Problems for chapter 13, due May3rd, 2012

3. The dipole moment for a general distribution of charges is defined as the sum

$$\mathbf{p} = \sum_i q_i \mathbf{r}_i,$$

where q_i and $\mathbf{r}_i$ are the charge and position, respectively, of the i^{th} charge, and the summation is over all the charges present. The choice of the origin of coordinates is arbitrary.

   a) Show that the above reduces to expression (8.1) for the special case of two equal and opposite charges. (Take an arbitrary origin.)
   b) Prove that if the charge system has an overall electrical neutrality, then the dipole moment is independent of the choice of origin.

4. Determine the dipole moment for the following charge distributions: 1.5 μcoul each at the points (0,3), (0,5), where the coordinate numbers are given in centimeters.

5. ***Polarization of sphere.*** A sphere of dielectric constant ϵ is placed in a uniform external electric field E_0. (a) What is the volume average electric field E in the sphere? (b) Show that the polarization in the sphere is $P = \chi E_0/[1 + (4\pi\chi/3)]$, where $\chi = (\epsilon - 1)/4\pi$. Hint: You do not need to calculate E_{loc} in this problem; in fact it is confusing to do so, because ϵ and χ are defined so that $P = \chi E$. We require E_0 to be unchanged by insertion of the sphere. We can produce a fixed E_0 by placing positive charges on one thin plate of an insulator and negative charges on an opposite plate. If the plates are always far from the sphere, the field of the plates will remain unchanged when the sphere is inserted between them. The results above are in CGS.

6. ***Ferroelectric criterion for atoms.*** Consider a system of two neutral atoms separated by a fixed distance a, each atom having a polarizability α. Find the relation between a and α for such a system to be ferroelectric. Hint: The dipolar field is strongest along the axis of the dipole.