Problems for chapter 9, due March 26, 2012

Hexagonal close-packed structure. Consider the first Brillouin zone of a crystal with a simple hexagonal lattice in three dimensions with lattice constants a and c. Let $\mathbf{G}_c$ denote the shortest reciprocal lattice vector parallel to the c axis of the crystal lattice. (a) Show that for a hexagonal-close-packed crystal structure the Fourier component $U(\mathbf{G}_c)$ of the crystal potential $U(\mathbf{r})$ is zero. (b) Is $U(2\mathbf{G}_c)$ also zero? (c) Why is it possible in principle to obtain an insulator made up of divalent atoms at the lattice points of a simple hexagonal lattice? (d) Why is it not possible to obtain an insulator made up of monovalent atoms in a hexagonal-close-packed structure?

Brillouin zones of two-dimensional divalent metal. A two-dimensional metal in the form of a square lattice has two conduction electrons per atom. In the almost free electron approximation, sketch carefully the electron and hole energy surfaces. For the electrons choose a zone scheme such that the Fermi surface is shown as closed.

Open orbits. An open orbit in a monovalent tetragonal metal connects opposite faces of the boundary of a Brillouin zone. The faces are separated by $G = 2 \times 10^8\ \mathrm{cm}^{-1}$. A magnetic field $B = 10^3$ gauss $= 10^{-1}$ tesla is normal to the plane of the open orbit. (a) What is the order of magnitude of the period of the motion in $\mathbf{k}$ space? Take $v \approx 10^8$ cm/sec. (b) Describe in real space the motion of an electron on this orbit in the presence of the magnetic field.

De Haas-van Alphen period of potassium. (a) Calculate the period $\Delta(1/B)$ expected for potassium on the free electron model. (b) What is the area in real space of the extremal orbit, for $B = 10$ kG $= 1$ T? The same period applies to oscillations in the electrical resistivity, known as the Shubnikow-de Haas effect.