairs of ions:

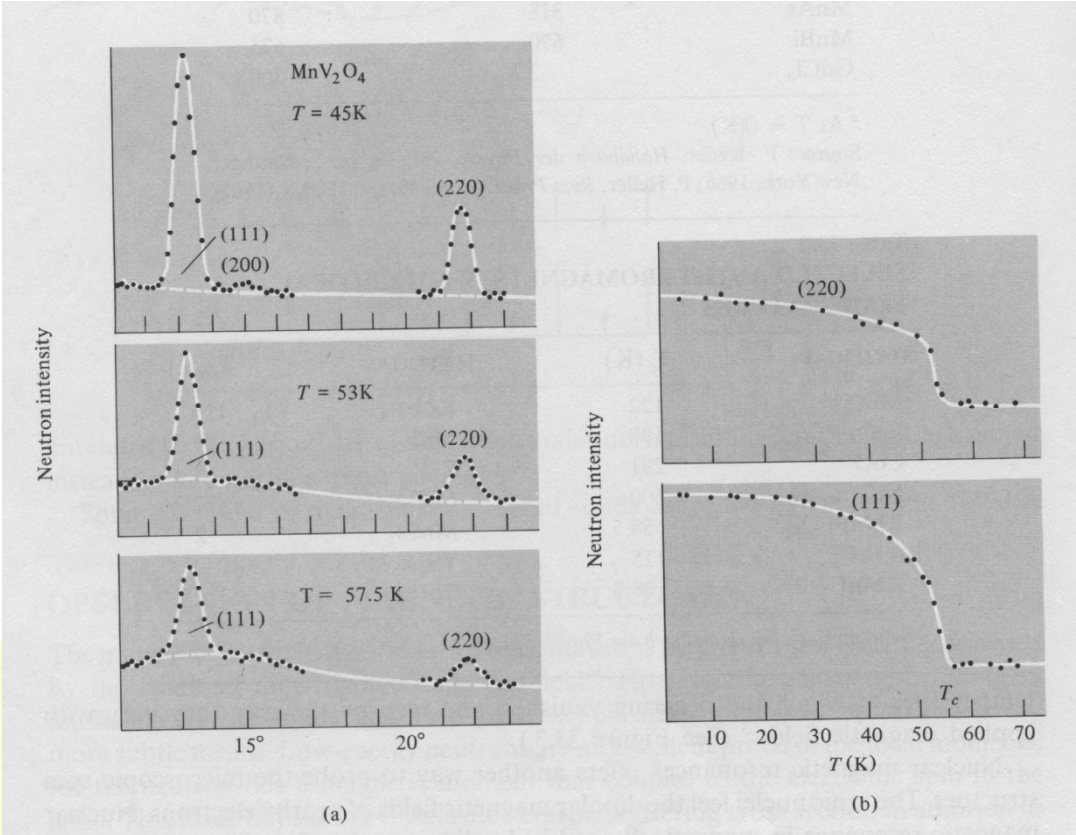
$$H^{spin} = -\sum_{p=1}^{N} 2J\vec{S}_p \cdot \vec{S}_{p+1} \qquad \text{If}$$

1. All magnetic ions are far enough apart that the overlap of their electronic wave functions is very small.

2. When the angular momentum of each ion contains an orbital as well as a spin part, the coupling in the spin Hamiltonian may depend on the absolute as well as the relative spin orientations.

The approximate connection between the exchange integral J and the Curie temperature T_C. For the atom under consideration has z nearest neighbors, each connected with the central atom by the exchange coupling J. For more distance neighbors we take J as zero

$$J = \frac{3k_B T_C}{2zS(S+1)}.$$



Thermodynamic properties at the onset of magnetic ordering

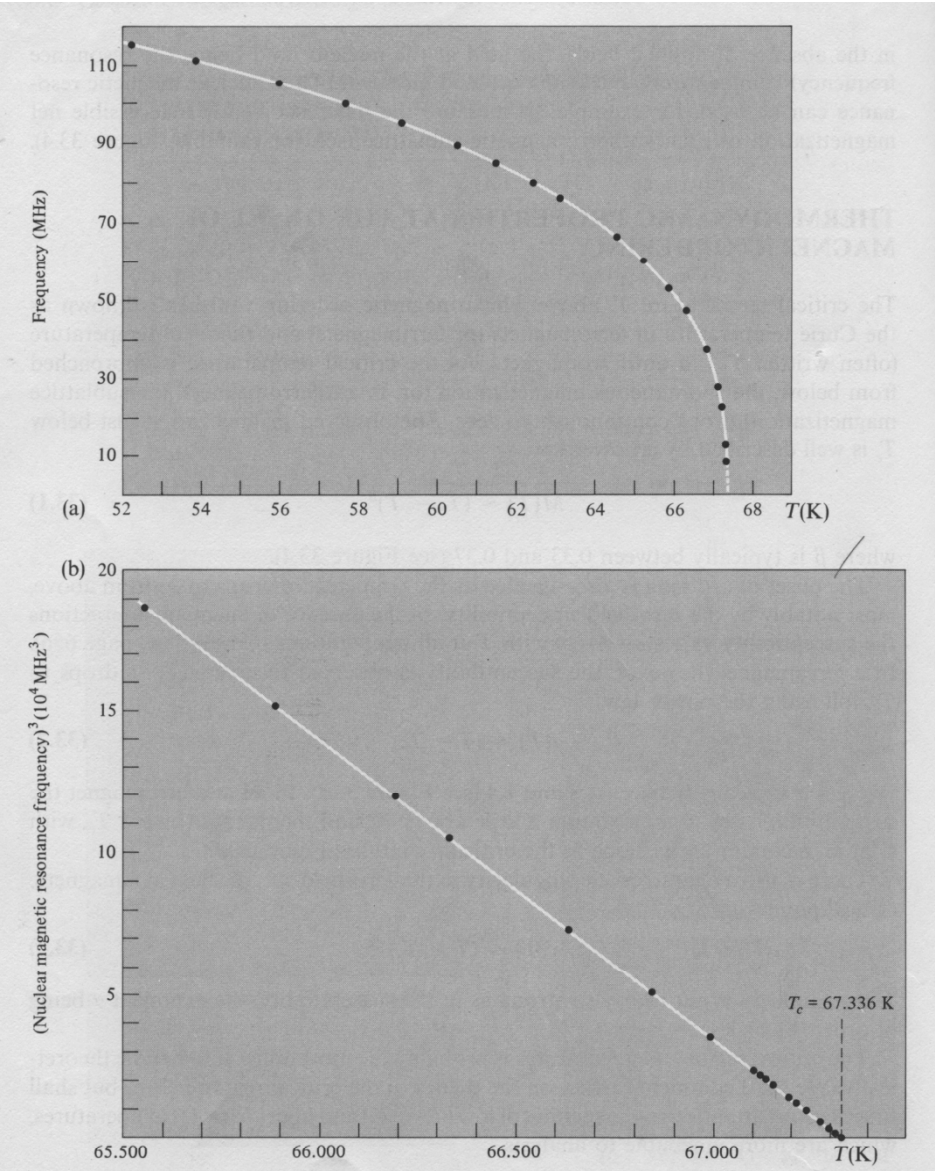
As the critical temperature is approached from below, the spontaneous magnetization drops continuously to zero, the observed magnetization just below T_c is given by:

$$M(T) \propto (T_c - T)^{\beta},$$

where β is typically between 0.33 and 0.37.

The onset of ordering is also signaled as the temperature drops to T_c from above, most notably by the zero-field susceptibility. In the absence of magnetic interactions the susceptibility varies inversely with T at all temperatures. In a ferromagnet, it diverges as T drops to T_c following the power law:

$$\chi(T) \propto (T_c - T)^{-\gamma}$$


Frequency (MHz)
110
90
70
50
30
10
(a)
52
54
56
58
60
62
64
66
68
T(K)
(b)
20
15
10
5
(Nuclear magnetic resonance frequency)^3 (10^4 MHz^3)
T_c = 67.336 K
65.500
66.000
66.500
67.000
T(K)

Magnons

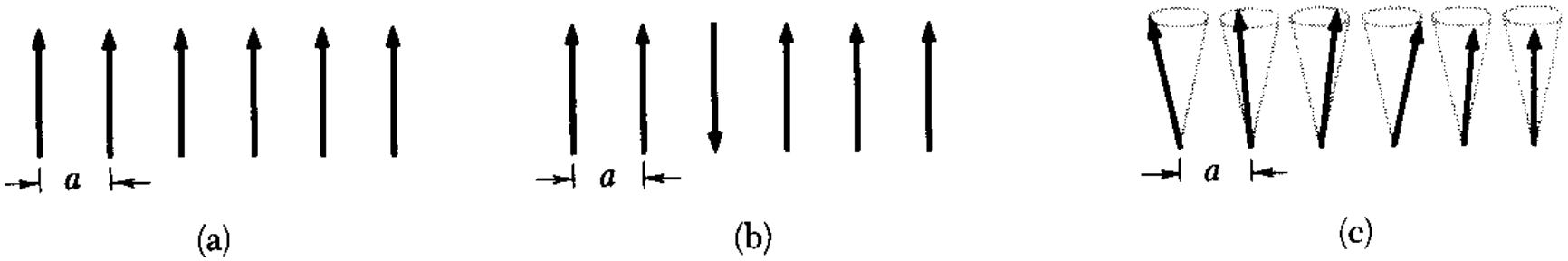
A magnon is a quantized spin wave. Consider N spins each of magnitude S on a line or a ring, with nearest neighbor spins coupled by the Heisenerg interaction:

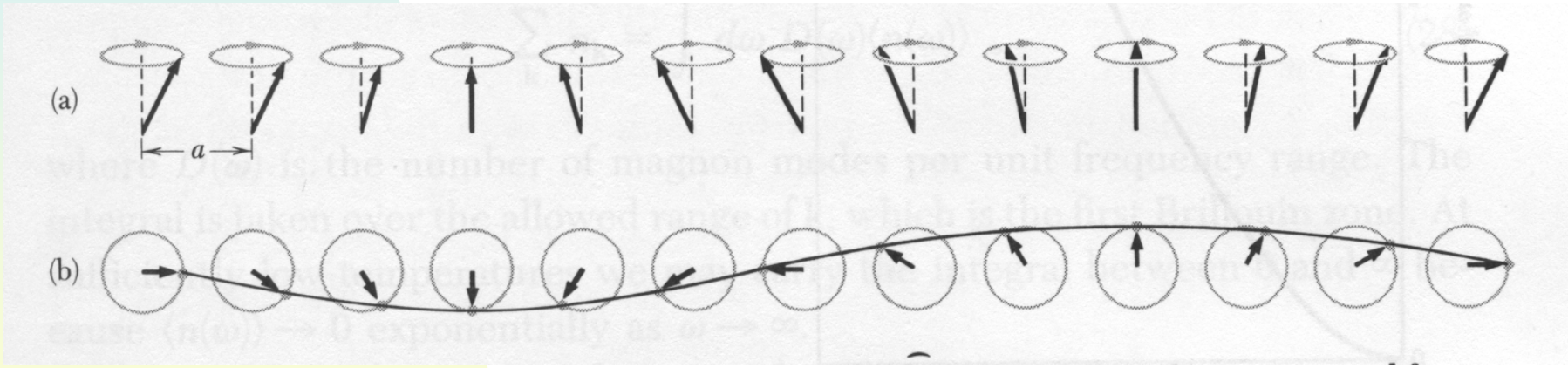
$$U = -2J\sum_{p=1}^{N}\vec{S}_p \cdot \vec{S}_{p+1}.$$

If $\vec{S}_p$ are classical vectors, then in the ground state $\vec{S}_p \cdot \vec{S}_{p+1} = S^2$ and the exchange energy of the system is $U_0 = -2NJS^2$.

For an excited state with one particular spin reversed, the total energy increased by $8JS^2$, so that $U_1 = U_0 + 8JS^2$.

For the pth spin, we have $-2J\vec{S}_p \cdot (\vec{S}_{p-1} + \vec{S}_{p+1})$.


a
(a)
a
(b)
a
(c)


(a)
a
(b)

The magnetic moment at site p as $\vec{\mu}_{\mathrm{p}} = -g\mu_B \vec{S}_p$.

we have $-\vec{\mu}_p \cdot [(-2J/g\mu_B)(\vec{S}_{p-1} + \vec{S}_{p+1})] = -\vec{\mu}_p \cdot \vec{B}_p$.

From mechanics the rate of change of the angular momentum $\hbar\vec{S}_p$ is equal to the torque $\vec{\mu}_p \times \vec{B}_p$ or $\hbar d\vec{S}_p / dt = \vec{\mu}_p \times \vec{B}_p$

$$d\vec{S}_p / dt = (-g\mu_B / \hbar)\vec{S}_p \times \vec{B}_p = (2J/\hbar)(\vec{S}_p \times \vec{S}_{p-1} + \vec{S}_p \times \vec{S}_{p+1})$$

In Cartesian components:

$$dS_p^x / dt = (2J/\hbar)[S_p^y(S_{p-1}^z + S_{p+1}^z) - S_p^z(S_{p-1}^y + S_{p+1}^y)]$$

If the amplitude of the excitations is small, $S_p^z = S$

$$dS_p^x / dt = (2JS/\hbar)[2S_p^y - (S_{p-1}^y + S_{p+1}^y)];$$

$$dS_p^y / dt = (2JS/\hbar)[2S_p^x - (S_{p-1}^x + S_{p+1}^x)];$$

$$dS_p^z / dt = 0.$$

Assuming traveling wave solutions of the form

$$S_p^x = u\exp[i(pka-\omega t)]; S_p^y = v\exp[i(pka-\omega t)].$$

$$-i\omega u = (2JS/\hbar)[2-(e^{-ika}+e^{ika})]v = (4JS/\hbar)(1-\cos ka)v;$$

$$-i\omega v = -(2JS/\hbar)[2-(e^{-ika}+e^{ika})]u = -(4JS/\hbar)(1-\cos ka)u;$$

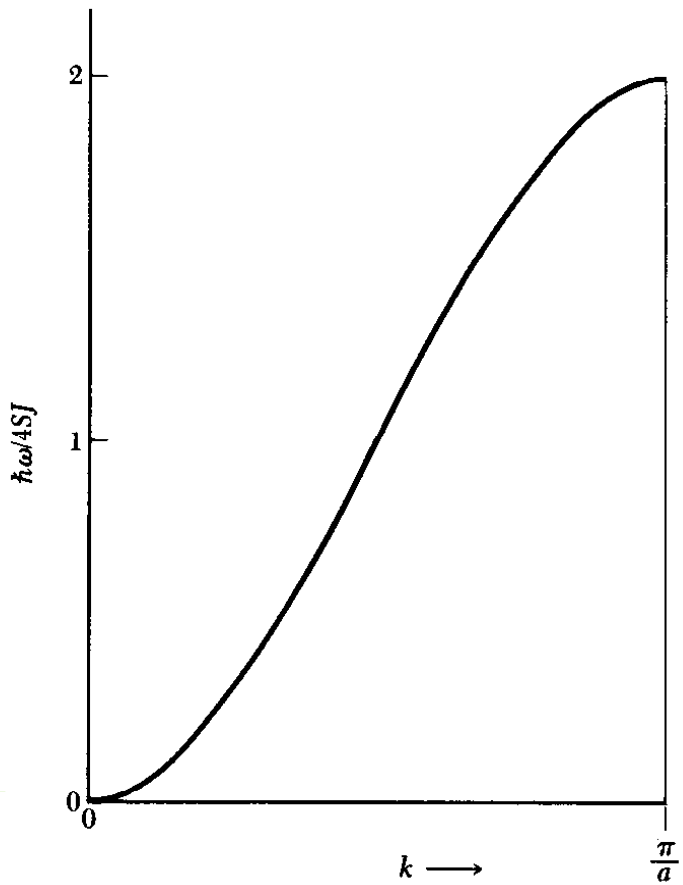
$$\begin{vmatrix} i\omega & (4JS/\hbar)(1-\cos ka) \\ -(4JS/\hbar)(1-\cos ka) & i\omega \end{vmatrix} = 0$$

Therefore $\hbar\omega = 4JS(1-\cos ka).$

At low momentum transfer and long wave lengths, we have

$$(1-\cos ka) \approx \frac{1}{2}(ka)^2$$

$$\hbar\omega \approx (2JSa^2)k^2.$$

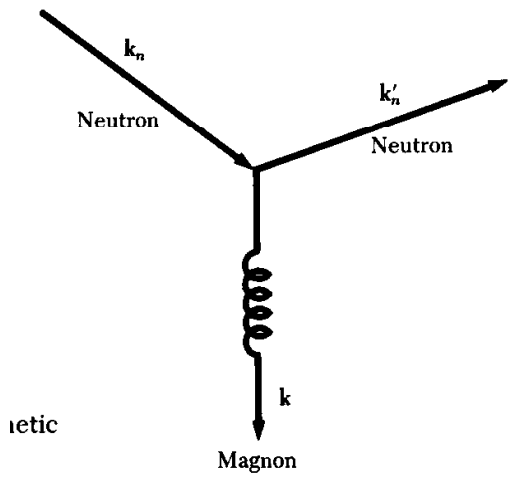


Quantization of spin waves

The energy of a mode of frequency ω_k with n_k magnons is given by

$$\varepsilon_{\mathrm{k}} = (n_k + \frac{1}{2})\hbar\omega_k$$

Neutron magnetic scattering



A neutron can be scattered elastically or inelastically by the magnetic structure. For inelastic neutron scattering, a neutron creates or annihilates a magnon. If the incident neutron has wavevector k_n and is scattered to $k_n^{'}$ with the creation of a magnon of wavevector k.

Conservation of energy

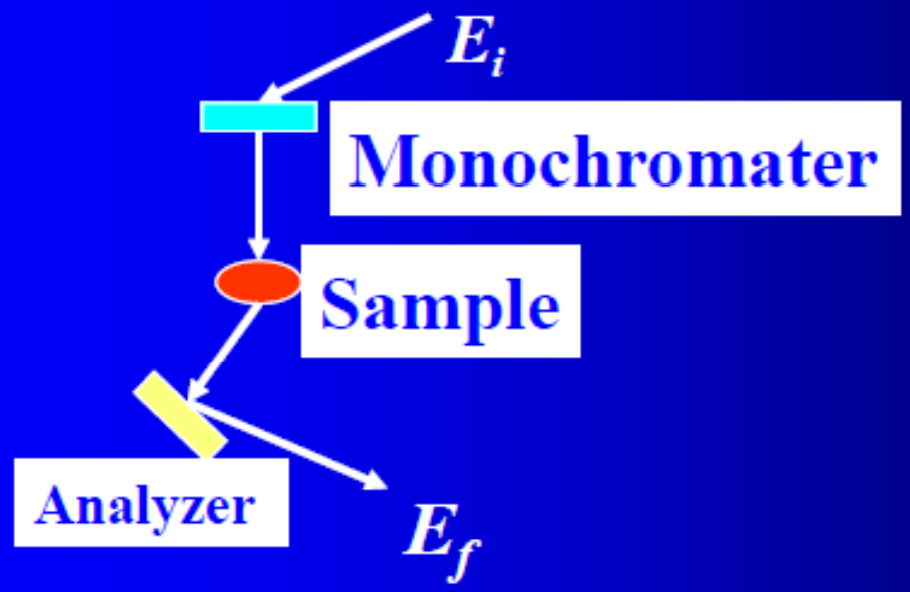
$$\frac{\hbar^2 k_n^2}{2M_n} = \frac{\hbar^2 k_n^{'2}}{2M_n} + \hbar\omega_k,$$ where $\hbar\omega_k$ is the energy of the magnon.

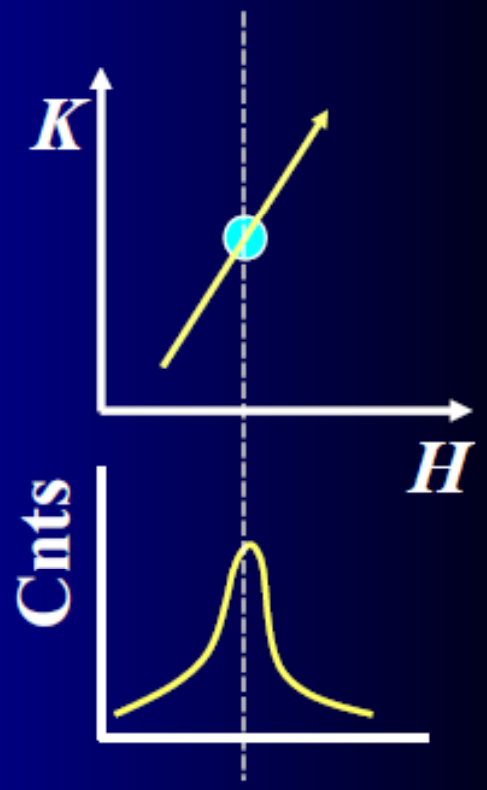
Neutron Scattering

- $k_i = 2\pi/\lambda$
- $E = h^2k^2/2m$

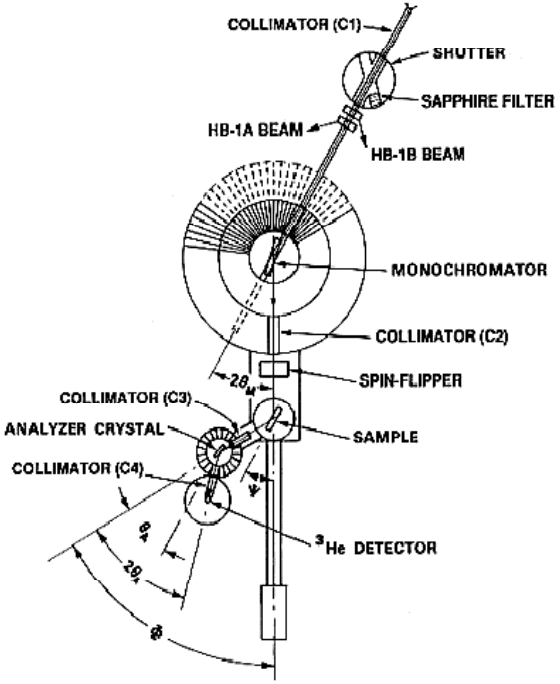
$$S(Q,w) \propto (n(\omega)+1)\chi''(Q,w)$$

Triple-axis Spectrometer

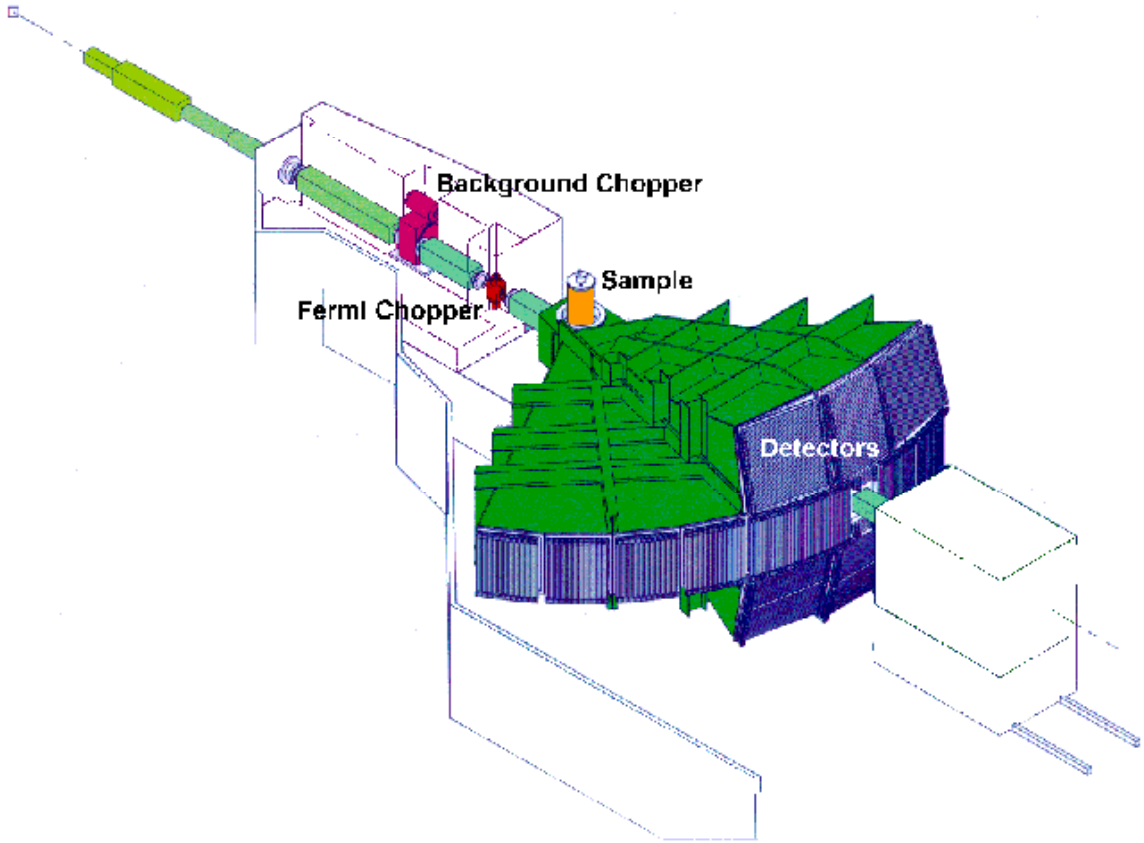




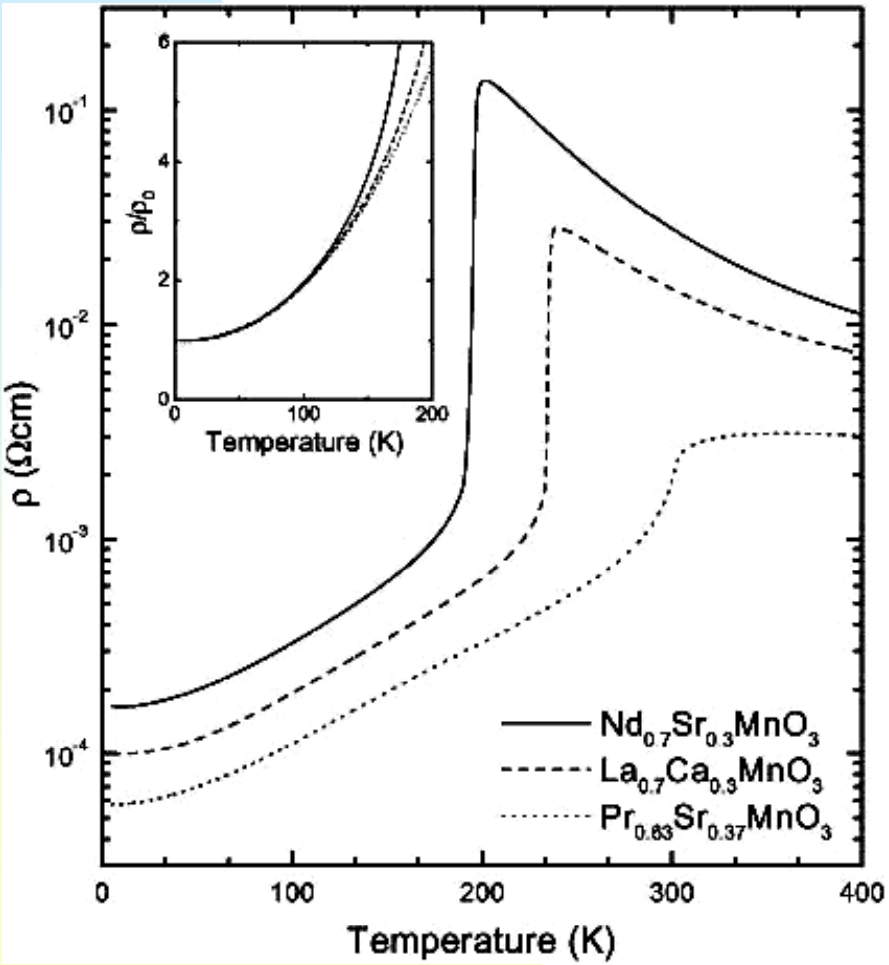
Triple Axis Spectrometer



MAPS at ISIS: 576 detectors; 147,456 total pixels; 36,864 spectra; 0.5 Gb data per scan
Time of flight technique is the future!

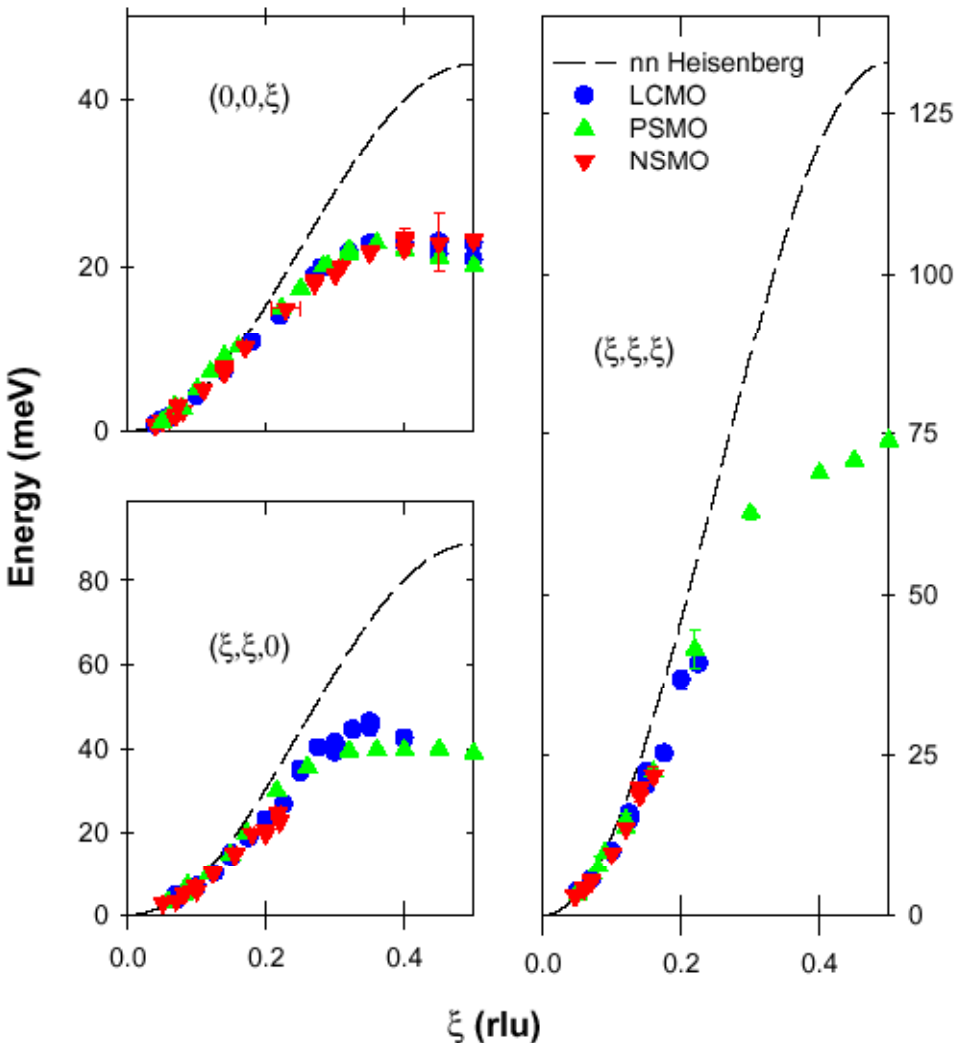


What to expect in standard ferromagnets?



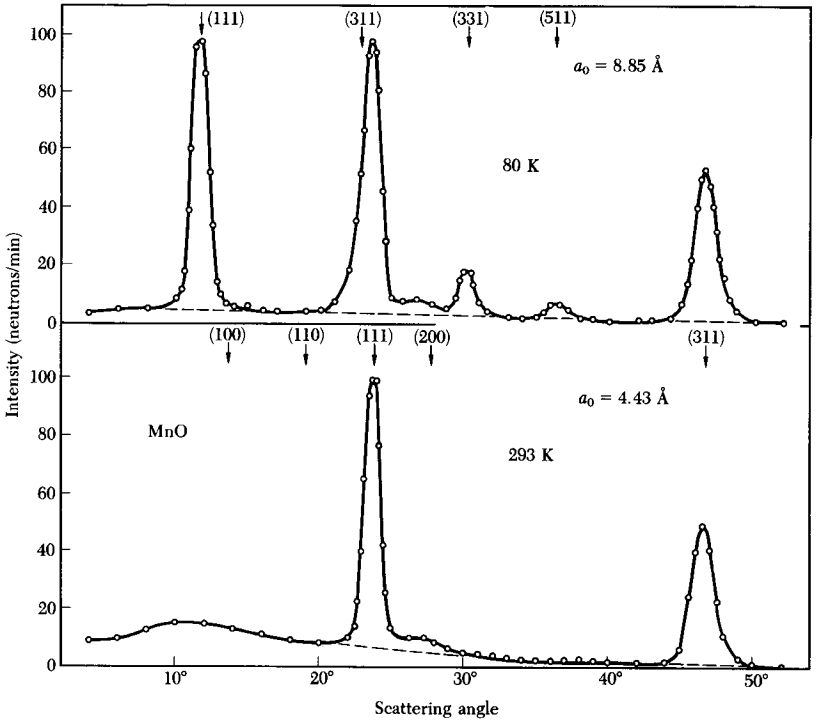
- Single crystals of
  $Nd_{0.7}Sr_{0.3}MnO_3$ (T_C=198K)
  $La_{0.7}Ca_{0.3}MnO_3$ (T_C=240K)
  $Pr_{0.63}Sr_{0.37}MnO_3$ (T_C=300K)
- Spin-wave excitations should be different for different materials.
- *D* should decrease with T_C.

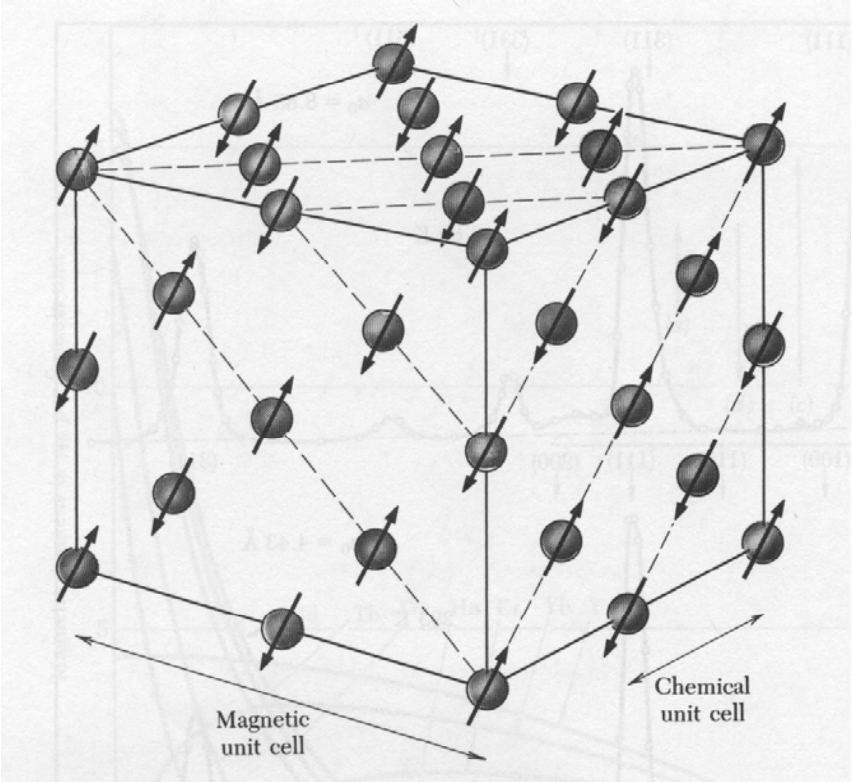
Measured spin-wave excitations



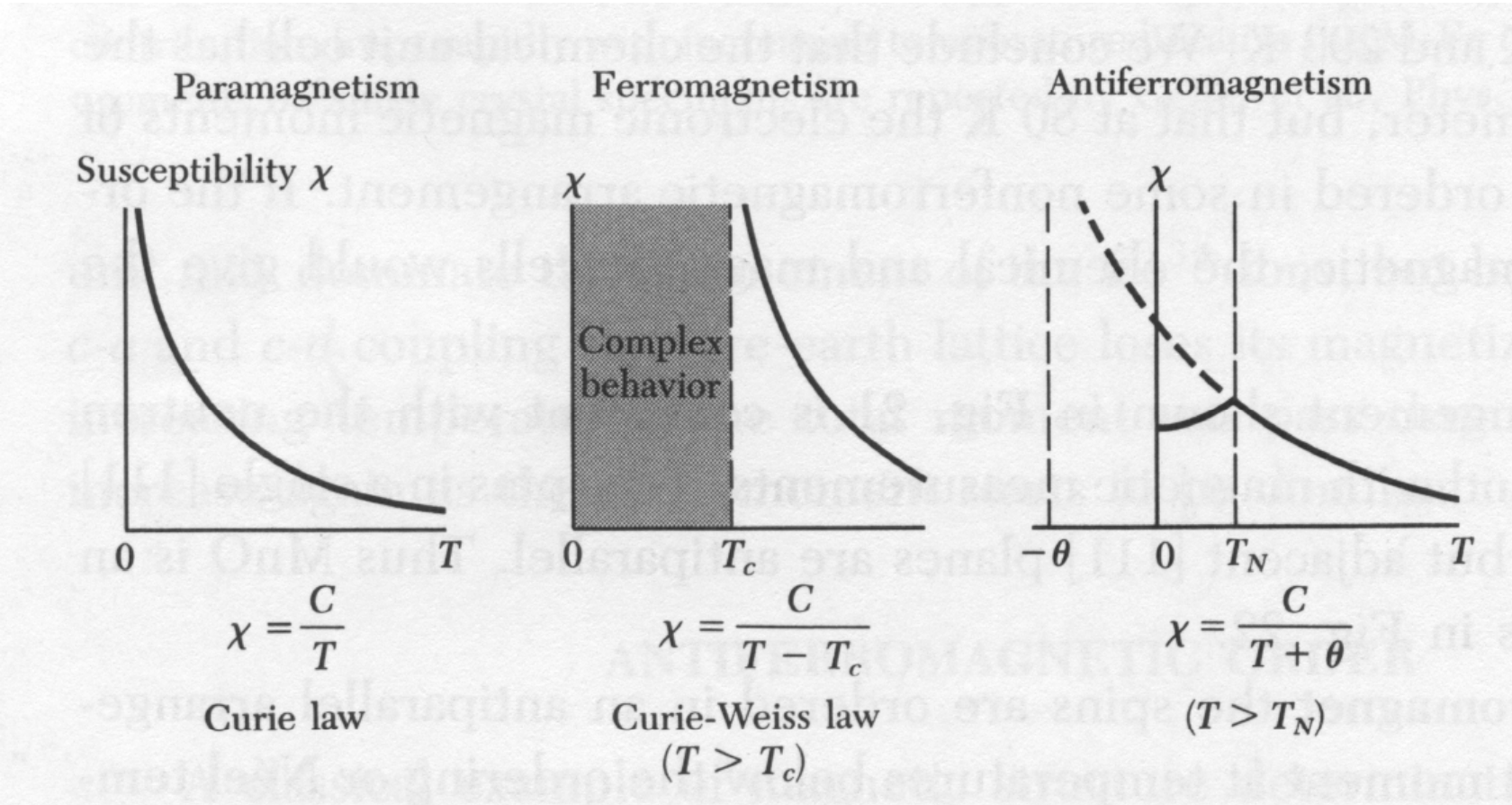
- Similar dispersions in spite of different T_C's.
- Energy softening near zone boundary.
- Heisenberg nn interaction cannot describe the spin-wave dispersions.

Antiferromagnetic order





In an antiferromagnet the spins are ordered in an antiparallel arrangement with zero net moment below the ordering Neel temperature.



The susceptibility in the paramagnetic region $T > T_N$ is $\chi = \dfrac{2C}{T + T_N}$.

The experimental results at $T > T_N$ are of the form $\chi = \dfrac{2C}{T + \theta}$.

Table 3 Antiferromagnetic crystals

Substance	Paramagnetic ion lattice	Transition temperature, T_N, in K	Curie-Weiss θ, in K	$\frac{\theta}{T_N}$	$\frac{\chi(0)}{\chi(T_N)}$
MnO	fcc	116	610	5.3	$\frac{2}{3}$
MnS	fcc	160	528	3.3	0.82
MnTe	hex. layer	307	690	2.25	
MnF_2	bc tetr	67	82	1.24	0.76
FeF_2	bc tetr	79	117	1.48	0.72
$FeCl_2$	hex. layer	24	48	2.0	<0.2
FeO	fcc	198	570	2.9	0.8
$CoCl_2$	hex. layer	25	38.1	1.53	
CoO	fcc	291	330	1.14	
$NiCl_2$	hex. layer	50	68.2	1.37	
NiO	fcc	525	~2000	~4	
Cr	bcc	308			

Antiferromagnetic Magnons

We obtain the dispersion relation of magnons in a one-dimensional antiferromagnet by making the appropriate substitutions in the treatment (16)–(22) of the ferromagnetic line. Let spins with even indices $2p$ compose sublattice A, that with spins up ($S^z = S$); and let spins with odd indices $2p + 1$ compose sublattice B, that with spins down ($S^z = -S$).

We consider only nearest-neighbor interactions, with J negative. Then (18) written for A becomes, with a careful look at (17),

$$dS^x_{2p}/dt = (2JS/\hbar)(-2S^y_{2p} - S^y_{2p-1} - S^y_{2p+1}) \ ; \tag{45a}$$

$$dS^y_{2p}/dt = -(2JS/\hbar)(-2S^x_{2p} - S^x_{2p-1} - S^x_{2p+1}) \ . \tag{45b}$$

The corresponding equations for a spin on B are

$$dS^x_{2p+1}/dt = (2JS/\hbar)(2S^y_{2p+1} + S^y_{2p} + S^y_{2p+2}) \ ; \tag{46a}$$

$$dS^y_{2p+1}/dt = -(2JS/\hbar)(2S^x_{2p+1} + S^x_{2p} + S^x_{2p+2}) \ . \tag{46b}$$

We form $S^+ = S^x + iS^y$; then

$$dS^+_{2p}/dt = (2iJS/\hbar)(2S^+_{2p} + S^+_{2p-1} + S^+_{2p+1}) \ ; \tag{47}$$

$$dS^+_{2p+1}/dt = -(2iJS/\hbar)(2S^+_{2p+1} + S^+_{2p} + S^+_{2p+2}) \ . \tag{48}$$

We look for solutions of the form

$$S^+_{2p} = u \exp[i2pka - iwt] \;; \quad S^+_{2p+1} = v \exp[i(2p+1)\,ka - iwt] \;, \tag{49}$$

so that (47) and (48) become, with $\omega_{ex} \equiv -4JS/\hbar = 4|J|S/\hbar$,

$$\omega u \quad = \tfrac{1}{2}\omega_{ex}(2u + ve^{-ika} + ve^{ika}) \;; \tag{50a}$$

$$-\omega v = \tfrac{1}{2}\omega_{ex}(2v + ue^{-ika} + ue^{ika}) \;. \tag{50b}$$

Equations (50) have a solution if

$$\begin{vmatrix} \omega_{ex} - \omega & \omega_{ex}\cos ka \\ \omega_{ex}\cos ka & \omega_{ex} + \omega \end{vmatrix} = 0 \;; \tag{51}$$

thus

$$\omega^2 = \omega^2_{ex}(1 - \cos^2 ka) \;; \quad \omega = \omega_{ex}|\sin ka| \;. \tag{52}$$

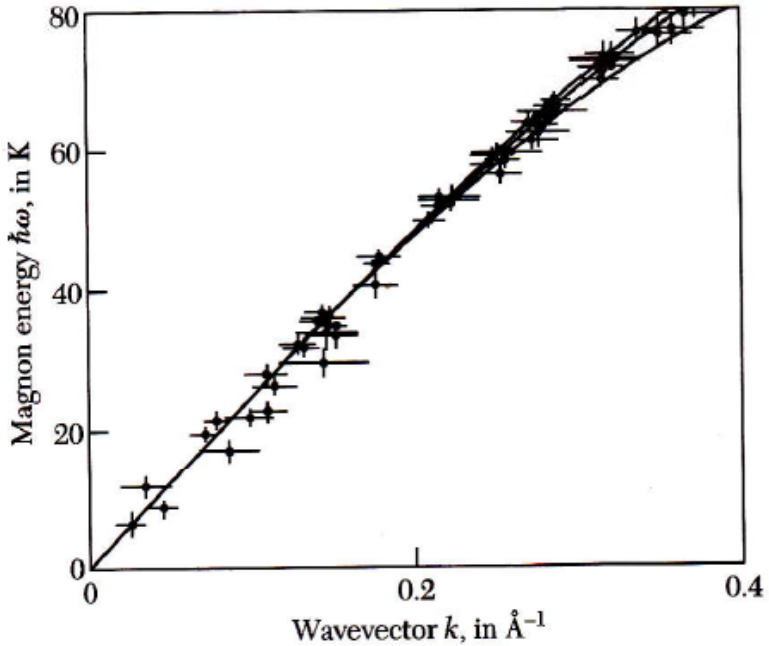
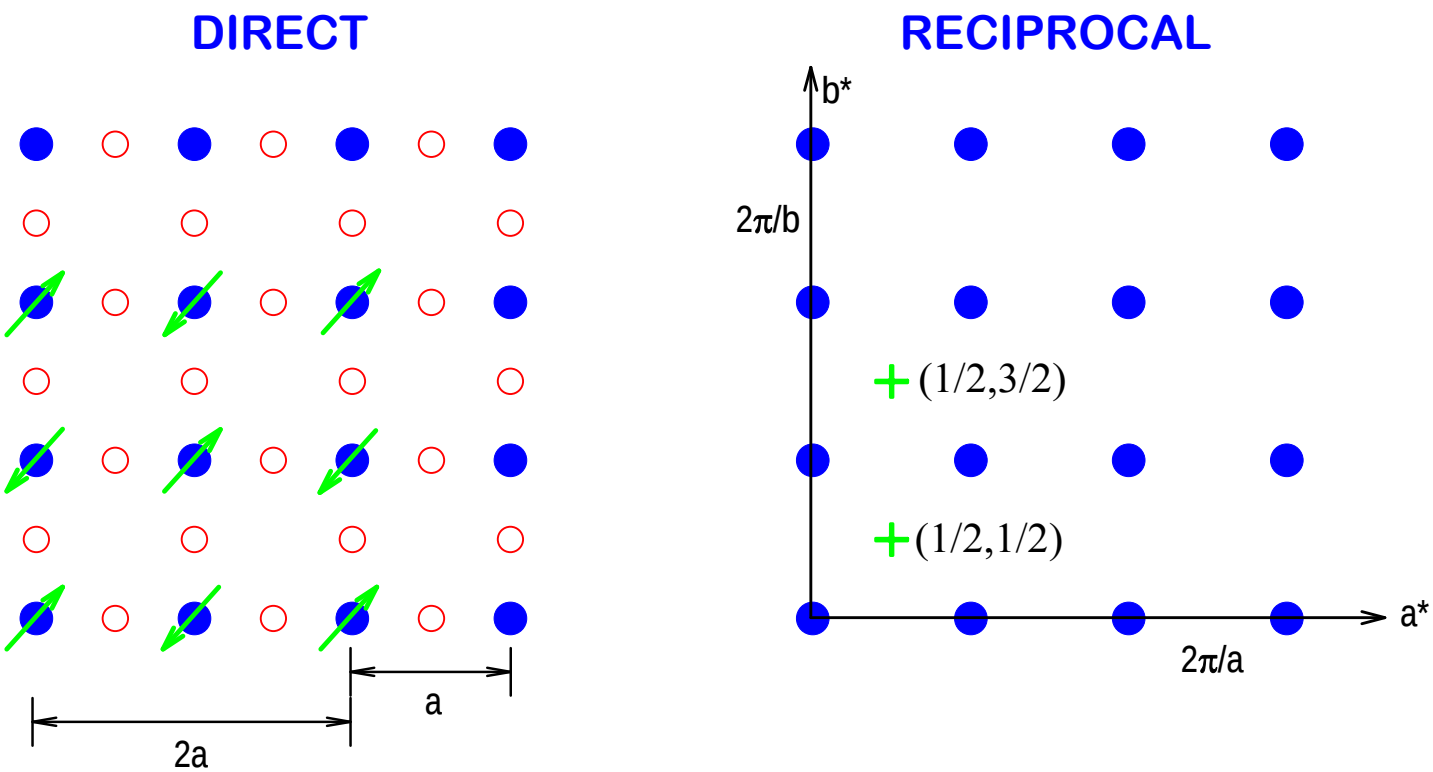


Figure 23 Magnon dispersion relation in the simple cubic antiferromagnet $RbMnF_3$ as determined at 4.2 K by inelastic neutron scattering. (After C. G. Windsor and R. W. H. Stevenson.)

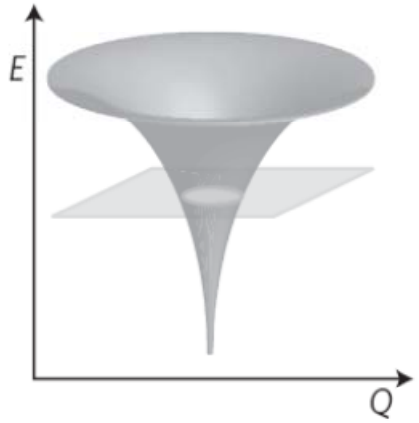
The dispersion relation for magnons in an antiferromagnet is quite different from (22) for magnons in a ferromagnet. For $ka \ll 1$ we see that (52) is linear in k: $\omega \cong \omega_{ex}|ka|$. The magnon spectrum of $RbMnF_3$ is shown in Fig. 23, as determined by inelastic neutron scattering experiments. There is a large region in which the magnon frequency is linear in the wavevector.

Real, reciprocal space of CuO_2 plane

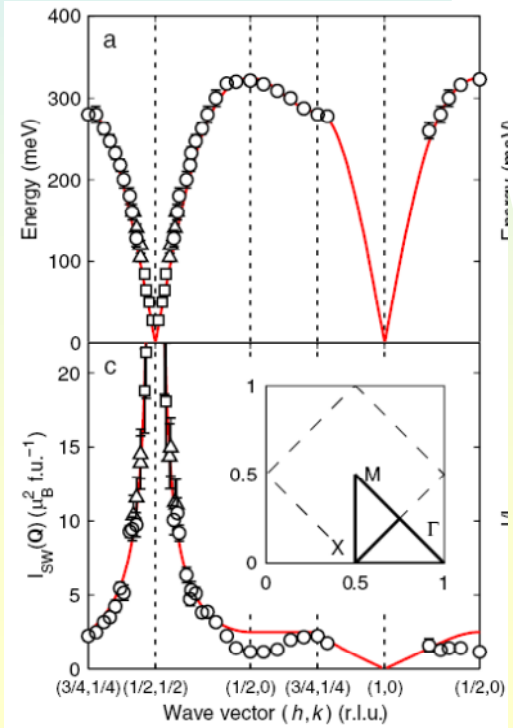


$La_{2-x}Sr_xCuO_4$ single layer, orthorhombic
$Nd_{2-x}Ce_xCuO_4$ single layer, tetragonal

Ground State of the Cuprates



N.S. Headings, *et. al.*



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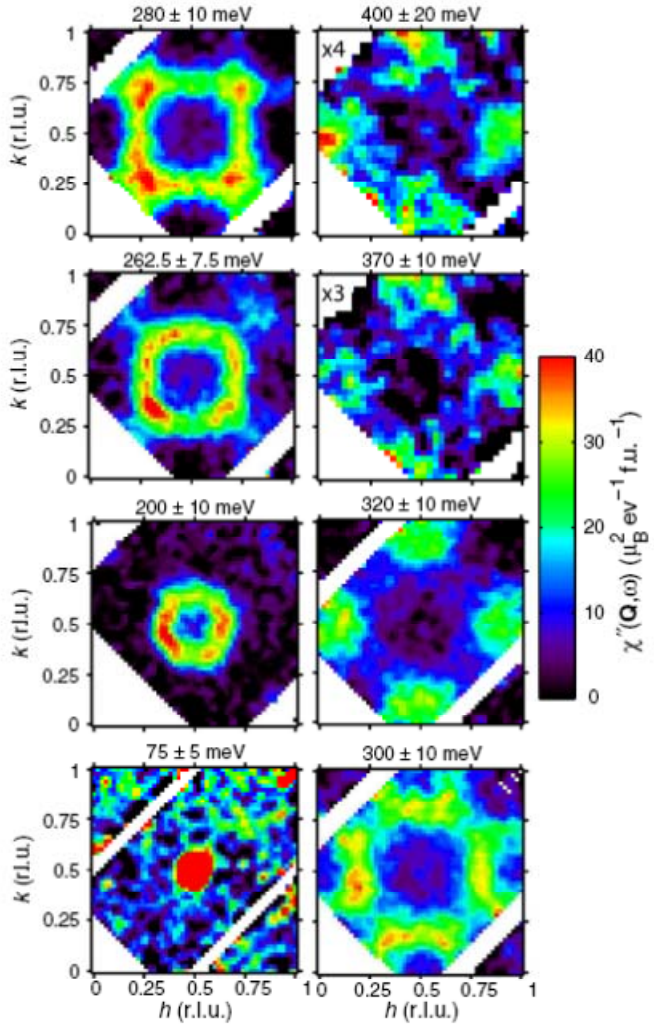

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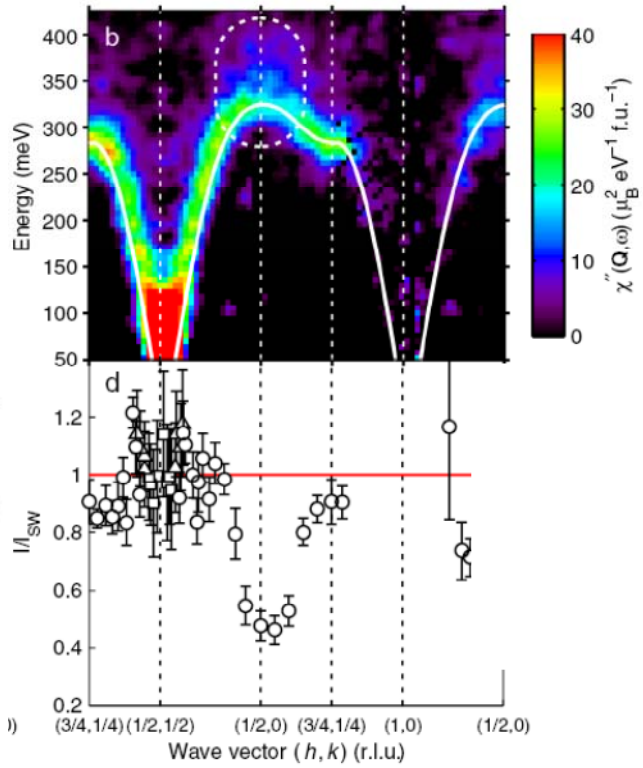
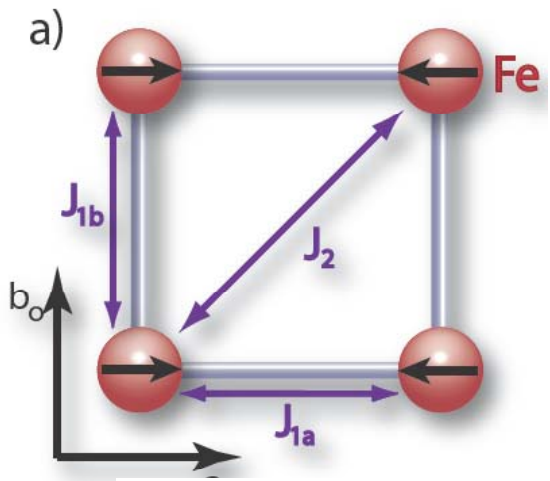


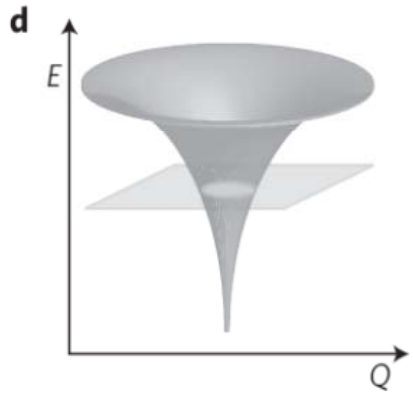
The Heisenberg Model

$$\frac{d^2\sigma}{d\Omega dE} = \frac{k_f}{k_i}\left(\frac{\gamma r_0}{2}\right)^2 g^2 f^2(Q) e^{-2W} \sum_{\alpha\beta} (\delta_{\alpha\beta} - Q_\alpha Q_\beta) S^{\alpha\beta}(Q,E)$$



$$\mathcal{H} = J_{1a} \sum_{\langle ij \rangle_a} \mathbf{S}_i \cdot \mathbf{S}_j + J_{1b} \sum_{\langle ij \rangle_b} \mathbf{S}_i \cdot \mathbf{S}_j + J_2 \sum_{\langle ik \rangle_{ab}} \mathbf{S}_i \cdot \mathbf{S}_k + J_{1c} \sum_{\langle i\ell \rangle_c} \mathbf{S}_i \cdot \mathbf{S}_\ell .$$

$$E(q) = \sqrt{A_q^2 - B_q^2}$$



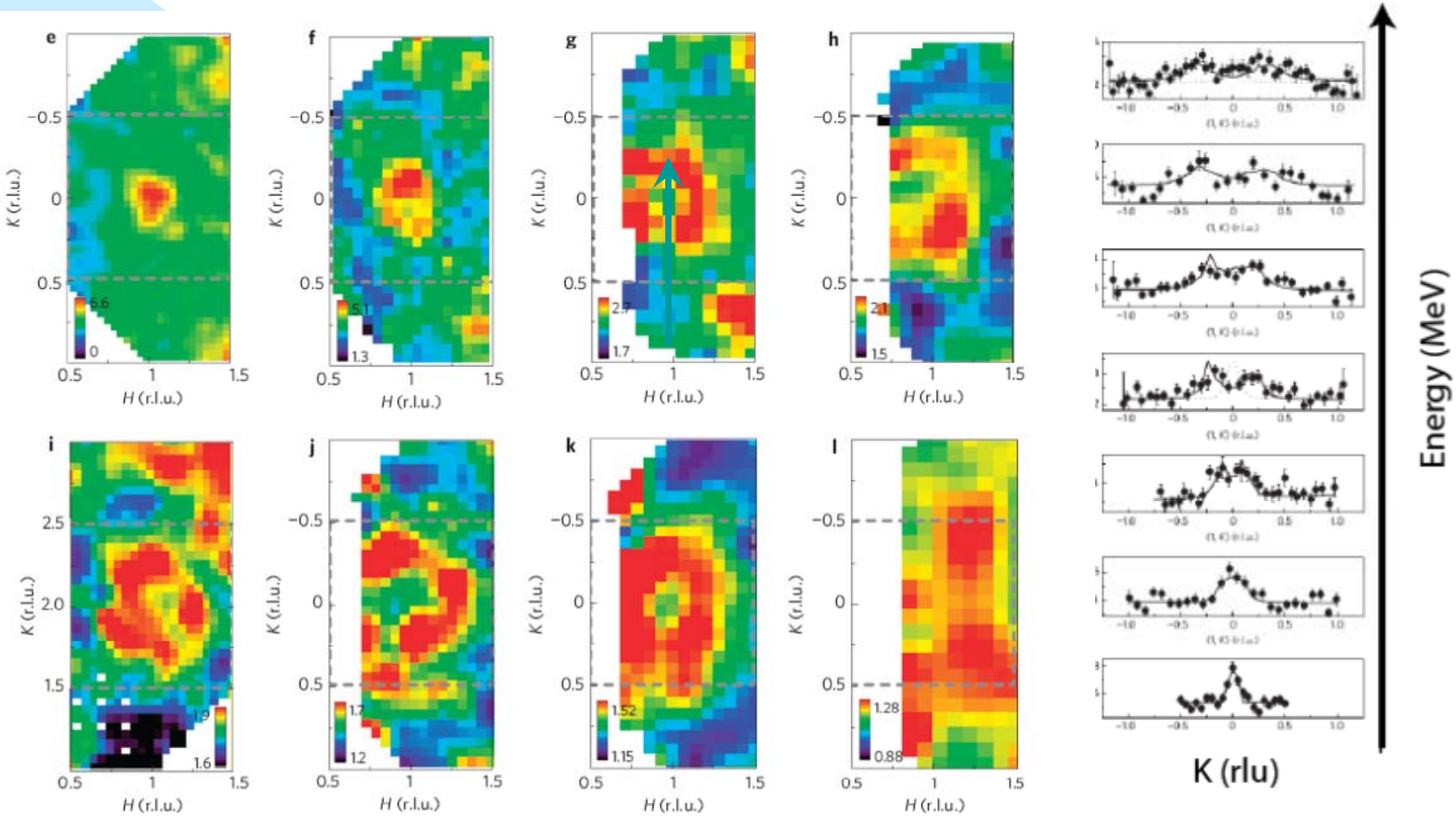
$$A_q = 2S[J_{1b}(\cos(\pi K) - 1) + J_{1a} + J_c + 2J_2 + J_s]$$

$$B_q = 2S[J_{1a}\cos(\pi H) + 2J_2\cos(\pi H)\cos(\pi K) + J_c\cos(\pi L)]$$

$$S^{yy}(Q,E) = S^{zz}(Q,E) = S_{\text{eff}} \frac{(A_q - B_q)}{E_0(1 - e^{-E/k_B T})} \frac{4}{\pi} \frac{\Gamma E E_0}{(E^2 - E_0^2)^2 + 4(\Gamma E)^2}$$

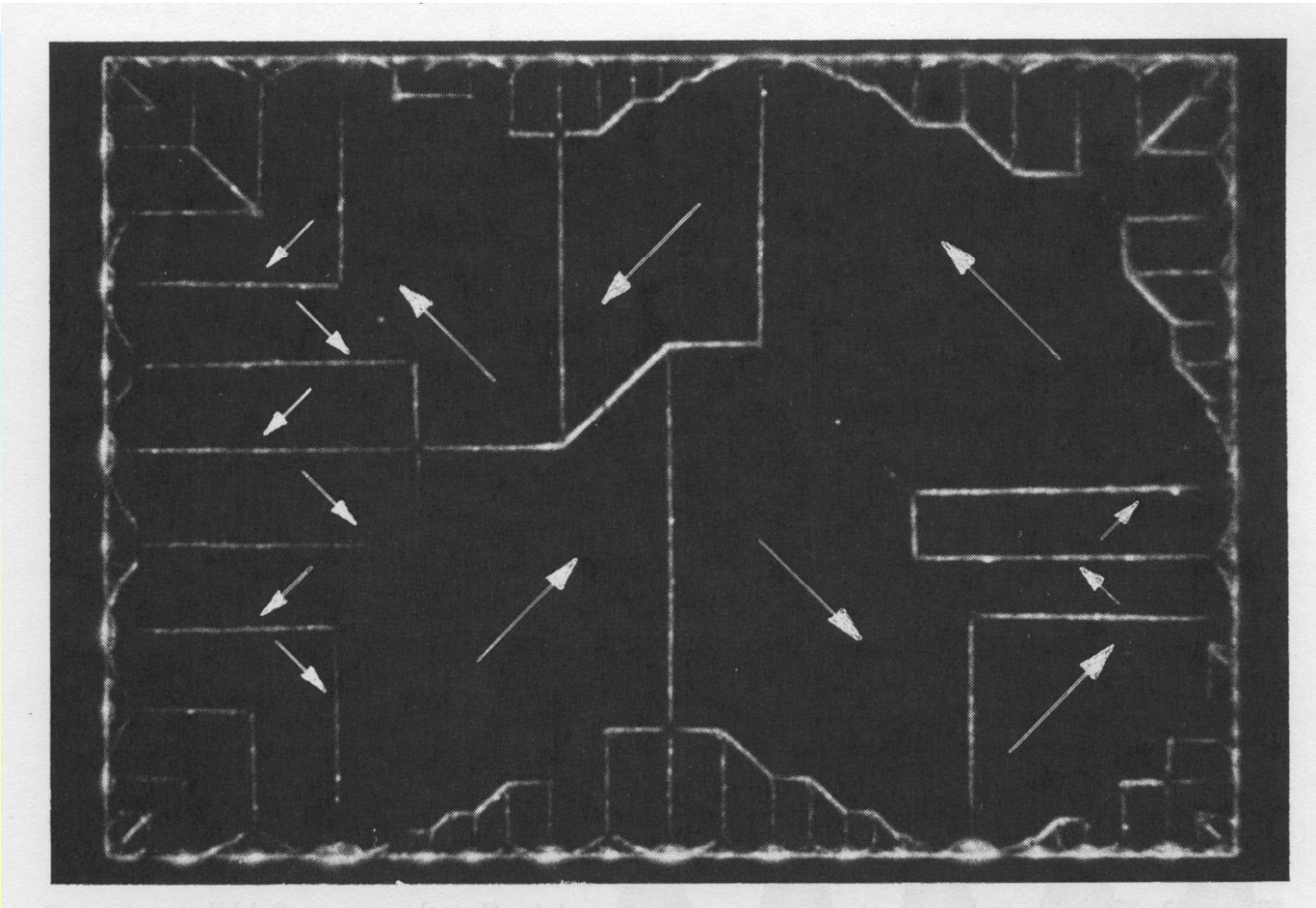
Low Temperature CaFe2As2

CaFe2As2



Jun Zhao et. al., Nature Physics 5, 555 (2009)

Ferromagnetic domains



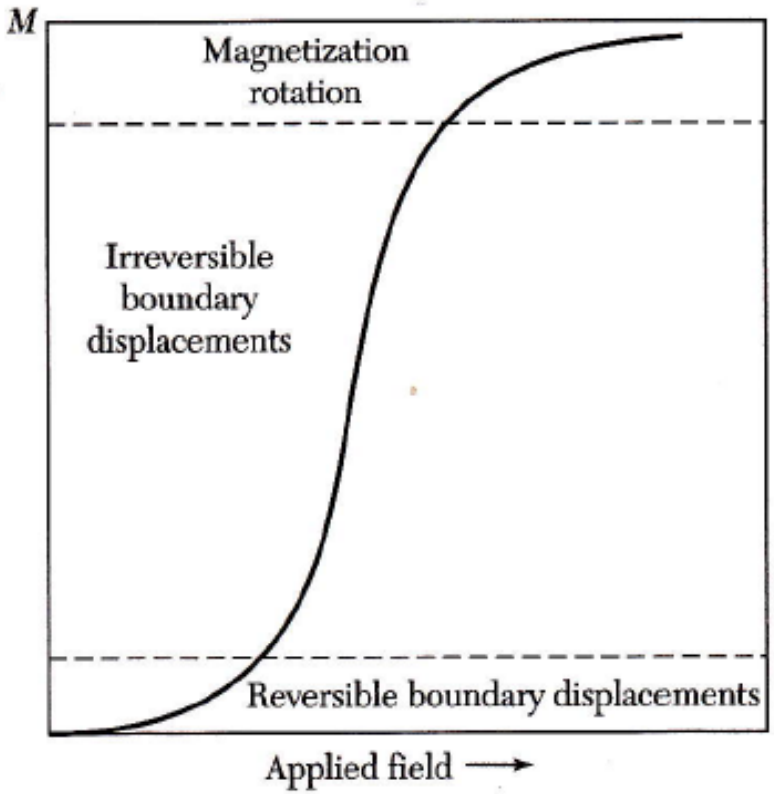


Figure 25 Representative magnetization curve, showing the dominant magnetization processes in the different regions of the curve.

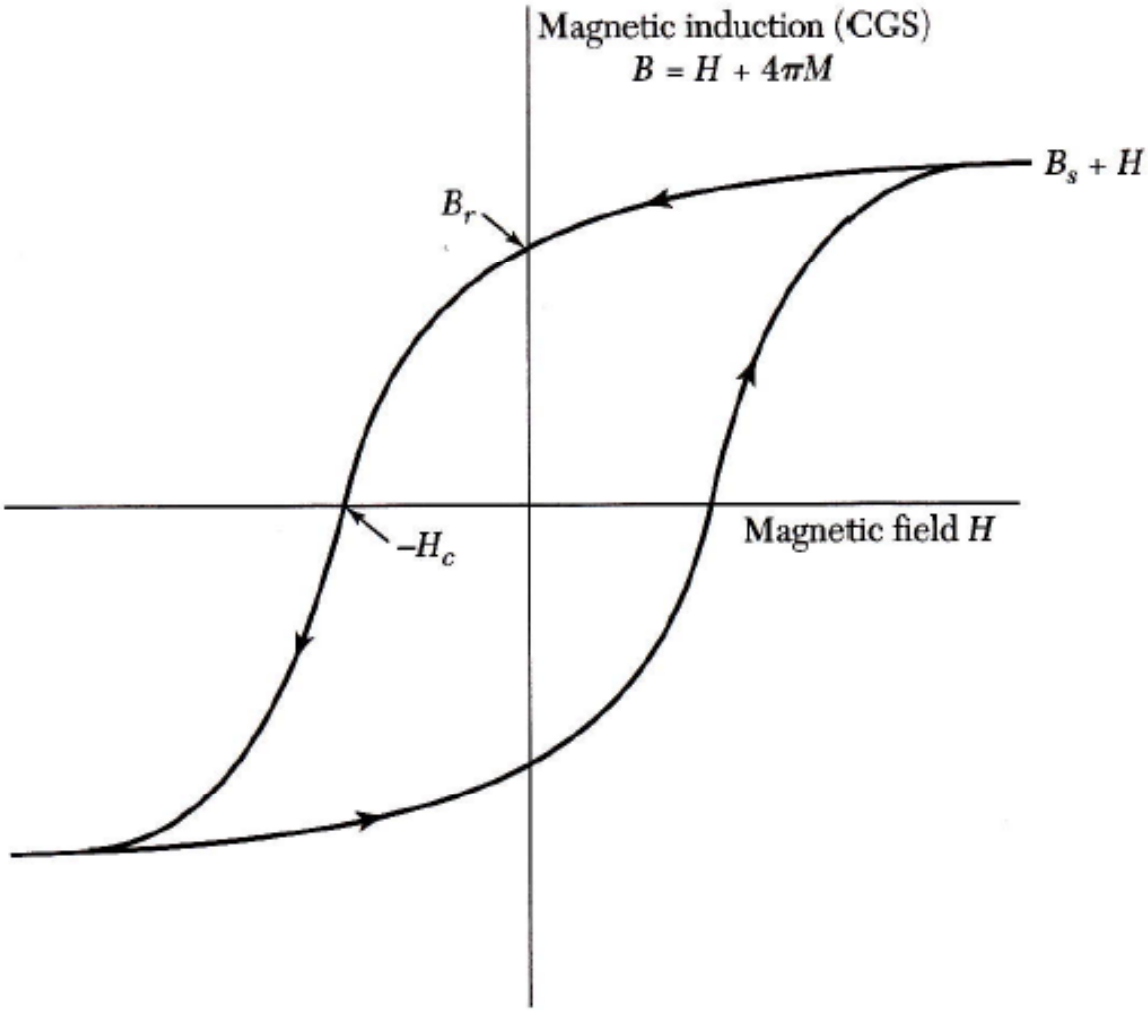
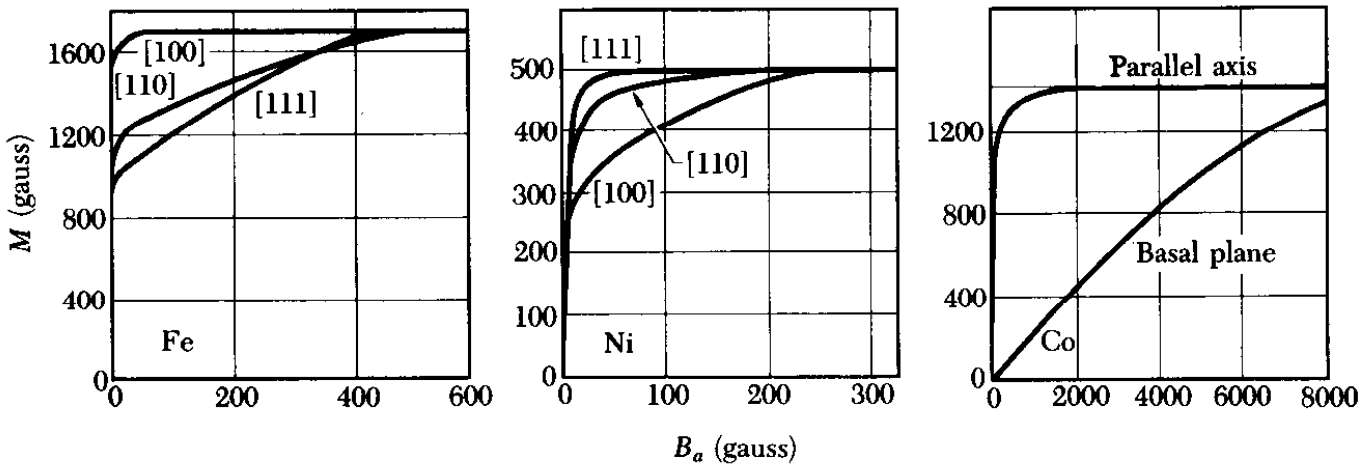
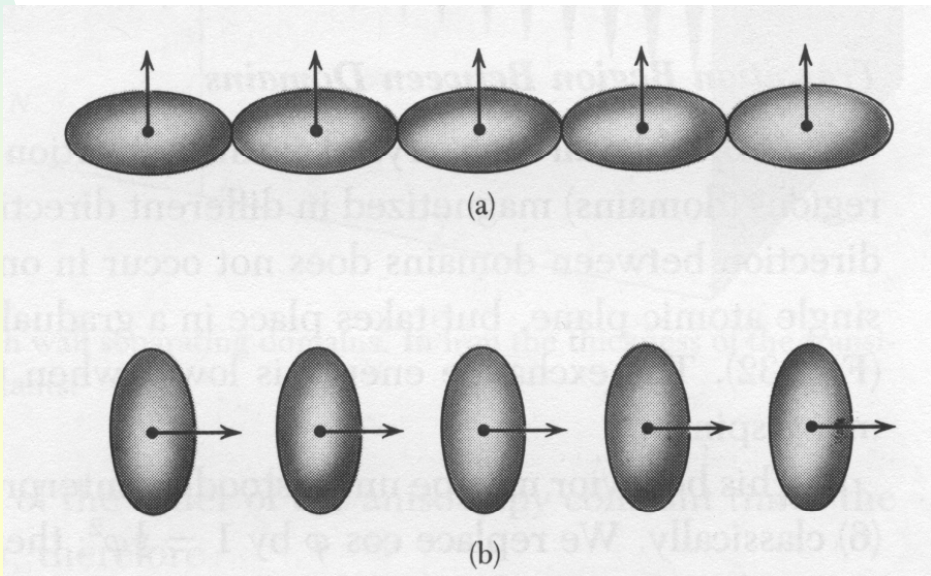


Figure 26 The technical magnetization curve (or hysteresis loop). The coercivity H_c is the reverse field that reduces B to zero; a related coercivity H_{ci} reduces M or $B - H$ to zero. The **remanence** B_r is the value of B at $H = 0$. The saturation induction B_s is the limit of $B - H$ at large H, and the saturation magnetization $M_s = B_s/4\pi$. In SI the vertical axis is $B = \mu_0(H + M)$.

Magnetic anisotropy





1. ***Magnon dispersion relation.*** Derive the magnon dispersion relation (24) for a spin S on a simple cubic lattice, $z = 6$. Hint: Show first that (18a) is replaced by

$$dS^x_{\boldsymbol{\rho}}/dt = (2JS/\hbar)(6S^y_{\boldsymbol{\rho}} - \sum_{\delta} S^y_{\boldsymbol{\rho}+\delta}) \ ,$$

where the central atom is at $\boldsymbol{\rho}$ and the six nearest neighbors are connected to it by six vectors $\boldsymbol{\delta}$. Look for solutions of the equations for $dS^x_{\boldsymbol{\rho}}/dt$ and $dS^y_{\boldsymbol{\rho}}/dt$ of the form $\exp(i\mathbf{k} \cdot \boldsymbol{\rho} - i\omega t)$.