Problems for chapter 4, due Feb. 10, 2012

1. Consider a dipole in an external electric field $\vec{E} = E_0\hat{y}$ where E_0 is a constant. For the four cases of orientation of the dipole (p is the magnitude of the dipole moment):

   (i) $\vec{p} = p\hat{x}$

   (ii) $\vec{p} = p\hat{y}$

   (iii) $\vec{p} = -p\hat{x}$

   (iv) $\vec{p} = -p\hat{y}$

   a. Find the potential energy of the dipole in this external field. Order your answers from highest to lowest potential energy.

   b. If the dipole were free to rotate in this external electric field, which orientation would be the equilibrium alignment? (as a hint, consider the analogy of a particle in the gravitational field of the earth; its potential energy increases with increasing altitude above the surface. If left free, in what direction of change of potential energy does it go?)

2. A point dipole $\vec{p} = p_0\hat{y}$ is located at the origin of the coordinates and $p_0 = 1.00 \times 10^{-15}$ C m. At a distance of 0.01 meters from the dipole, find the electric field at each of the following locations and, on a diagram, sketch in the direction of the field at each point:

   a. along the positive x-axis.

   b. along the positive y-axis.

   c. along the negative x-axis.

   d. along the negative y-axis.

   e. at a point in the first quadrant in the x-y plane, making an angle of 45° with both the positive-x and positive-y axes.

5. *Linear ionic crystal.* Consider a line of $2N$ ions of alternating charge $\pm q$ with a repulsive potential energy A/R^n between nearest neighbors. (a) Show that at the equilibrium separation

(CGS) $$U(R_0) = -\frac{2Nq^2 \ln 2}{R_0}\left(1 - \frac{1}{n}\right) .$$

(b) Let the crystal be compressed so that $R_0 \to R_0(1 - \delta)$. Show that the work done in compressing a unit length of the crystal has the leading term $\frac{1}{2}C\delta^2$, where

(CGS) $$C = \frac{(n-1)q^2 \ln 2}{R_0} .$$

To obtain the results in SI, replace q^2 by $q^2/4\pi\epsilon_0$. Note: We should not expect to obtain this result from the expression for $U(R_0)$, but we must use the complete expression for $U(R)$.