PHYS 342/555
Condensed Matter in a Nutshell

Instructor: Dr. Pengcheng Dai

Professor of Physics

The University of Tennessee

(Room 407A, Nielsen, 974-1509)

(Office hours: TR 1:10PM-2:00 PM)

Lecture room 314 Nielsen

Chapter 10: Superconductivity

Lecture in pdf format will be available at:

http://www.phys.utk.edu

Electron-electron interaction

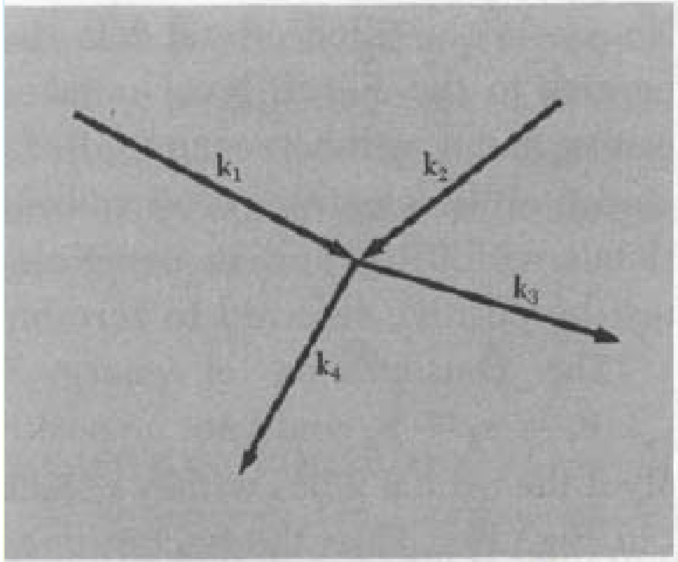
The effects of electron-electron interactions are usually described within the framework of the Landau theory of a Fermi liquid.
A Fermi gas is a system of noninteracting fermions; the same system with interactions is a Fermi liquid.

Consider that we gradually turning on the interactin between electrons, they will have two effects:
1. The energies of each on-electron level will be modified.

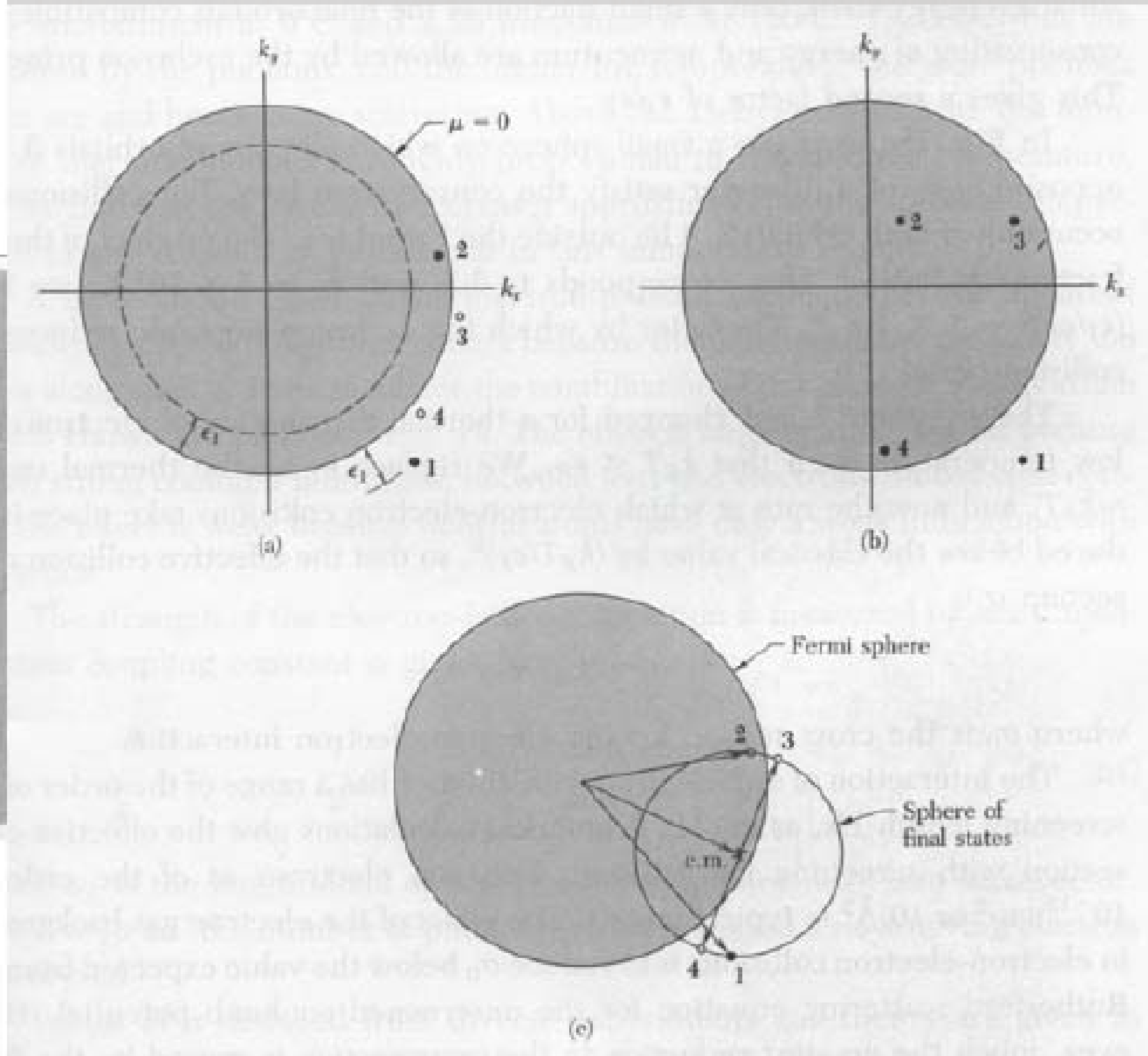
2. Electron will be scattered in and out of the single electron level, which are no longer stationary. Whether this scattering is serious enough to invalidate the independent electron picture depends on how rapid the rate of scattering is. If the scattering rate is low, electron-electron relaxation time is much larger than other relaxation time, then we can ignore it and use the independent electron theory with modified mass.

In metal, although conduction electrons are crowded together only 2 A apart, they travel a long distance before colliding with each other due to

1. Exclusion principle.
2. The screening of the coulomb interaction between two electrons.



k_1
k_2
k_3
k_4



k_y
$\mu = 0$
k_x
2
3
4
1
ϵ_1
(a)
(b)
Fermi sphere
Sphere of final states
c.m.
(c)

Suppose N electron state consists of a filled Fermi sphere (at $T=0$) plus a single excited electron in a level with $\varepsilon_1 > \varepsilon_F$.

In order for this electron to be scattered, it must interact with an electron of energy ε_2, which must be less than ε_F. The exclusion principle requires that these two electrons can only scatter into unoccupied levels, whose energies ε_3 and ε_4 must be greater than ε_F. Or $\varepsilon_2 < \varepsilon_F, \varepsilon_3 > \varepsilon_F, \varepsilon_4 > \varepsilon_F$.

In addition, energy conservation requires that

$\varepsilon_1 + \varepsilon_2 = \varepsilon_3 + \varepsilon_4$.

If ε_1 is exactly ε_F, $\varepsilon_2, \varepsilon_3, \varepsilon_4$ must also be ε_F. Thus the allowed wave vectors occupy a region of K space of zero volume. The life time of an electron at the Fermi surface at $T=0$ is infinite.

When ε_1 is different from ε_F, some phase space becomes available since the other three energies can now vary within a shell of thickness of order $|\varepsilon_1 - \varepsilon_F|$ about the Fermi surface, leading to a scattering rate of order $(\varepsilon_1 - \varepsilon_F)^2$.

If the excited electron is superimposed not on a filled Fermi surface, but on a thermal equilibrium distribution of electrons at nonzero T. There will be partially occupied levels in a shell of width $k_B T$ about ε_F. This provides an additional range of choice of order $k_B T$, and therefore leads to a scattering rate as $(k_B T)^2$. At temperature T, an electron of energy ε_1 near the Fermi surface has a scattering rate

$$\frac{1}{\tau} = a(\varepsilon_1 - \varepsilon_F)^2 + b(k_B T)^2,$$

where the coefficients a and b are independent of ε_1 and T.

Thus the electron life time due to electron-electron scattering can be made as large as one wishes by reducing T and by considering electrons sufficiently close to the Fermi surface.

Assume that the temperature dependence of τ is taken into account by a factor $1/T^2$. We expect from lowest-order pertrubation theory that τ will depend on the electron-electron interaction through the square of the Fourier transform of the interaction potential.

$$\frac{1}{\tau} \propto (k_B T)^2 \left(\frac{4\pi e^2}{k_0^2} \right)^2 \propto (k_B T)^2 \left(\frac{\pi^2 \hbar^2}{m k_F} \right)^2 ,$$

Fermi liquid theory: quasiparticles

If the independent electron picture is a good approximation, then at least for levels near the Fermi energy, electron-electron scattering will not invalidate this picture. If electron-electron interactions are strong, what happens?

Landau suggested that we can use the independent "quasiparticles" that obey the exclusion principle. The independent electron picture is quite likely to be valid if

1. We are only dealing with electrons within $k_B T$ of ε_F.
2. We are deadling with "quasiparticles"
3. We allow for the effects of interaction on the ε vs K relation.

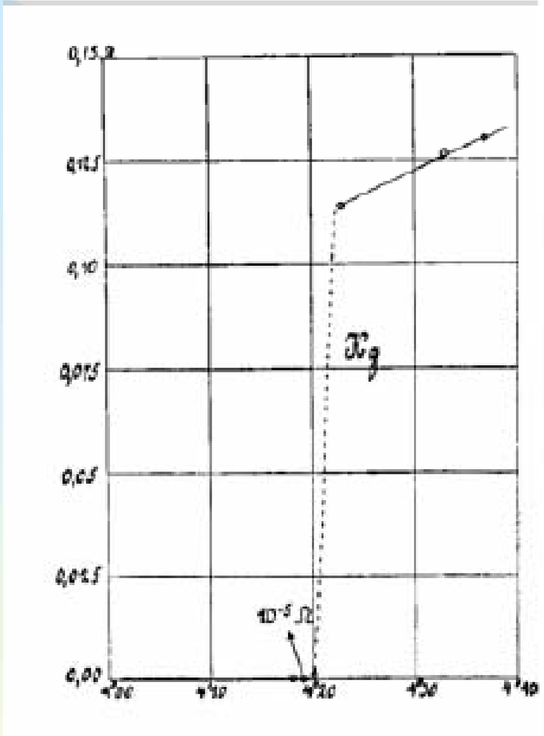
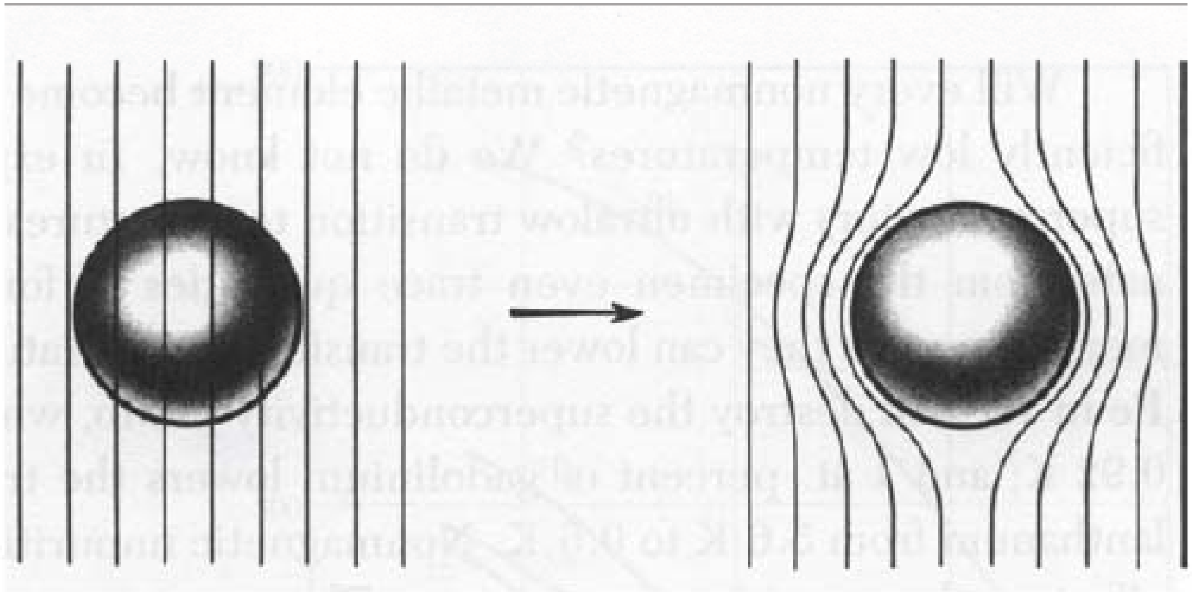


Figure 1. Resistance in ohms of a specime



Meissner effect: When a specimen is placed in a magnetic field and then cooled through the transition temperature for superconductivity, the magnetic flux originally present is rejected from the specimen.

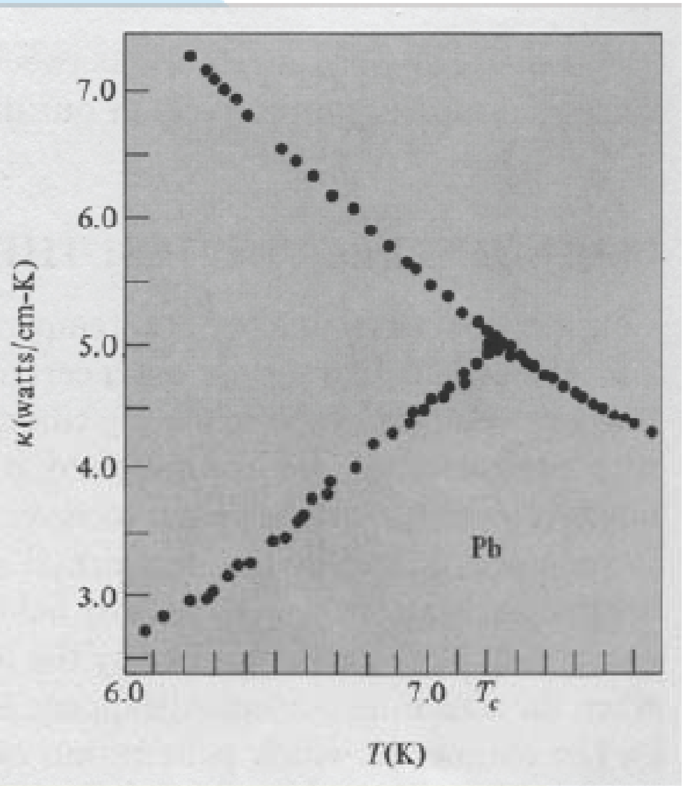
Properties of a superconductor:

1. A superconductor can behave as if it had no measurable DC electrical resistivity.

2. A superconductor can behave as a perfect diamagnet. A sample in thermal equilibrium in an applied magnetic field, provided the field is not too strong, carries electrical surface currents.

3. A superconductor usually behaves as if there were a gap in energy of width 2Δ centered about Fermi energy, in the set of allowed one-electron levels. Thus an electron of energy ε can be accommodated by a superconductor only if $\varepsilon - \varepsilon_F$ exceeds Δ. The energy gap increases in size as the temperature drops, leveling off to a maximum value at low T.

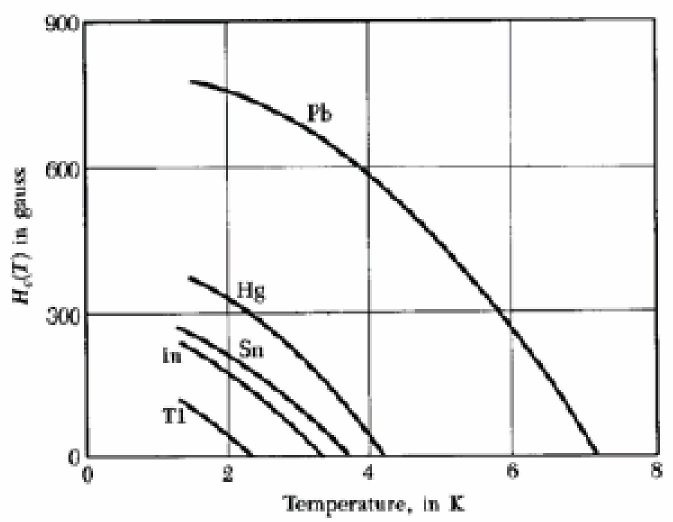
Thermoelectric properties



In the independent electron approximation good electrical conductors are also good conductors of heat, since the conduction electrons transport entropy as well as electric charge. Superconductors are poor thermal conductors. An electric current at uniform temperature in a superconductor is not accompanied by a thermal current, as it would be in a normal metal. This indicates that those electrons that participate in the persistent current carry no entropy.

Meissner effect

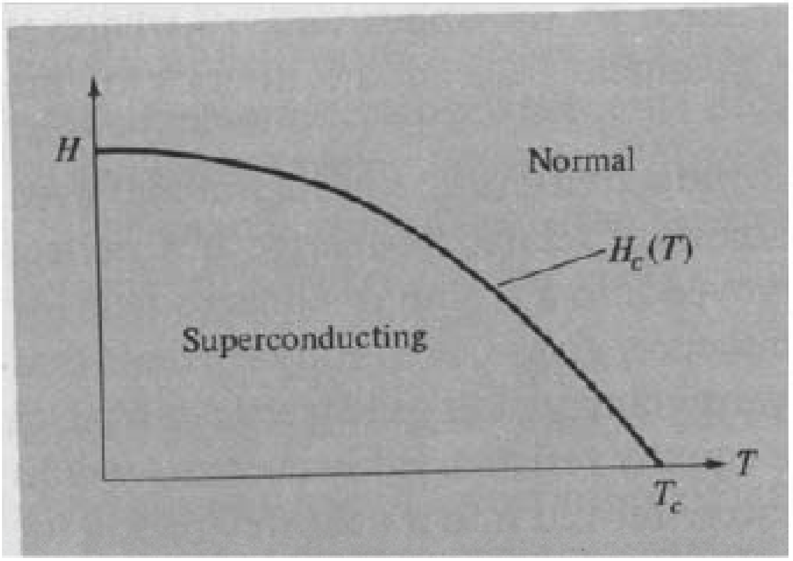
Meissner effect: When a specimen is placed in a magnetic field and then cooled through the transition temperature for superconductivity, the magnetic flux originally present is rejected from the specimen. Thus the transition, when it occurs in a magnetic field, is accompanied by the appearance of whatever surface currents are required to cancel the magnetic field in the interior of the specimen.



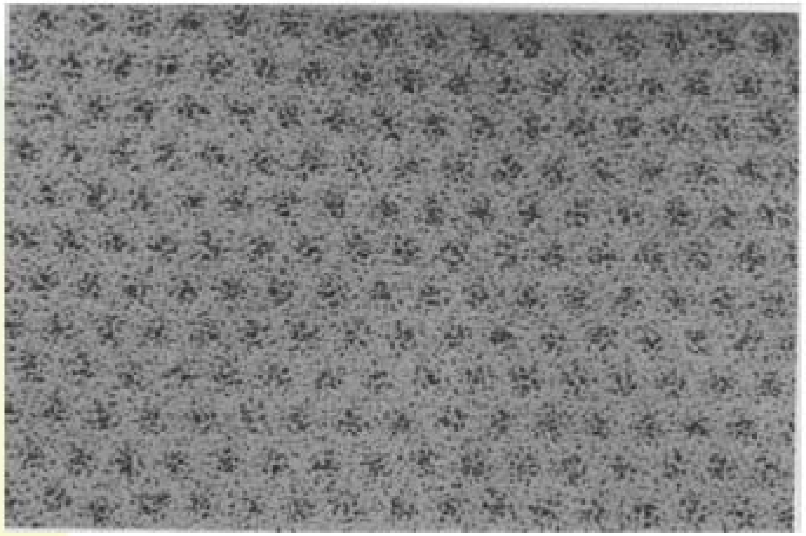
Destruction of Superconductivity by magnetic fields

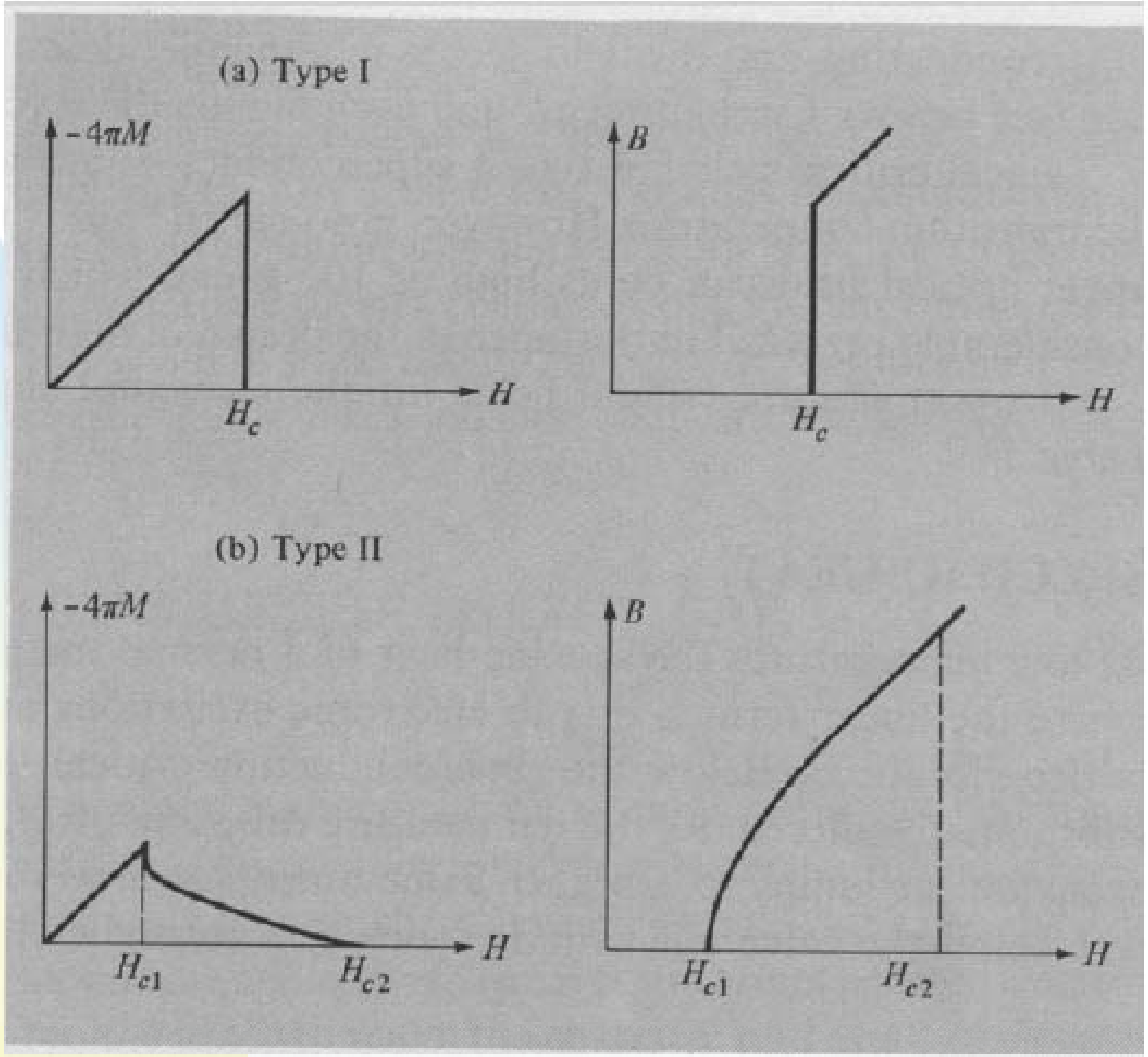
A sufficiently strong magnetic field will destroy superconductivity. The critical field at which superconductivity is suppressed is called $H_c(T)$ and is a function of temperature.

Type I: Below a critical field $H_c(T)$ that increases as T falls below T_c, there is no penetration of flux; when the applied field exceeds $H_c(T)$ the entire specimen reverts to the normal state and the field penetrates perfectly.



Type II: Below a lower critical field $H_{c1}(T)$ there is no penetration of flux; when the applied field exceeds an upper critical field $H_{c2}(T) > H_{c1}(T)$, the entire specimen reverts to the normal state and the field penetrates perfectly. When the applied field strength is between $H_{c1}(T)$ and $H_{c2}(T)$, there is partial penetration of flux, and the sample develops a rather complicated microscopic structure of both normal and superconducting regions, known as the mixed state.

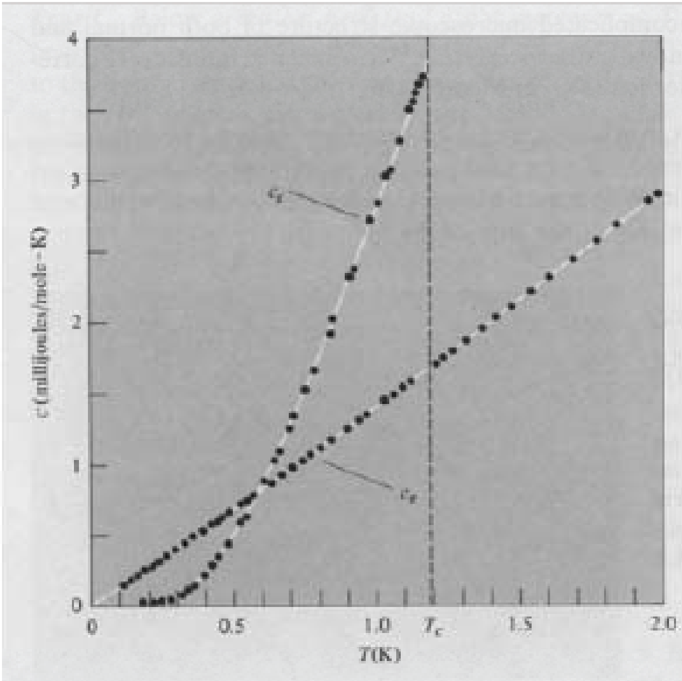




(a) Type I
-4πM
H
Hc
B
H
Hc
(b) Type II
-4πM
H
Hc1
Hc2
B
H
Hc1
Hc2

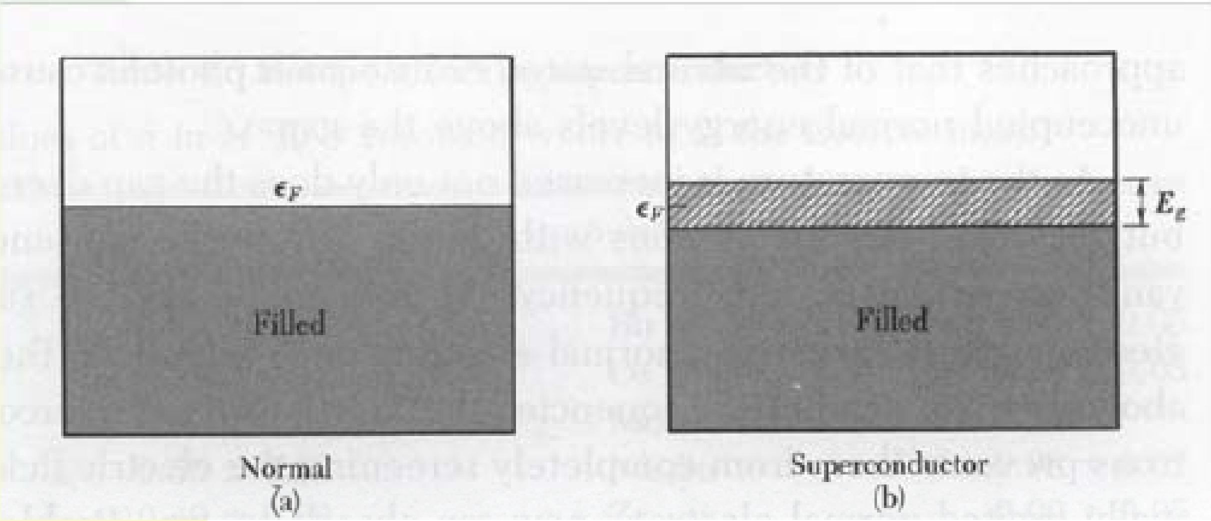
Specific heat

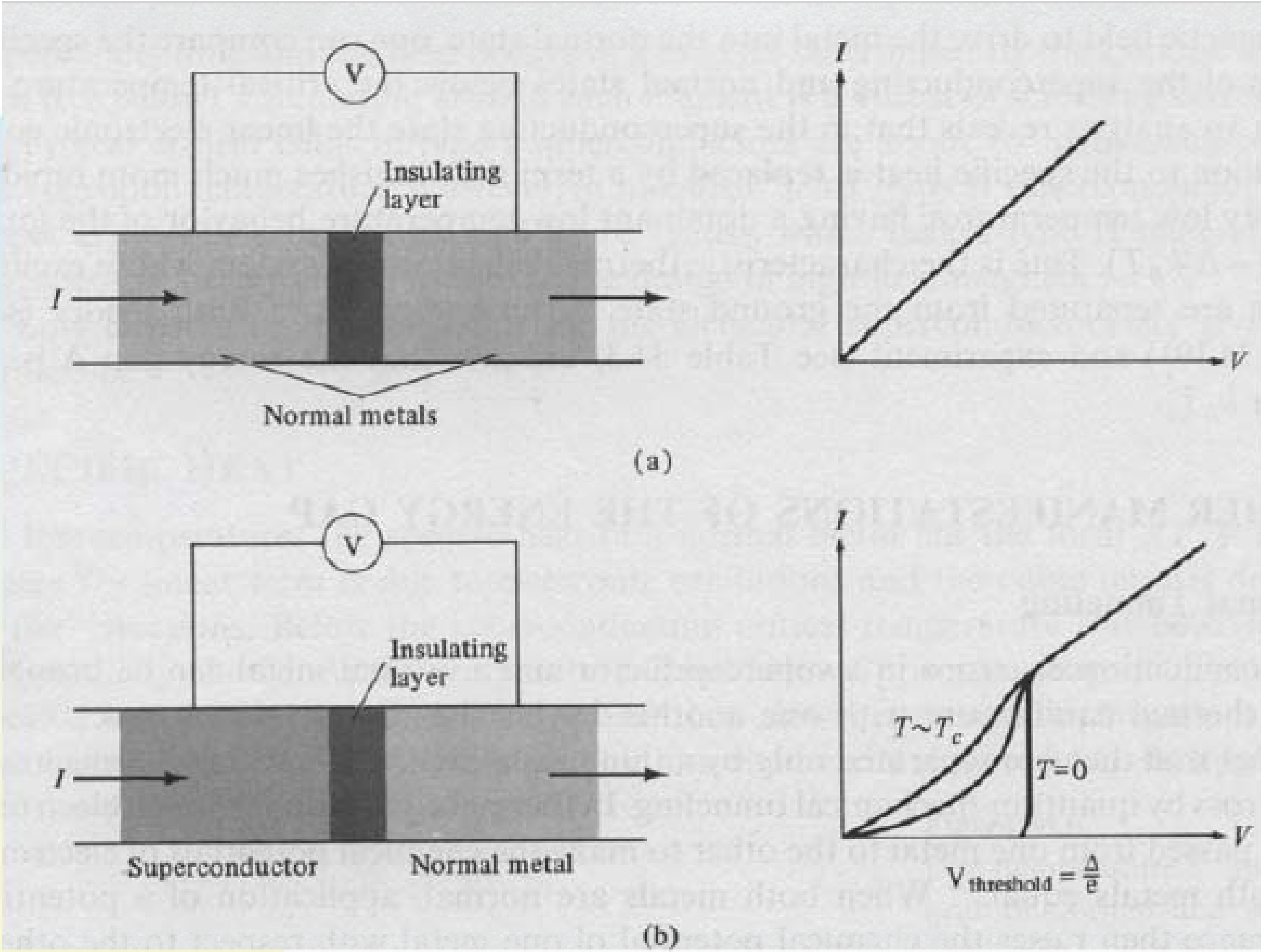
At low temperature the specific heat of a normal metal has the form $AT + BT^3$, where the linear term is due to electronic excitations and the cubic term is due to lattice vibrations. The exponential decay of the specific heat indicates the characteristic thermal behavior of a system whose excited levels are separated from the ground state by an energy 2Δ.



Energy Gap: Normal Tunneling

The conduction electrons in a superconductor and a normal metal can be brought into thermal equilibrium with one another by placing the metals into such close contact that they are separated only by a thin insulating layer, which the electrons can cross by quantum-mechanical tunneling. When one of the metals is superconducting, then no current is observed to flow until the potential V reaches a threshold value, $eV = \Delta$.




V
Insulating layer
I
Normal metals
I
V
(a)
V
Insulating layer
I
Superconductor
Normal metal
I
T~T_c
T=0
V
V threshold = Δ/e
(b)

Isotope effect

The critical temperature of superconductors varies with isotopic mass. The experimental results find: $M^{\alpha}T_c = const.$

From the isotope effect, we know that lattice vibrations and hence electron-lattice interactions are deeply involved in superconductivity.

Table 4 Isotope effect in superconductors

Experimental values of α in $M^{\alpha}T_c$ = constant, where M is the isotopic mass.

Substance	α	Substance	α
Zn	0.45 ± 0.05	Ru	0.00 ± 0.05
Cd	0.32 ± 0.07	Os	0.15 ± 0.05
Sn	0.47 ± 0.02	Mo	0.33
Hg	0.50 ± 0.03	Nb_3Sn	0.08 ± 0.02
Pb	0.49 ± 0.02	Zr	0.00 ± 0.05

The London equation

The assumption of London model: in a superconductor at $T < T_c$, only a fraction $n_s(T)/n$ of the total number of conduction electrons are capable of participating in a supercurrent. $n_s(T)$ is the density of superconducting electrons (superfluid density). The remaining fraction of electrons are "normal fluid" density $n - n_s$ that cannot carry an electric current without normal dissipation.

Suppose that an electric field momentarily arises within a superconductor. The superconducting electrons will be freely accelerated without dissipation: $m\frac{d\vec{v}_s}{dt} = -e\vec{E}$.

Since the current density carried by these electrons is

$\vec{j} = -e\vec{v}_s n_s$, we have $\frac{d}{dt}\vec{j} = \frac{n_s e^2}{m}\vec{E}$.

AC conductivity $\vec{j}(\omega) = \sigma(\omega)\vec{E}(\omega), \sigma(\omega) = i\frac{n_s e^2}{m\omega}$.

Faraday's law of induction, $\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$,

$$\frac{\partial}{\partial t}\left(\nabla \times \vec{j} + \frac{n_s e^2}{mc}\vec{B}\right) = 0.$$

Consider Maxwell equation $\nabla \times \vec{B} = \frac{4\pi}{c}\vec{j}$, that determines the magnetic fields and current densities that can exist within a perfect conductor.

$\nabla^2 \vec{B} = \frac{4\pi n_s e^2}{mc^2}\vec{B}, \nabla^2 \vec{j} = \frac{4\pi n_s e^2}{mc^2}\vec{j}$. These equatios predict that currents and magnetic fields in superconductors can exist only within a layer of thickness Λ of the surface, where Λ is the London penetration depth.

$$\Lambda = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{1/2} = 41.9\left(\frac{r_s}{a_0}\right)^{3/2}\left(\frac{n}{n_s}\right)^{1/2}.$$

Thus the London equation implies the Meissner effect, along with a specific picture of the surface current that screen out the applied field. These currents occur within a surface layer of thickness of $10^2 - 10^3$ Å.

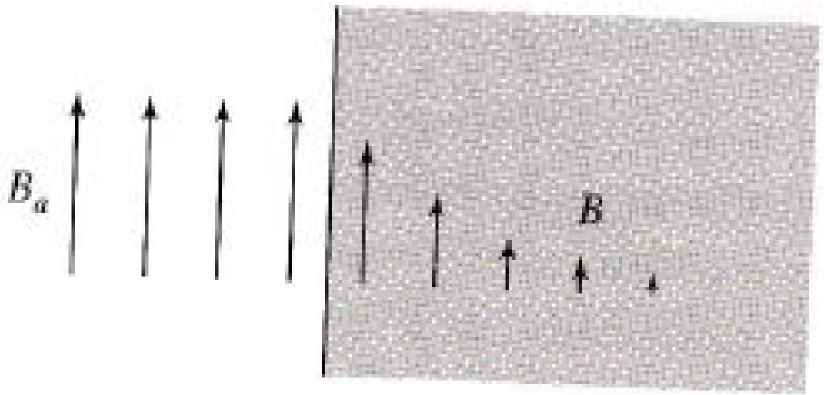


Figure 13 Penetration of an applied magnetic field into a semi-infinite superconductor. The penetration depth λ is defined as the distance in which the field decreases by the factor e^{-1}. Typically, $\lambda \approx 500$ Å in a pure superconductor.

occupy the space on the positive side of the x axis, as in Fig. 13. If $B(0)$ is the field at the plane boundary, then the field inside is

$$B(x) = B(0) \exp(-x/\lambda_L) \ , \qquad (14)$$

Table 5 Calculated intrinsic coherence length and London penetration depth, at absolute zero

Metal	Intrinsic Pippard coherence length ξ_0, in 10^{-6} cm	London penetration depth λ_L, in 10^{-6} cm	λ_L/ξ_0
Sn	23.	3.4	0.16
Al	160.	1.6	0.010
Pb	8.3	3.7	0.45
Cd	76.	11.0	0.14
Nb	3.8	3.9	1.02

After R. Meservey and B. B. Schwartz.

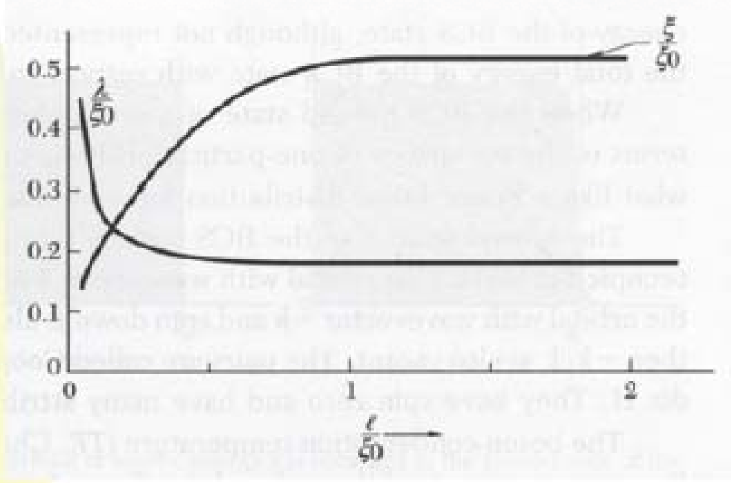
Coherence length

The coherence length is a measure of the distance within which the superconducting electron concentration cannot change drastically in a spatially-varying magnetic field.

We define an intrinsic coherence length ξ_0 related to the critical modulation by $\xi_0 = 1/q_0$. We have

$\xi_0 = \hbar^2 k_F / 2mE_g$.

The coherence length and the actual penetration depth depend on the mean free path of the electrons measured in the normal state.



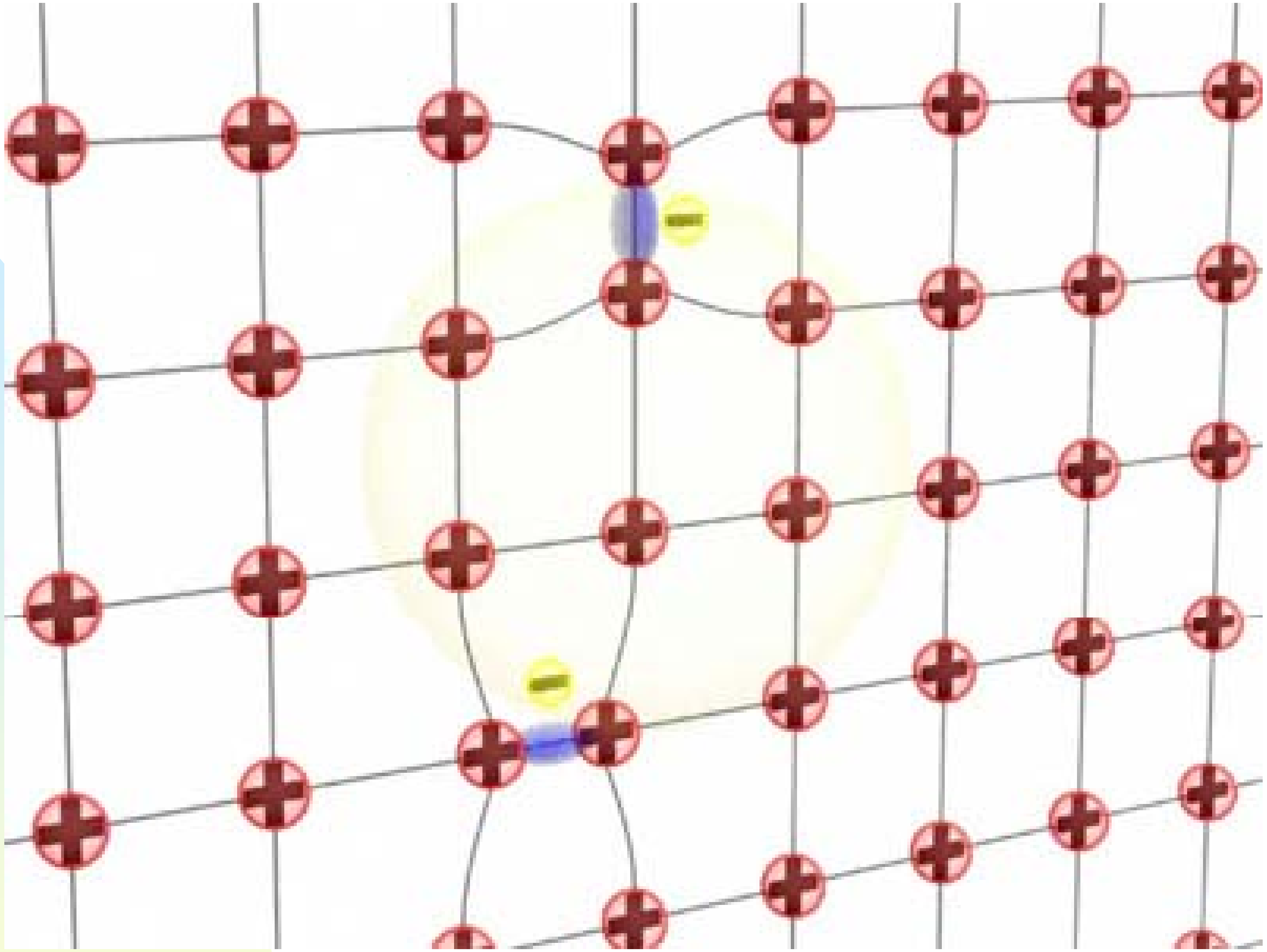
Microscopic theory: qualitative features

Bardeen, Cooper, Schrieffer theory of superconductivity:

1. A net attractive interaction between electrons in the neighborhood of the Fermi surface. Although the direct electrostatic interaction is repulsive, it is possible for the ionic motion to "overscreen" the Coulomb interaction, leading to a net attraction.
2. The electron-lattice-electron interaction leads to an energy gap of the observed magnitude. The indirect interaction proceeds when one electron interacts with the lattice and deforms it; a second electron sees the deformed lattice and adjusts itself to take advantage of the deformation to lower its energy. Thus the net attractive force.
3. The penetration depth and the coherence length are natural consequences of the BCS theory.

4. The criterion for the T_c of an element or alloy involves the electron density of orbital $D(\varepsilon_F)$ of one spin at the Fermi level and the electron lattice interaction U, which can be estimated from the electrical resistivity. For $UD(\varepsilon_F) \ll 1$ the BCS theory predicts $T_c = 1.13\theta e^{-1/UD(\varepsilon_F)}$. Where θ is the Debye temperature and U is an attractive interaction. The result for T_c is satisfied at least qualitatively by the experimental data. The higher the resistivity at room temperature the higher is U, and thus the more likely it is that a metal will be a superconductor when cooled.

5. Mangetic flux through a superconducting ring is quantized and the effective unit of charge is $2e$ rather than e. The BCS ground state involves pairs of electrons.



http://www.magnet.fsu.edu/education/tutorials/magnetacademy/superconductivity101/fullarticle.html

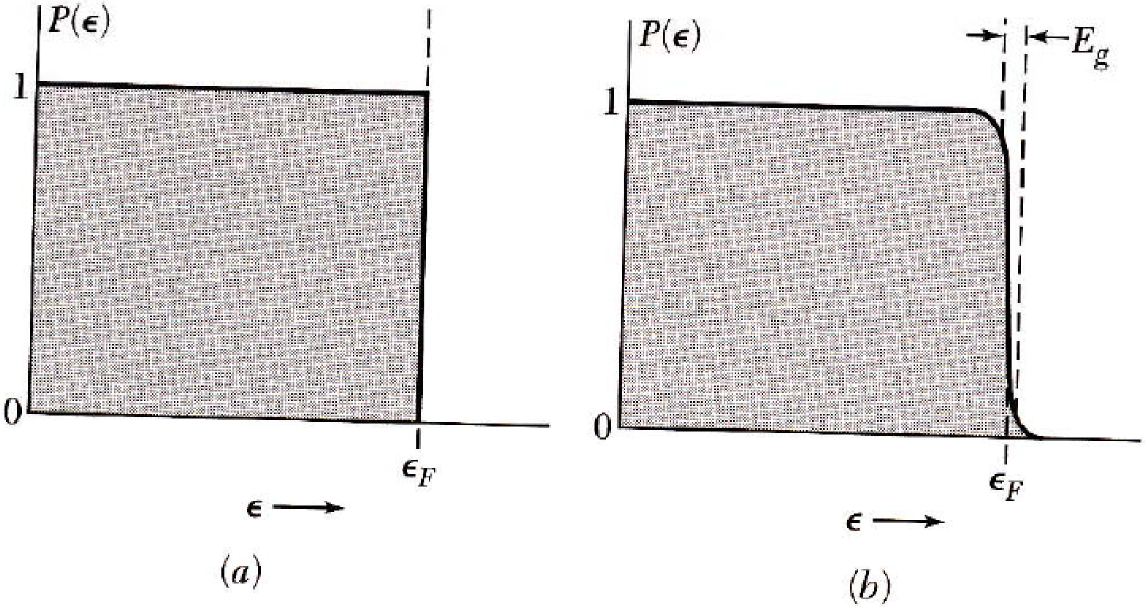
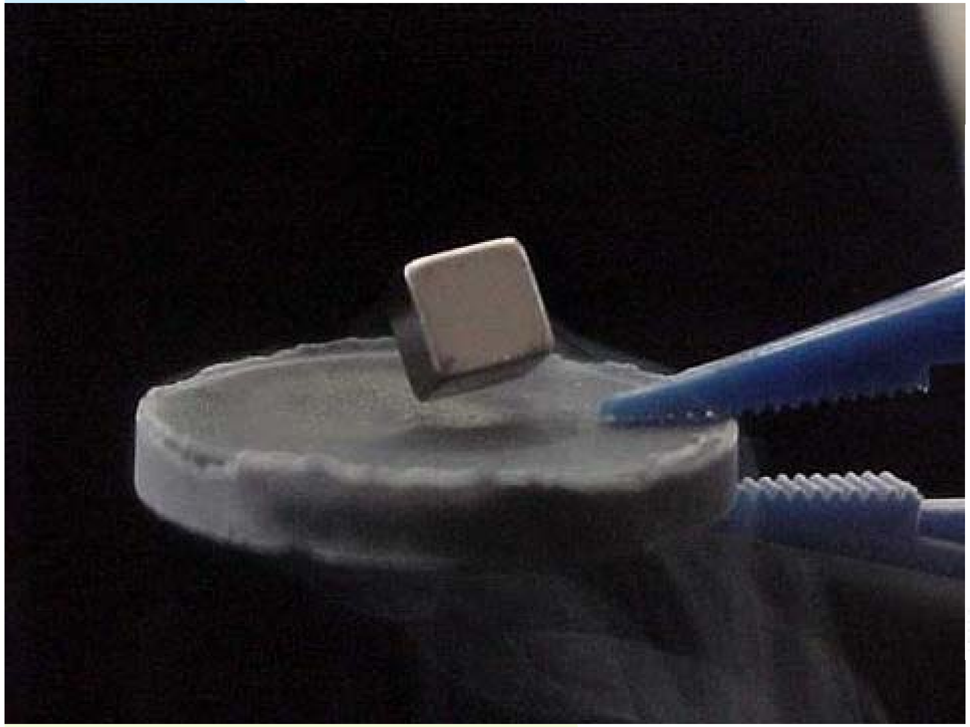
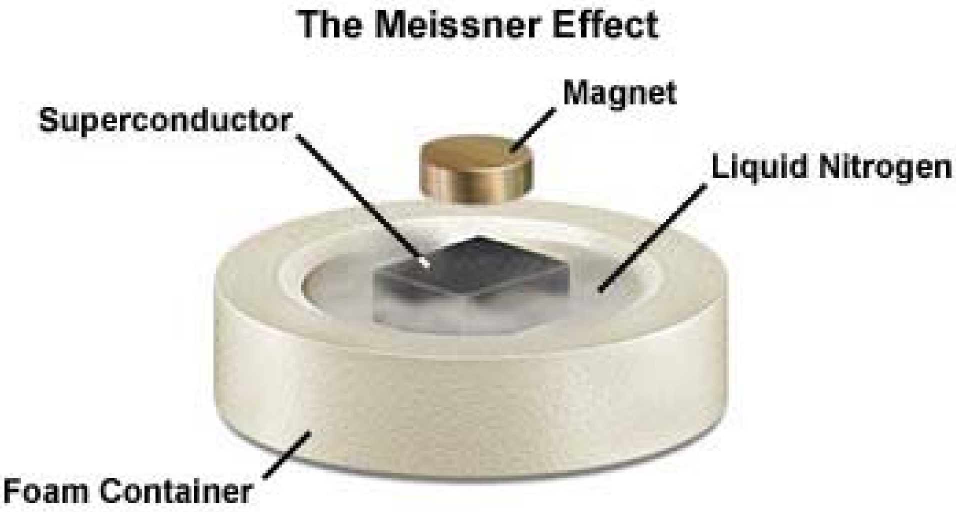


Figure 15 (a) Probability P that an orbital of kinetic energy ϵ is occupied in the ground state of the noninteracting Fermi gas; (b) the BCS ground state differs from the Fermi state in a region of width of the order of the energy gap E_g. Both curves are for absolute zero.

The formation of the BCS ground state is suggested by Fig. 15. The BCS state in (b) contains admixtures of one-electron orbitals from above the Fermi energy ϵ_F. At first sight the BCS state appears to have a higher energy than the Fermi state: the comparison of (b) with (a) shows that the kinetic energy of the BCS state is higher than that of the Fermi state. But the attractive potential energy of the BCS state, although not represented in the figure, acts to lower the total energy of the BCS state with respect to the Fermi state.

When the BCS ground state of a many-electron system is described in terms of the occupancy of one-particle orbitals, those near ϵ_F are filled somewhat like a Fermi-Dirac distribution for some finite temperature.




The Meissner Effect
Magnet
Superconductor
Liquid Nitrogen
Foam Container

- BCS superconductors

$$k_B T_c = 1.13\hbar\omega_D \cdot e^{-\frac{1}{N(0)|V|}}$$

$N(0)$ is the density of electronic levels for a single spin population in the normal metal and ω and V_0 are the parameters of the model Hamiltonian. Because of the exponential dependence, the effective coupling V_0 cannot be determined precisely enough to permit very accurate computations of the critical temperature.

- Higher T_c is achieved with:
  - Larger ω_D → lighter elements
  - Larger N(0) → van Hove singularity *etc.*
  - Larger |V| → stronger electron-phonon int.

Energy gap

The zero temperature energy gap:

$$\Delta(0) = 2\hbar\omega_D \cdot e^{-\frac{1}{N(0)|V|}}$$

The fundamental formula for the relationship between T_c and gap independent of the phenomenological parameters:

$$\frac{\Delta(0)}{k_B T_c} = 1.76,$$

The BCS theory also predicts that near the critical temperature the energy gap vanishes according to the universal law

$$\frac{\Delta(T)}{\Delta(0)} = 1.74\left(1 - \frac{T}{T_c}\right)^{1/2}, T \approx T_c.$$

Table 34.3

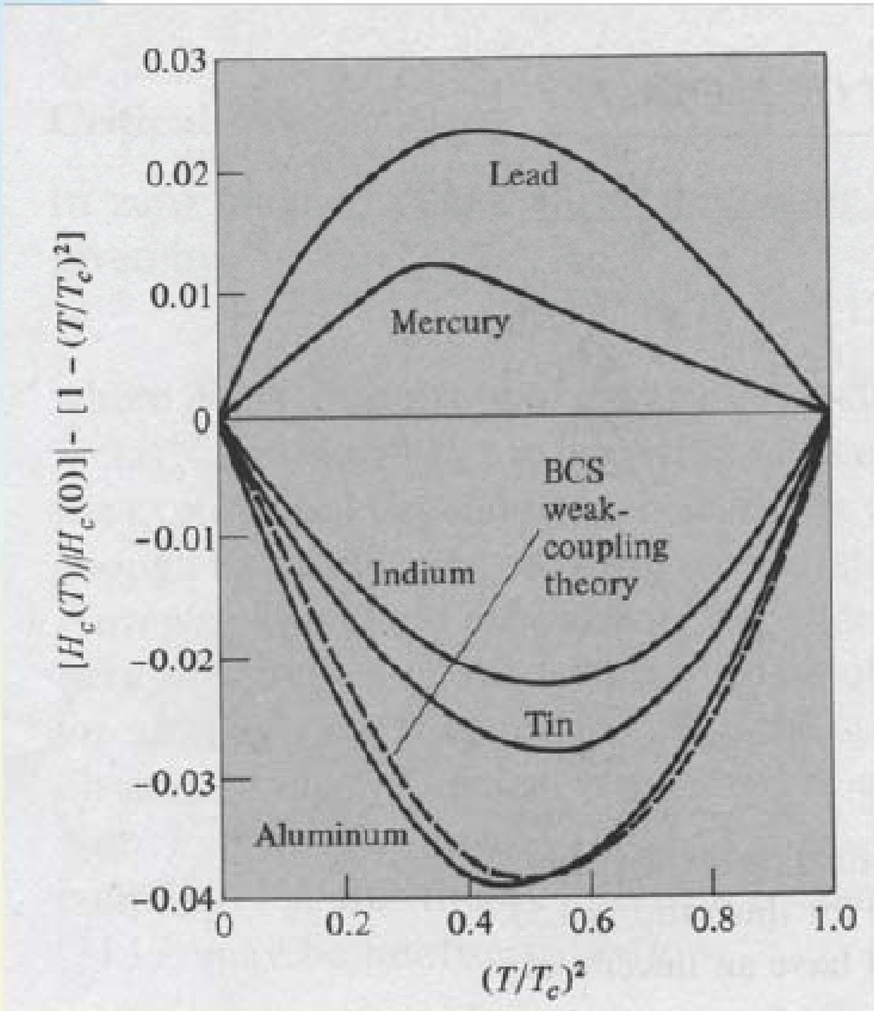
MEASURED VALUES[a] OF $2\Delta(0)/k_B T_c$

ELEMENT	$2\Delta(0)/k_B T_c$
Al	3.4
Cd	3.2
Hg (α)	4.6
In	3.6
Nb	3.8
Pb	4.3
Sn	3.5
Ta	3.6
Tl	3.6
V	3.4
Zn	3.2

[a] $\Delta(0)$ is taken from tunneling experiments. Note that the BCS value for this ratio is 3.53. Most of the values listed have an uncertainty of ± 0.1.

Source: R. Mersevey and B. B. Schwartz, *Superconductivity*, R. D. Parks, ed., Dekker, New York, 1969.

Critical Field



The elementary BCS prediction for $H_c(T)$ is often expressed in terms of the deviation from the empirical law:

$$\frac{H_c(T)}{H_c(0)} \approx 1 - \left(\frac{T}{T_c}\right)^2 .$$

The quantity $[H_c(T)/H_c(0)] - [1-(T/T_c)^2]$ is shown for several superconductors below.

Specific heat

At the critical temperature (in zero magnetic field) the elementary BCS theory predicts a discontinuity in the specific heat:

$$\left.\frac{c_s - c_n}{c_n}\right|_{T_c} = 1.43.$$

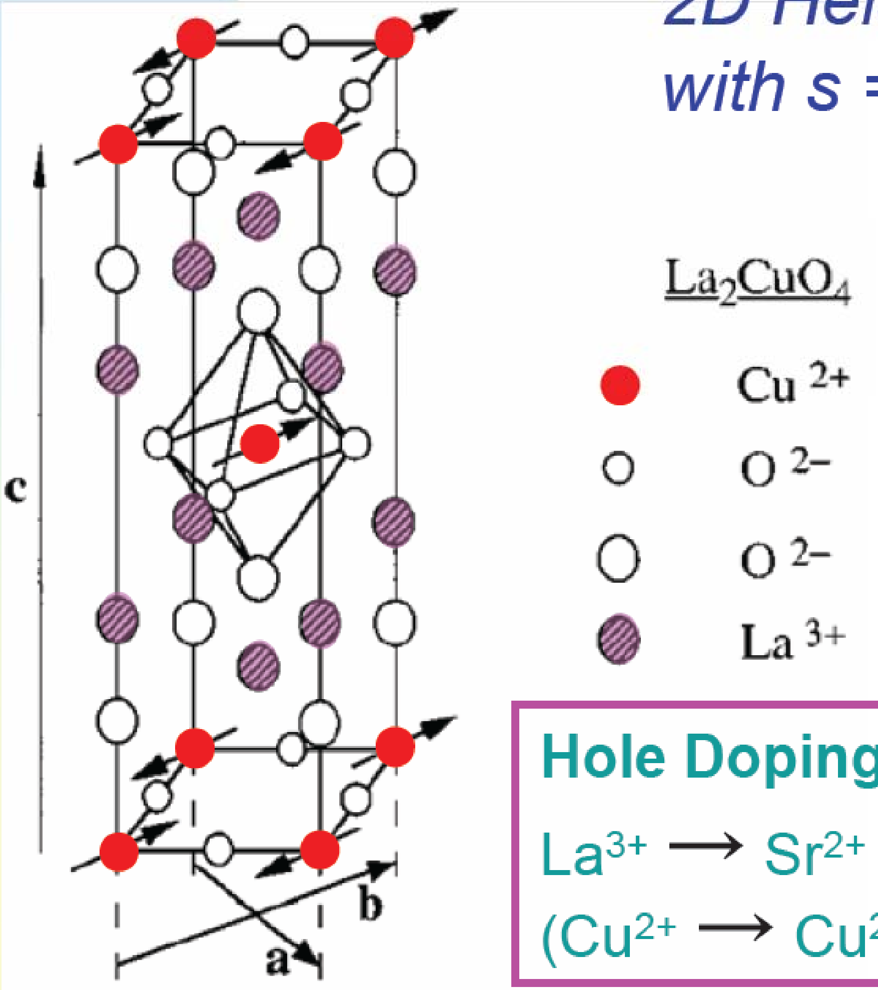
The low-temperature electronic specific heat can also be cast in a parameter independent form,

$$\frac{c_s}{\gamma T_c} = 1.34\left(\frac{\Delta(0)}{T}\right)^{3/2} e^{-\Delta(0)/T},$$

where γ is the coefficient of the linear term in the specific heat of the metal in the normal state.

High-temperature superconductors

- Parent Cuprate La_2CuO_4

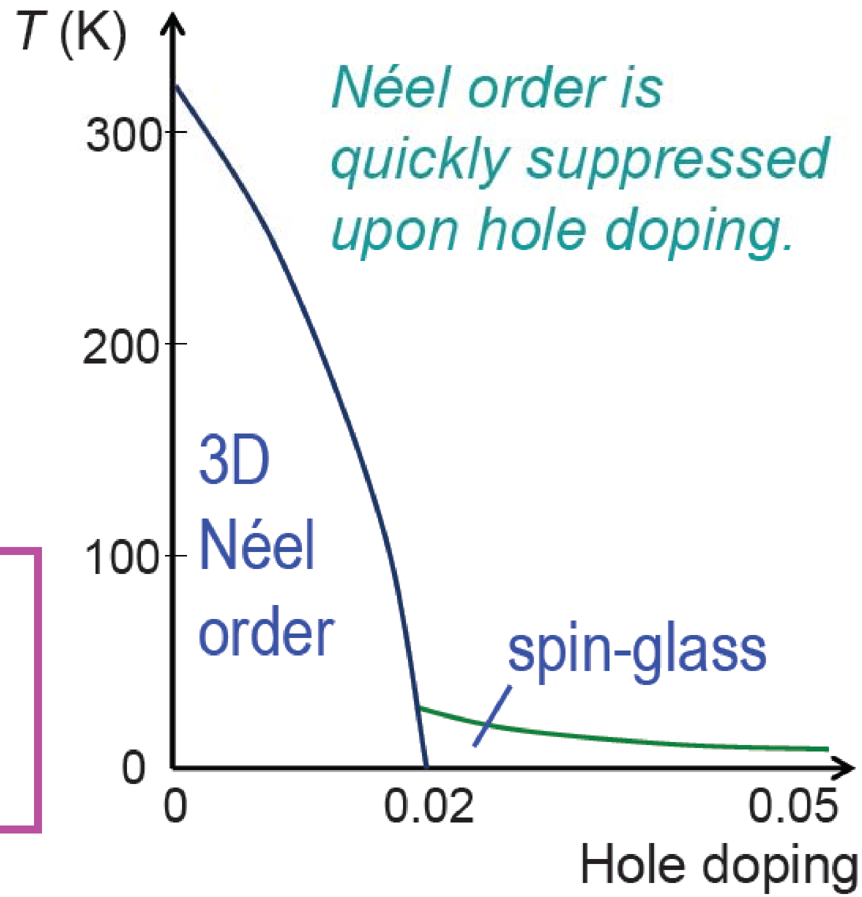


2D Heisenberg Antiferromagnet with s = 1/2 on a square lattice

Hole Doping:

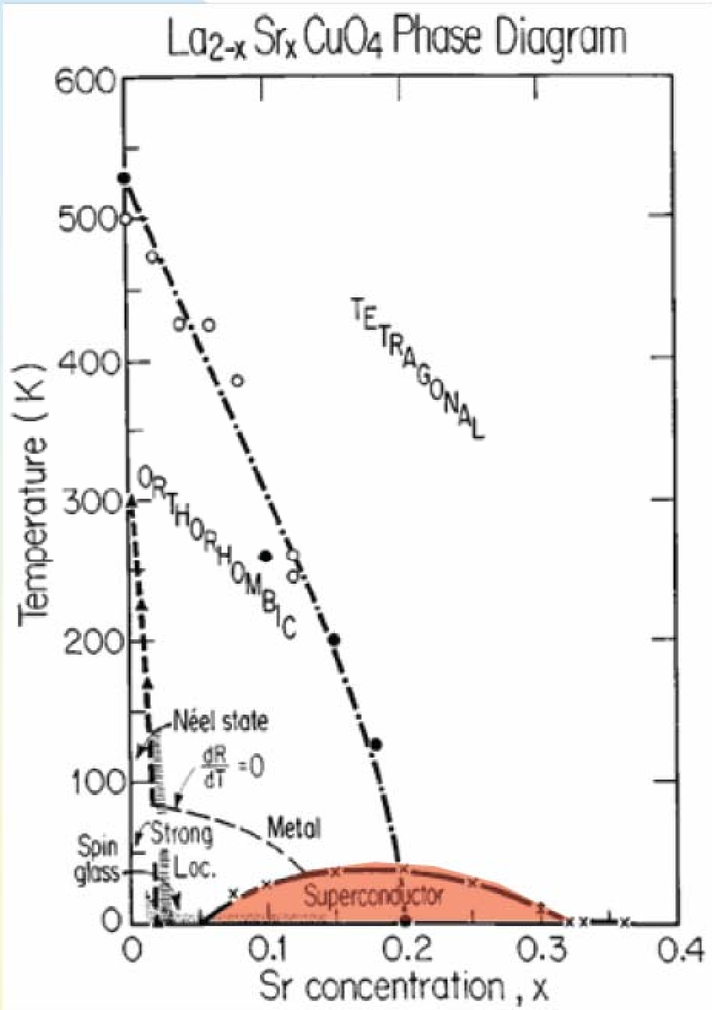
$La^{3+} \longrightarrow Sr^{2+}$

$(Cu^{2+} \longrightarrow Cu^{2+\delta})$



High-T_c Superconductivity

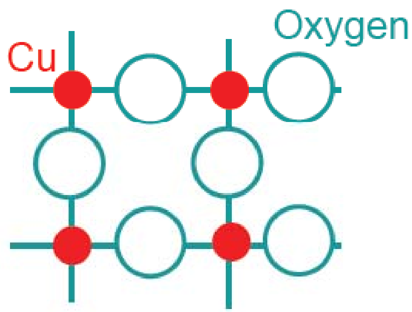
- Superconductivity shows up when moderately doped.



- Layered structure

→ Essentially 2D transport

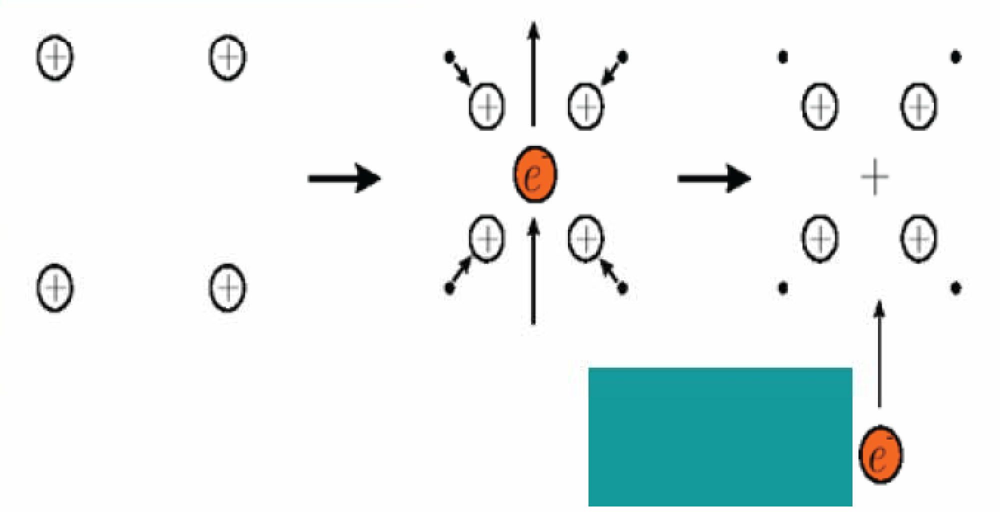
Basic building block: CuO_2 plane



Only <u>one band</u> is relevant to the transport in the CuO_2 plane. (Cu 3d – O 2p mixture)

Electrons form Cooper pairs

Conventional low temperature SC

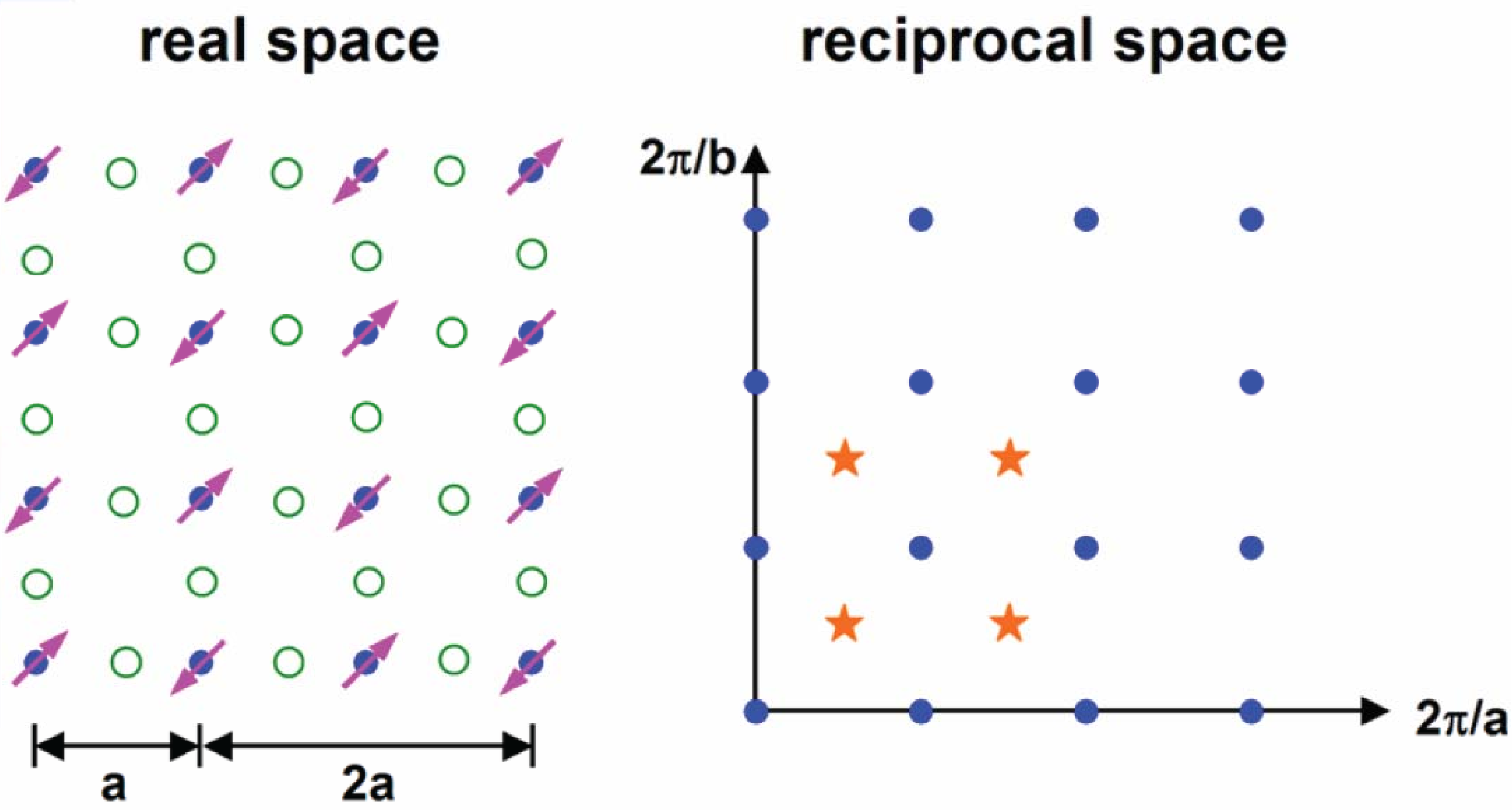


- The binding of electrons into Cooper pairs is essential.

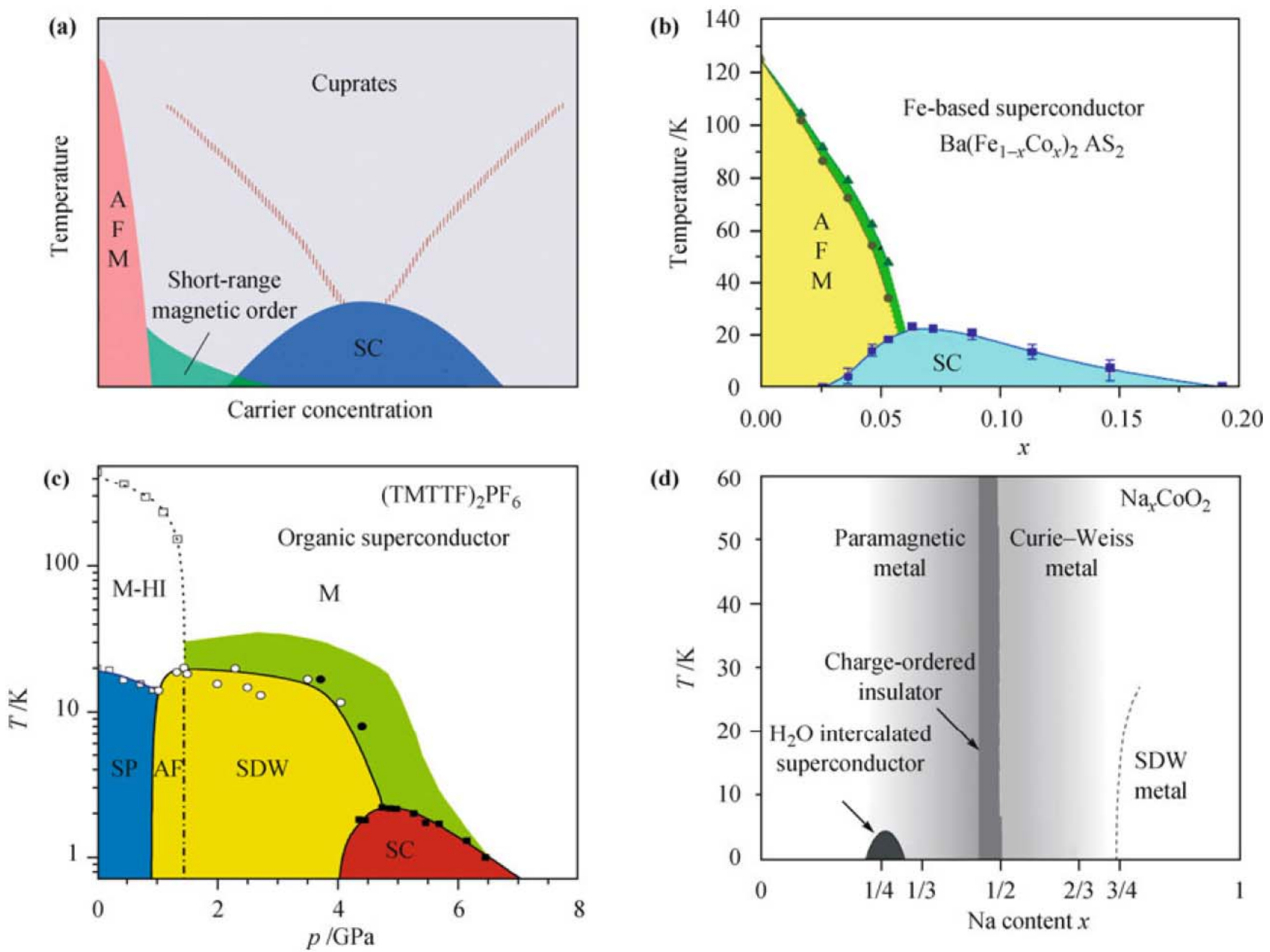
$$S = \uparrow\downarrow = 0$$

- Long-range phase coherence among the pairs is also required to have superconductivity.
- Electron pairing is mediated by electron-phonon interaction.

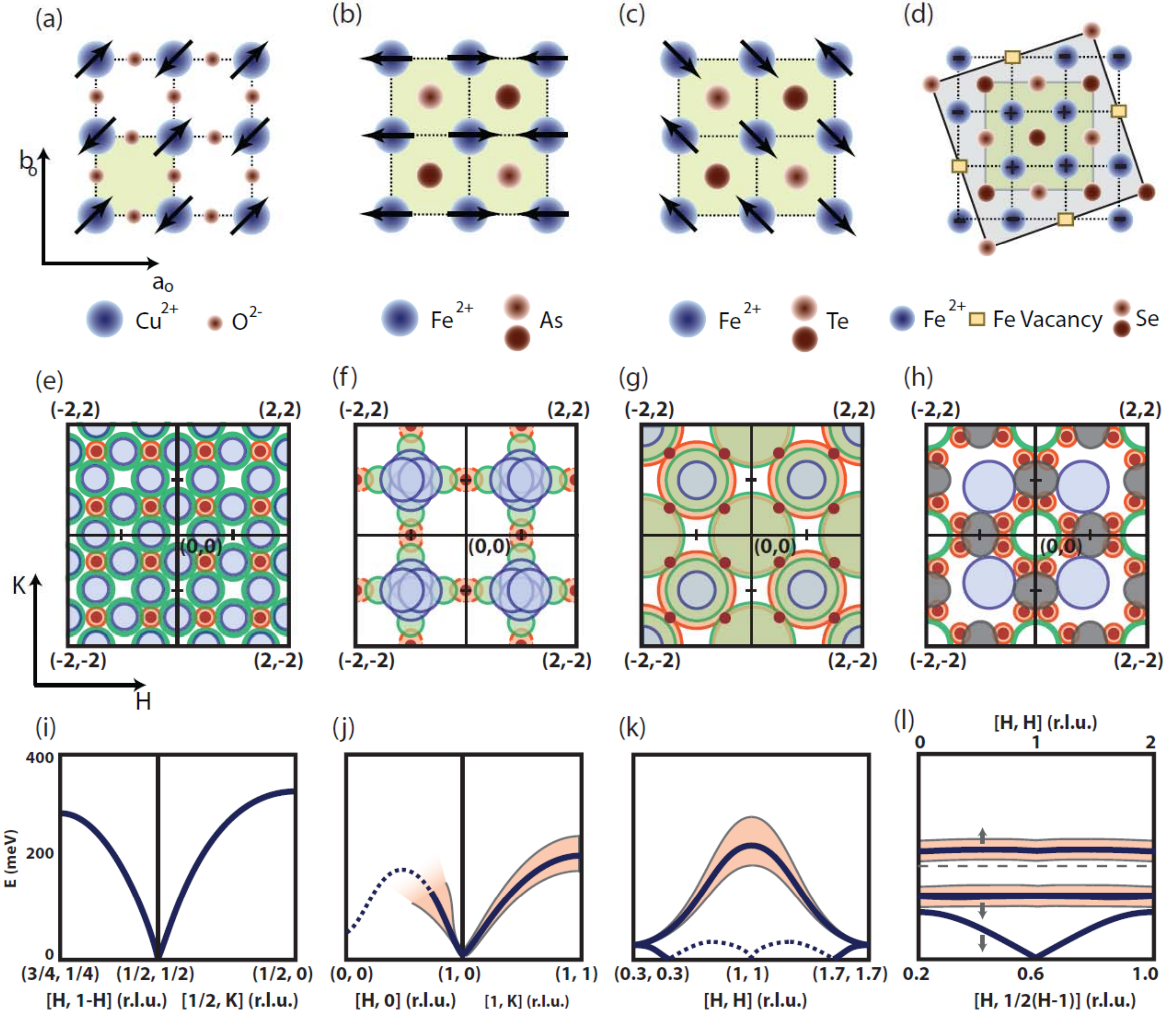
Real, reciprocal space of CuO_2 plane

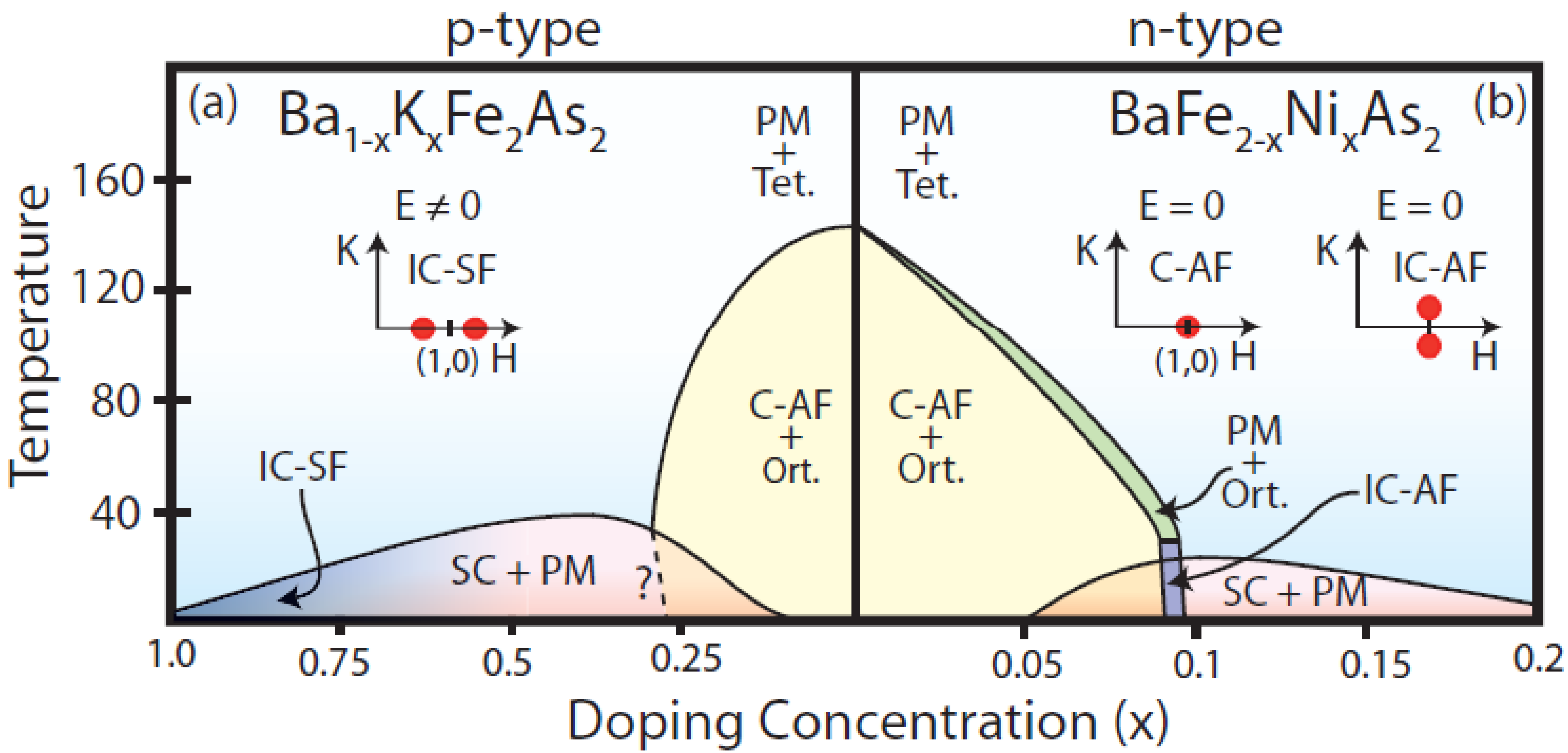


$La_{2-x}Sr_xCuO_4$ single layer, orthorhombic
$Nd_{2-x}Ce_xCuO_4$ single layer, tetragonal



(a)
Cuprates
Temperature
A
F
M
Short-range
magnetic order
SC
Carrier concentration
(b)
Fe-based superconductor
Ba(Fe1−xCox)2 AS2
Temperature /K
A
F
M
SC
x
(c)
(TMTTF)2PF6
Organic superconductor
M-HI
M
SP
AF
SDW
SC
T /K
p /GPa
(d)
NaxCoO2
Paramagnetic
metal
Curie–Weiss
metal
Charge-ordered
insulator
H2O intercalated
superconductor
SDW
metal
T /K
Na content x


(a)
(b)
(c)
(d)
b_o
a_o
Cu^{2+}
O^{2-}
Fe^{2+}
As
Fe^{2+}
Te
Fe^{2+}
Fe Vacancy
Se
(e)
(f)
(g)
(h)
(-2,2)
(2,2)
(-2,-2)
(2,-2)
(0,0)
K
H
(i)
(j)
(k)
(l)
E (meV)
400
200
0
(3/4, 1/4)
(1/2, 1/2)
(1/2, 0)
[H, 1-H] (r.l.u.)
[1/2, K] (r.l.u.)
(0, 0)
(1, 0)
(1, 1)
[H, 0] (r.l.u.)
[1, K] (r.l.u.)
(0.3, 0.3)
(1, 1)
(1.7, 1.7)
[H, H] (r.l.u.)
[H, H] (r.l.u.)
0
1
2
0.2
0.6
1.0
[H, 1/2(H-1)] (r.l.u.)



p-type
n-type
(a)
$Ba_{1-x}K_xFe_2As_2$
$BaFe_{2-x}Ni_xAs_2$
(b)
PM + Tet.
PM + Tet.
E ≠ 0
E = 0
E = 0
K
IC-SF
C-AF
IC-AF
(1,0) H
(1,0) H
H
C-AF + Ort.
C-AF + Ort.
PM + Ort.
IC-SF
IC-AF
SC + PM
SC + PM
?
160
120
80
40
Temperature
1.0
0.75
0.5
0.25
0.05
0.1
0.15
0.2
Doping Concentration (x)

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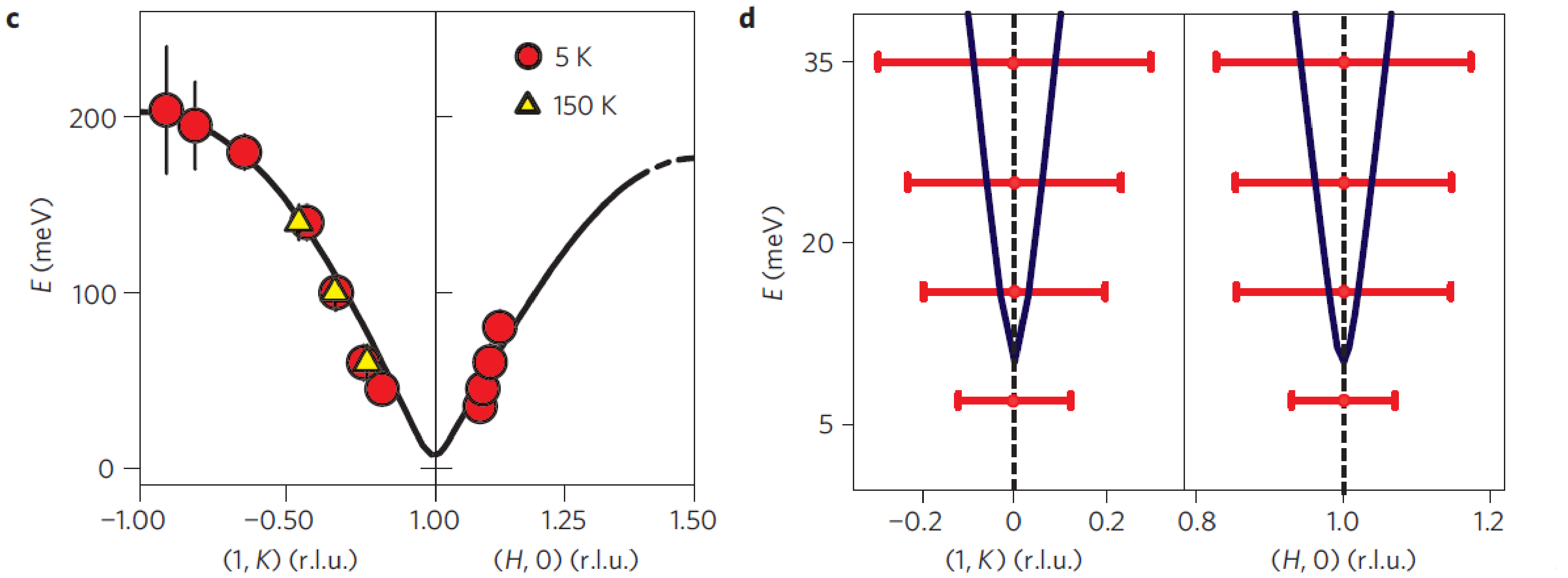
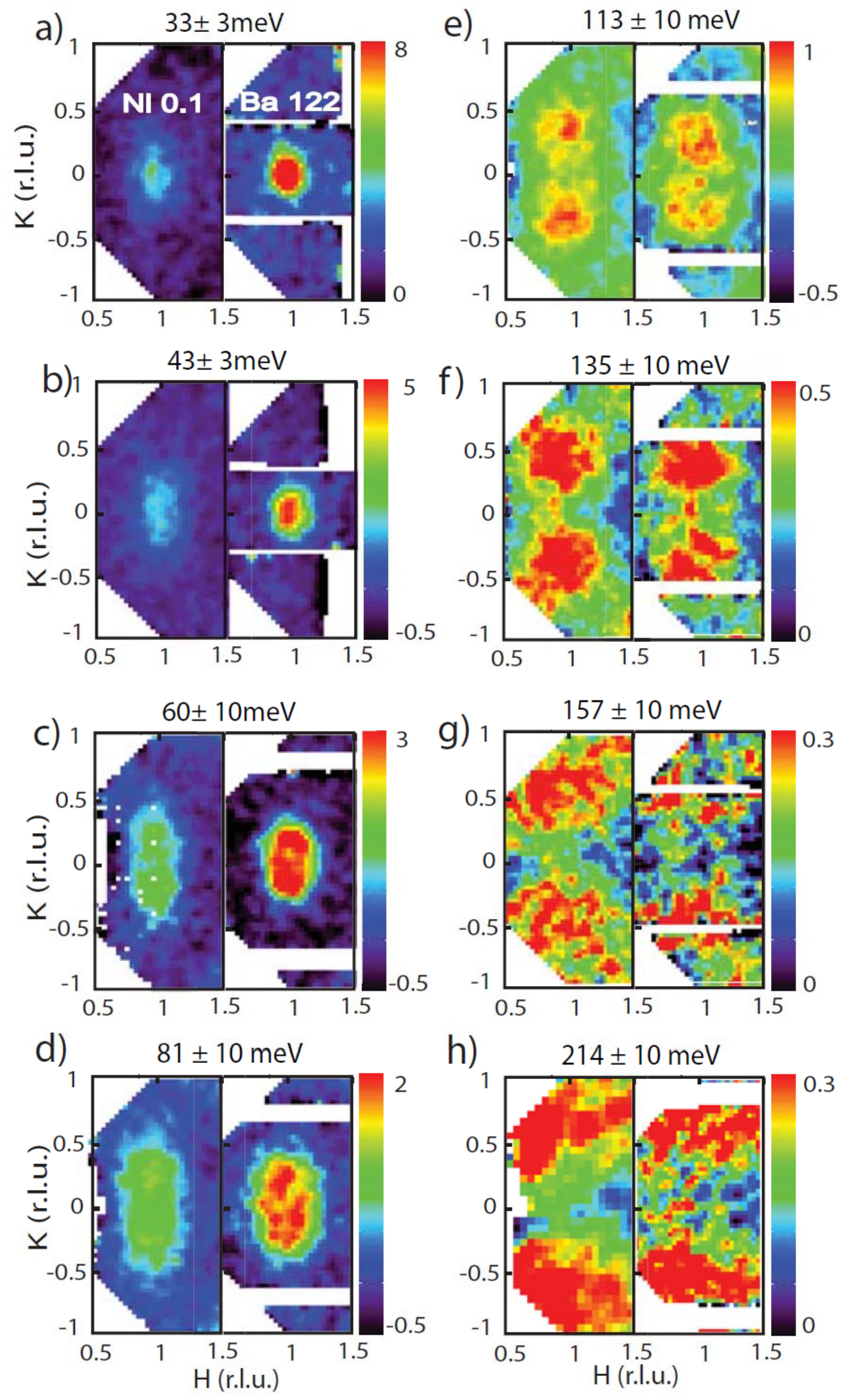
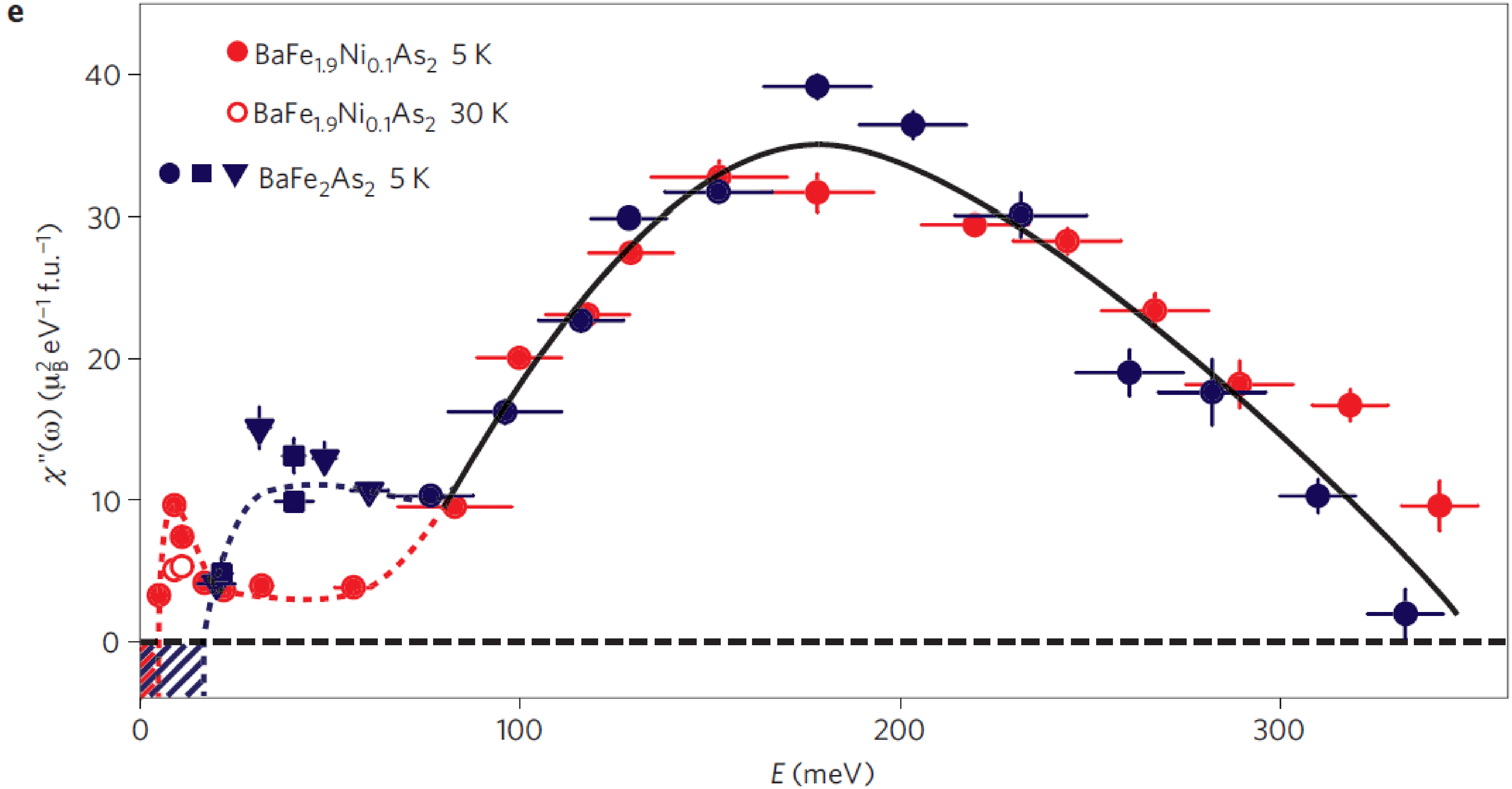
Nature of magnetic excitations in superconducting $BaFe_{1.9}Ni_{0.1}As_2$

Mengshu Liu[1], Leland W. Harriger[1], Huiqian Luo[2], Meng Wang[1,2], R. A. Ewings[3], T. Guidi[3], Hyowon Park[4], Kristjan Haule[4], Gabriel Kotliar[4], S. M. Hayden[5] and Pengcheng Dai[1,2]*





a)
33± 3meV
NI 0.1
Ba 122
b)
43± 3meV
c)
60± 10meV
d)
81 ± 10 meV
e)
113 ± 10 meV
f)
135 ± 10 meV
g)
157 ± 10 meV
h)
214 ± 10 meV
K (r.l.u.)
H (r.l.u.)


e
BaFe1.9Ni0.1As2 5 K
BaFe1.9Ni0.1As2 30 K
BaFe2As2 5 K
χ''(ω) (μB2 eV−1 f.u.−1)
40
30
20
10
0
100
200
300
E (meV)