Commutation relations

Below are listed several commutation relations.

$\mathbf{p}$ = momentum, $\mathbf{r}$ = position coordinate. Components p_i, x_j.
$[p_i, x_j] = -i\hbar\delta_{ij}$ which is equivalent to $[\mathbf{a}\cdot\mathbf{p}, \mathbf{b}\cdot\mathbf{r}] = -i\hbar\ \mathbf{a}\cdot\mathbf{b}$

$\mathbf{L} = \mathbf{r} \times \mathbf{p}$
$[L_i, L_j] = i\hbar\varepsilon_{ijk}L_k$ where summation over k is implied and ε_{ijk} is the antisymmetric tensor with elements that vanish if any two indices are identical, $\varepsilon_{123} = 1$ and all other indices are ± 1 depending upon whether the indices are an even or odd permutation of 123. Sometimes this is stated

$$[L_x, L_y] = i\hbar L_z, \quad \text{cycliclly}$$

The commutation relation can also be written
$[\mathbf{a}\cdot\mathbf{L}, \mathbf{b}\cdot\mathbf{L}] = i\hbar\mathbf{a}\times\mathbf{b}\cdot\mathbf{L}$
These commutation relations hold for any angular momentum operator $\mathbf{j}$. They hold for the spin-1/2 operator $\mathbf{S} = i\hbar\frac{1}{2}\boldsymbol{\sigma}$ of chapter I of the text.

Problem assignment III.

(1) Do Problem 2.1 of the text.

(2) Verify Eqs. 1.266 and 1.267.

(3) Show that $\mathbf{J} = \mathbf{L} + \mathbf{S}$ commutes with the Dirac Hamiltonian $H_0 = c\boldsymbol{\alpha} \cdot p + \beta mc^2$

(4) Suppose that the potential $V(r) = -g/r$ is a 4-scaler potential. Write Dirac's equation including this potential. Show that Dirac's equation is equivalent to the Klein-Gordan equation

$$[p^2 + (mc^2 - V)^2]\psi = E^2\psi.$$

Note that this equation can be solved much like the Schrödinger equation.