

PHYS 342/555

Condensed Matter in a Nutshell

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Chapter 13: Plasmons, Polaritons, and polarons

Lecture in pdf format will be available at:

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Chapter 13 Plasmons, Polaritons, and Polarons

The dielectric constant ϵ of electrostatics is defined in terms of the electric field E and the polarization P , the dipole moment density:

$$\vec{D} = \vec{E} + 4\pi\vec{P} = \epsilon\vec{E}; \quad \epsilon \text{ is the relative permittivity.}$$

Suppose a positively charged particle is placed at a fixed position inside the electron gas. It will then attract electrons and create a surplus negative charge around it, thus reducing its field.

If ρ_{ext} is the particle's charge density without screening, the full charge density should be $\rho = \rho_{\text{ext}} + \rho_{\text{ind}}$.

\vec{D} is related to the external applied charge density ρ_{ext} and \vec{E} is related to the total charge density $\rho = \rho_{\text{ext}} + \rho_{\text{ind}}$.

$$\text{div}\vec{D} = \text{div}\epsilon\vec{E} = 4\pi\rho_{\text{ext}}$$

$$\text{div}\vec{E} = 4\pi(\rho_{\text{ext}} + \rho_{\text{ind}}).$$

Define $\varepsilon(\vec{K})$ such that:

$$\vec{D}(\vec{K}) = \varepsilon(\vec{K})\vec{E}(\vec{K}); \quad \varepsilon(\vec{K}) \text{ is the relative permittivity.}$$

Then we have

$$\text{div}\vec{E} = \text{div}\sum E(\vec{K})e^{i\vec{K}\cdot\vec{r}} = 4\pi\sum \rho(K)e^{i\vec{K}\cdot\vec{r}}.$$

$$\text{div}\vec{D} = \text{div}\sum \varepsilon(\vec{K})E(\vec{K})e^{i\vec{K}\cdot\vec{r}} = 4\pi\sum \rho_{\text{ext}}(K)e^{i\vec{K}\cdot\vec{r}}$$

$$\varepsilon(\vec{K}) = \rho_{\text{ext}}(K) / \rho(K) = 1 - \rho_{\text{ind}}(K) / \rho(K)$$

The electrostatic potentials satisfy the Poisson equation

$$\nabla^2\varphi_{\text{ext}} = -4\pi\rho_{\text{ext}}; \quad \text{and} \quad \nabla^2\varphi = -4\pi\rho.$$

$$\frac{\varphi_{\text{ext}}(\vec{K})}{\varphi(\vec{K})} = \frac{\rho_{\text{ext}}(\vec{K})}{\rho(\vec{K})} = \varepsilon(\vec{K}).$$

Plasma optics

The equation of motion of a free electron in an electric field:

$$m \frac{d^2 x}{dt^2} = -eE. \text{ if } x \text{ and } E \text{ have the time dependence } e^{-i\omega t}, \text{ then}$$
$$-\omega^2 mx = -eE; \quad x = eE / m\omega^2.$$

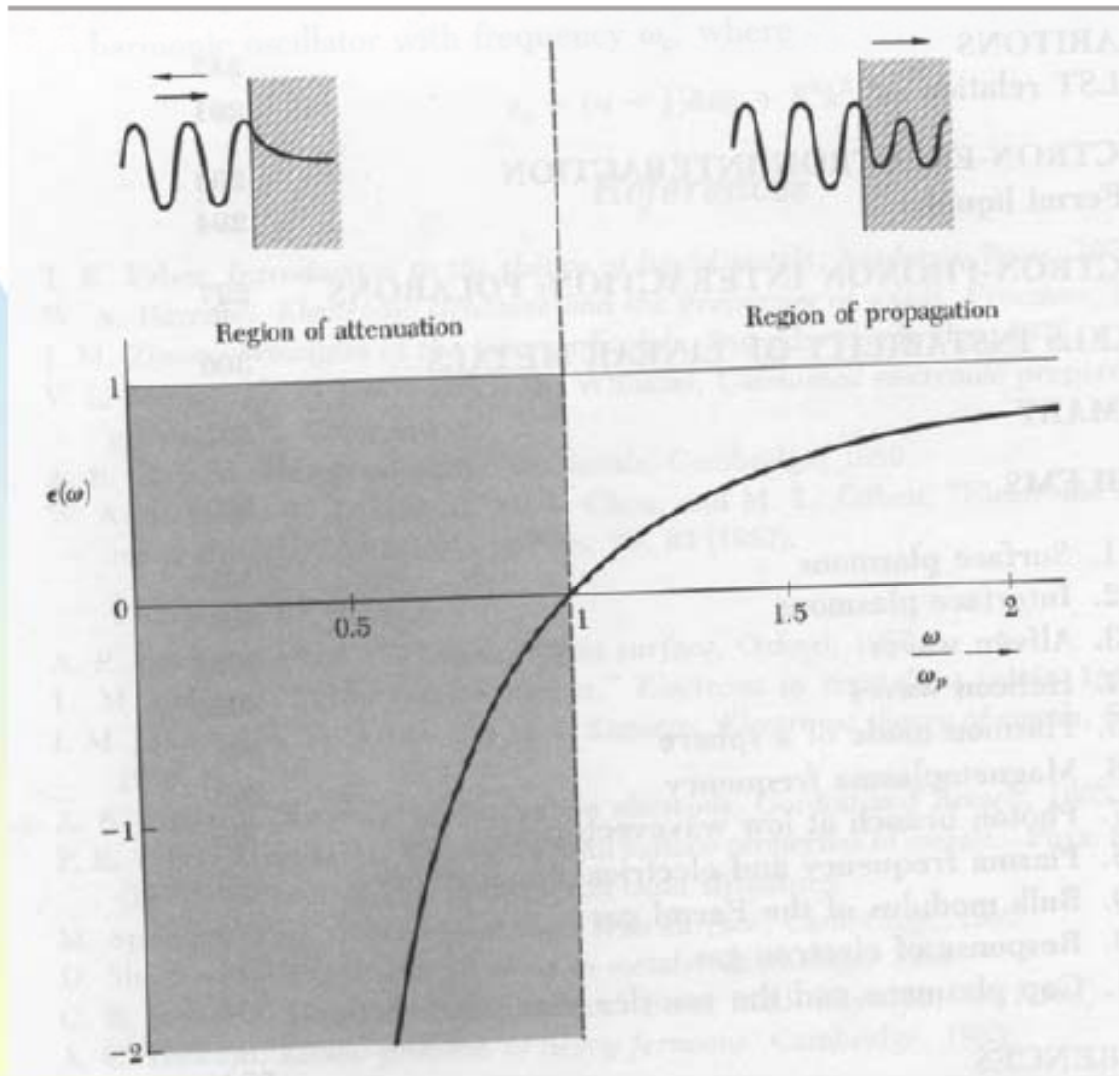
The dipole moment of one electron is $-ex = -e^2 E / m\omega^2$, and the polarization defined as the dipole moment per unit volume is

$P = -nex = -ne^2 E / m\omega^2$, the dielectric function is then

$$\varepsilon(\omega) \equiv \frac{D(\omega)}{E(\omega)} \equiv 1 + 4\pi \frac{P(\omega)}{E(\omega)} = 1 - \frac{4\pi ne^2}{m\omega^2} \equiv 1 - \frac{\omega_p^2}{\omega^2}, \omega_p = 4\pi ne^2 / m.$$

If positive ion core background has a dielectric constant $\varepsilon(\infty)$,

$$\varepsilon(\omega) = \varepsilon(\infty) - 4\pi ne^2 / m\omega^2 = \varepsilon(\infty)[1 - \tilde{\omega}_p^2 / \omega^2].$$



Dispersion relation for electromagnetic waves

Maxwell's equation for a nonmagnetic isotropic medium,

$$\frac{\partial^2 \vec{D}}{\partial t^2} = c^2 \nabla^2 \vec{E}.$$

If $\vec{E} \propto e^{i\vec{k}\cdot\vec{r}} e^{-i\omega t}$ and $\vec{D} = \varepsilon(\omega, \vec{K}) \vec{E}$; then

$$\varepsilon(\omega, \vec{K}) \omega^2 = c^2 K^2;$$

If ε is real and >0 : \vec{K} is real and a transverse electromagnetic wave propagates with the phase velocity $c / \varepsilon^{1/2}$.

If ε is real and <0 : \vec{K} is imaginary and the wave is damped with a characteristic length $1/|\vec{K}|$.

If ε is complex. For ω real, \vec{K} is complex and the waves are damped in space.

$\varepsilon = \infty$. This means the system has a finite response in the absence of an applied force; thus the poles of $\varepsilon(\omega, K)$ define the frequencies of the free oscillations of the medium.

$\varepsilon = 0$. Longitudinally polarized waves are possible only at the zero of ε .

Dispersion relation for electromagnetic waves

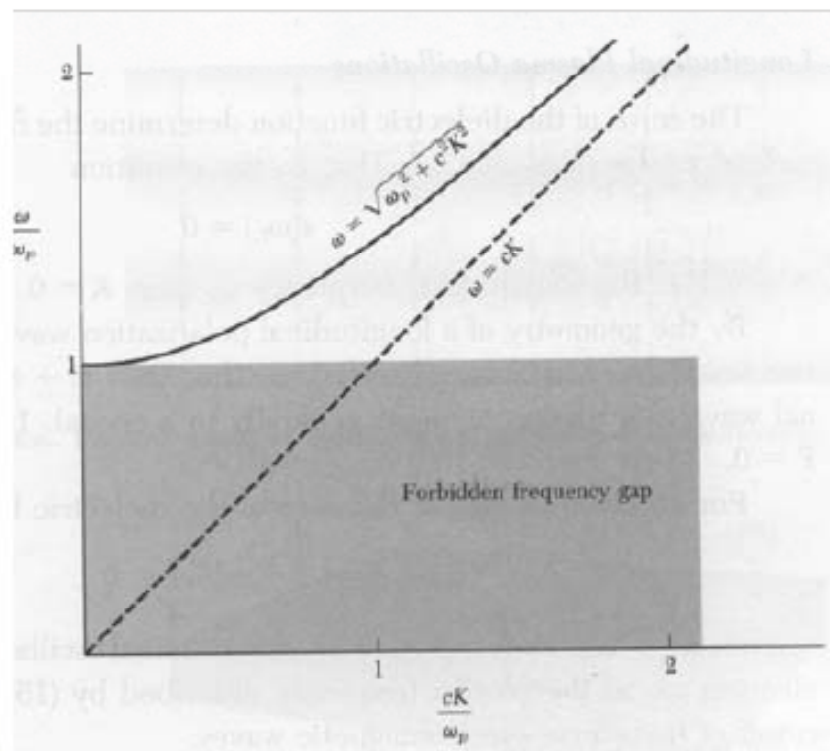
The dispersion relationship

$$\varepsilon(\omega)\omega^2 = \varepsilon(\infty)[\omega^2 - \tilde{\omega}_p^2] = c^2 K^2.$$

For $\omega < \tilde{\omega}_p$ we have $K^2 < 0$, so that K is imaginary. Waves incident on the medium in the frequency range $0 < \omega < \tilde{\omega}_p$ do not propagate, but will be totally reflected.

An electron gas is transparent when $\omega > \tilde{\omega}_p$, the dispersion relation is then

$\omega^2 = \tilde{\omega}_p^2 + c^2 K^2 / \varepsilon(\infty)$; this describes transverse electromagnetic waves in a plasma.



n , electrons/cm ³	10^{22}	10^{18}	10^{14}	10^{10}
ω_p , s ⁻¹	5.7×10^{15}	5.7×10^{13}	5.7×10^{11}	5.7×10^9
λ_p , cm	3.3×10^{-5}	3.3×10^{-3}	0.33	33

Longitudinal plasma oscillations

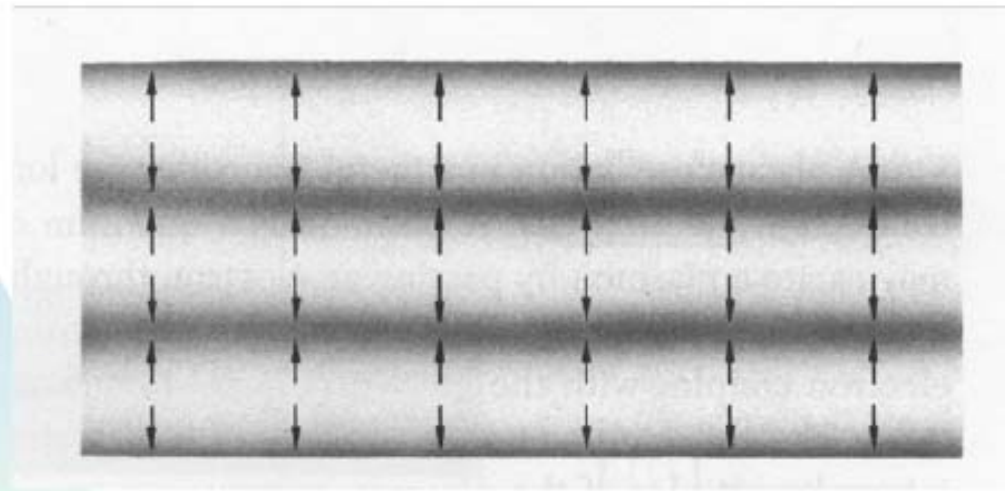
The zero of the dielectric function determine the frequencies of the longitudinal modes of oscillation. $\varepsilon(\omega_L)=0$

$\varepsilon(\omega_L) = 1 - \omega_p^2 / \omega_L^2 = 0$. When $\omega_L = \omega_p$, there is a free longitudinal oscillation mode of an electron gas at the plasma frequency as the low-frequency cutoff of transverse electron magnetic waves.

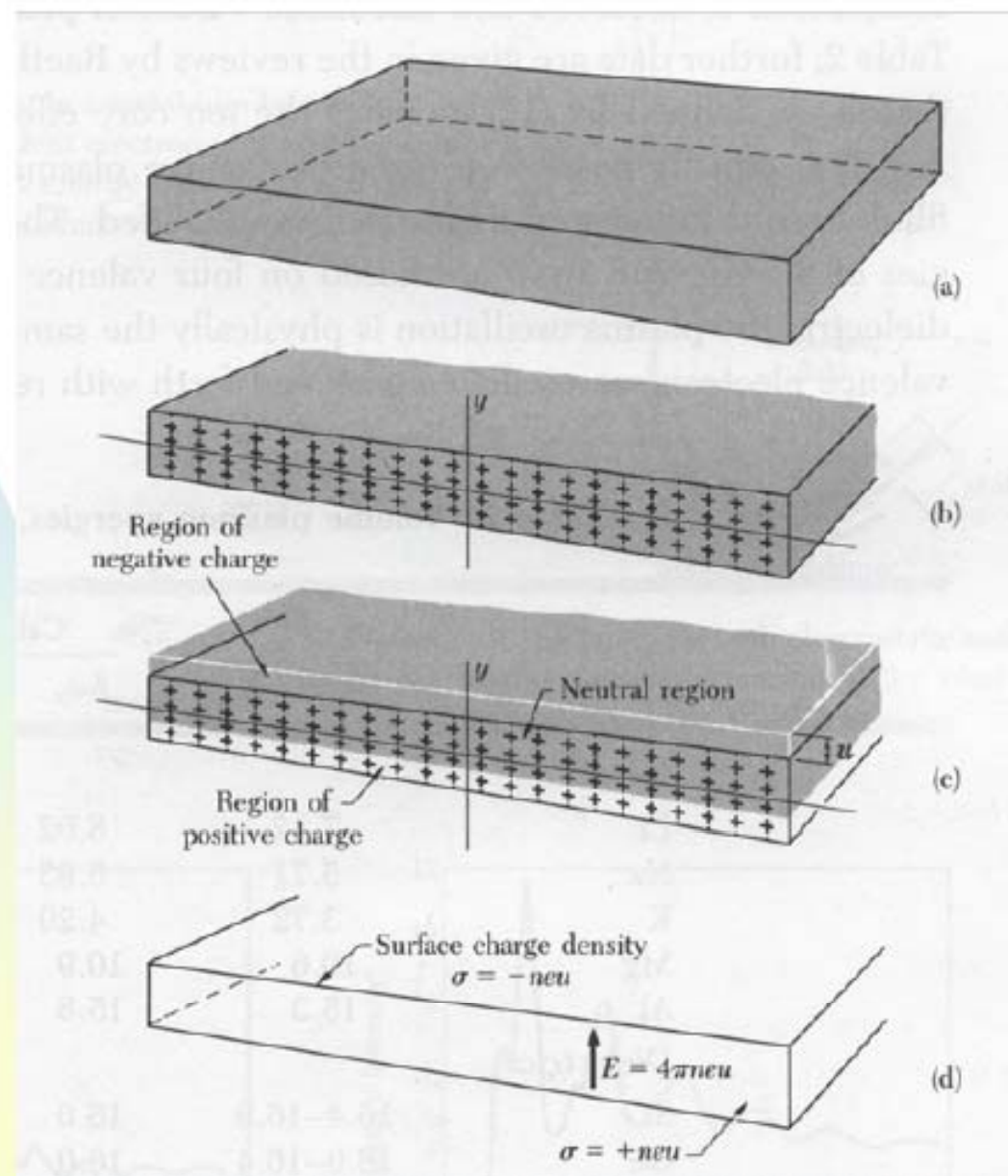
The equation of motion is

$$nm \frac{d^2 u}{dt^2} = -neE = -4\pi n^2 e^2 u,$$

$$\frac{d^2 u}{dt^2} + \omega_p^2 u = 0; \quad \omega_p = \left(\frac{4\pi n e^2}{m} \right)^{1/2}.$$

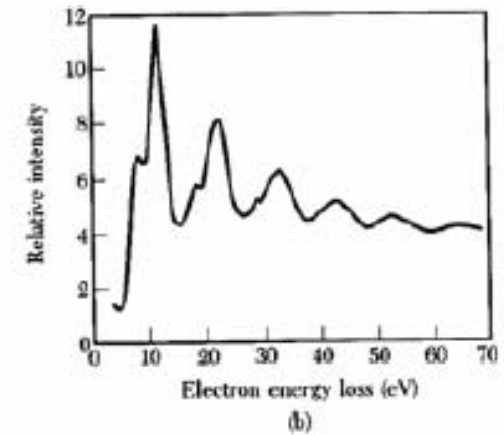
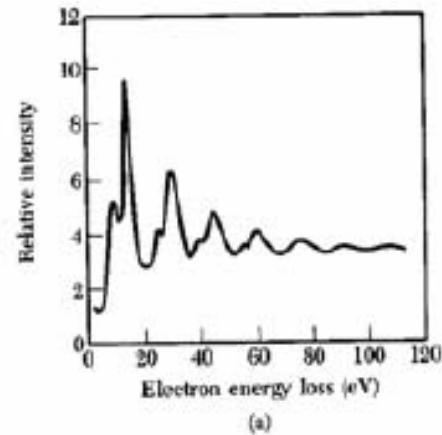
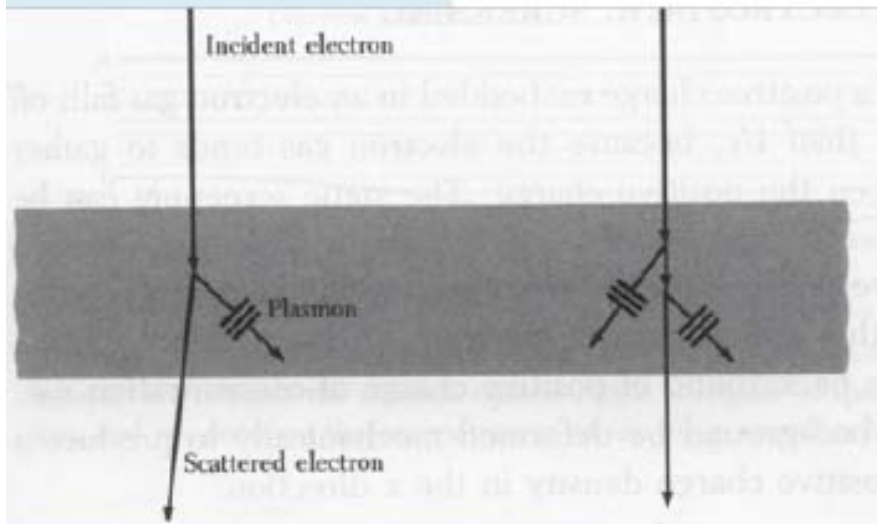


The displacement establishes a surface charge density $-neu$ on the upper surface of the slab and $+neu$ on the lower surface. An electric field $E = 4\pi neu$ is produced inside the slab.



Plasmons

A plasma oscillation in a metal is a collective longitudinal excitation of the conduction gas. A plasmon is a quantum of a plasma oscillations.



Electrostatic screening

The electric field of a positive charge embedded in an electron gas falls off with increasing r faster than $1/r$ because of the screen effect.

The static screening can be described by $\varepsilon(0, \vec{K})$. For a uniform gas of electrons of charge concentration $-n_0e$ superimposed on a background of positive charge of concentration n_0e .

$$\rho^+(x) = n_0e + \rho_{ext}(\vec{K}) \sin Kx$$

$\rho_{ext}(\vec{K}) \sin Kx$ is the electrostatic field applied to the electron gas.

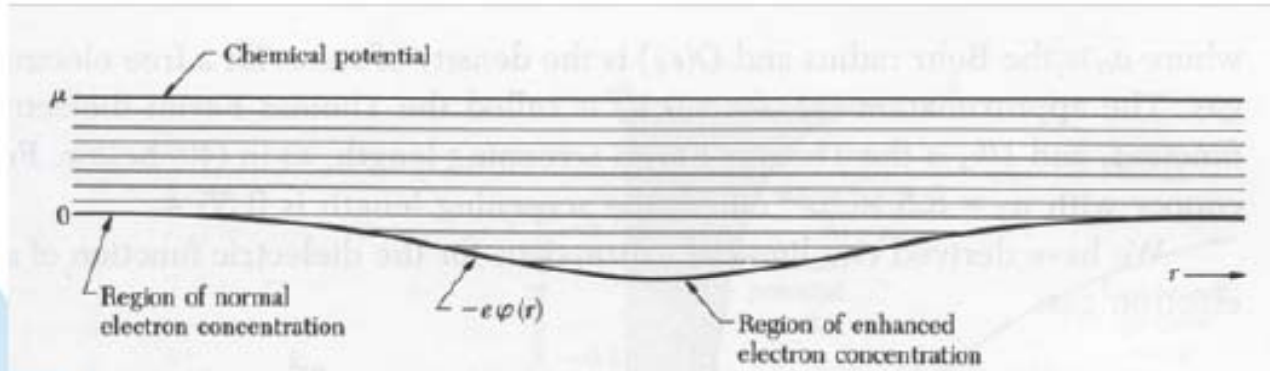
For positive charge, $\varphi = \varphi_{ext}(K) \sin Kx$; $\rho = \rho_{ext}(K) \sin Kx$.

$$K^2 \varphi_{ext}(K) = 4\pi \rho_{ext}(K).$$

The amplitude of the total electrostatic potential $\varphi(K) = \varphi_{ext}(K) + \varphi_{ind}(K)$.

Total charge density variation $\rho(K) = \rho_{ext}(K) + \rho_{ind}(K)$

$$K^2 \varphi(K) = 4\pi \rho(K).$$



Thomas-Fermi approximation: a local internal chemical potential can be defined as a function of the electron concentration at that point. In a region of no electrostatic contribution to the chemical potential, we have

$$\mu = \varepsilon_F^0 = \frac{\hbar^2}{2m} \left(3\pi^2 \frac{N}{V} \right)^{2/3} = \frac{\hbar^2}{2m} \left(3\pi^2 n_0 \right)^{2/3} \text{ at zero temperature.}$$

In the region where the electrostatic potential is $\varphi(x)$, we have

$$\mu = \varepsilon_F(x) - e\varphi(x) \cong \frac{\hbar^2}{2m} \left(3\pi^2 n \right)^{2/3} - e\varphi(x) \cong \frac{\hbar^2}{2m} \left(3\pi^2 n_0 \right)^{2/3}$$

At $q \ll k_F$, by a Taylor expansion of ε_F , we have

$$\frac{d\varepsilon_F}{dn_0} [n(x) - n_0] \cong e\varphi(x)$$

$$n(x) - n_0 \cong \frac{3}{2} n_0 \frac{e\varphi(x)}{\varepsilon_F} \text{ or } \rho_{ind}(K) = -(3n_0 e^2 / 2\varepsilon_F) \varphi(K).$$

$$\rho_{ind}(K) = -(6\pi n_0 e^2 / \varepsilon_F K^2) \rho(K),$$

$$\varepsilon(0, K) = 1 - \rho_{ind}(K) / \rho(K) = 1 + k_s^2 / K^2;$$

$$\text{Where } k_s^2 \equiv 6\pi n_0 e^2 / \varepsilon_F$$

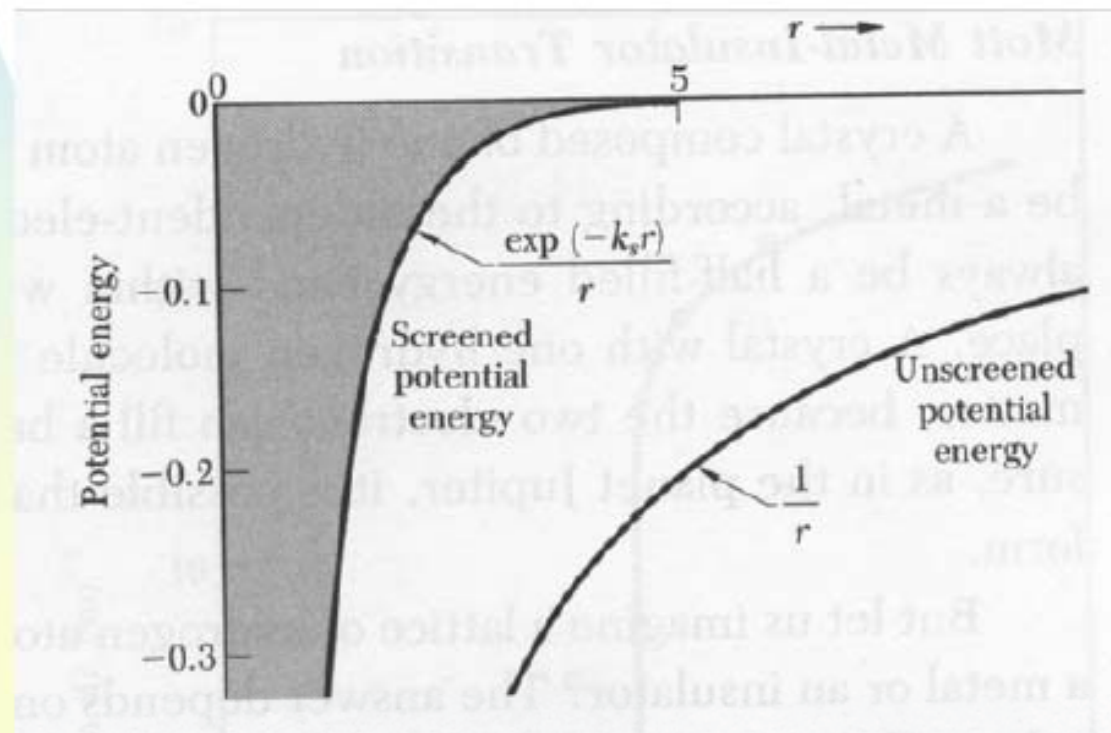
Screened Coulomb potential

The Poisson equation for the unscreened coulomb potential is

$$\nabla^2 \varphi_0 = -4\pi q \delta(r),$$

The unscreen potential is $\varphi_0 = q/r$.

The screen potential is $\varphi(r) = (q/r) \exp(-k_s r)$.



Mott Metal-Insulator Transition

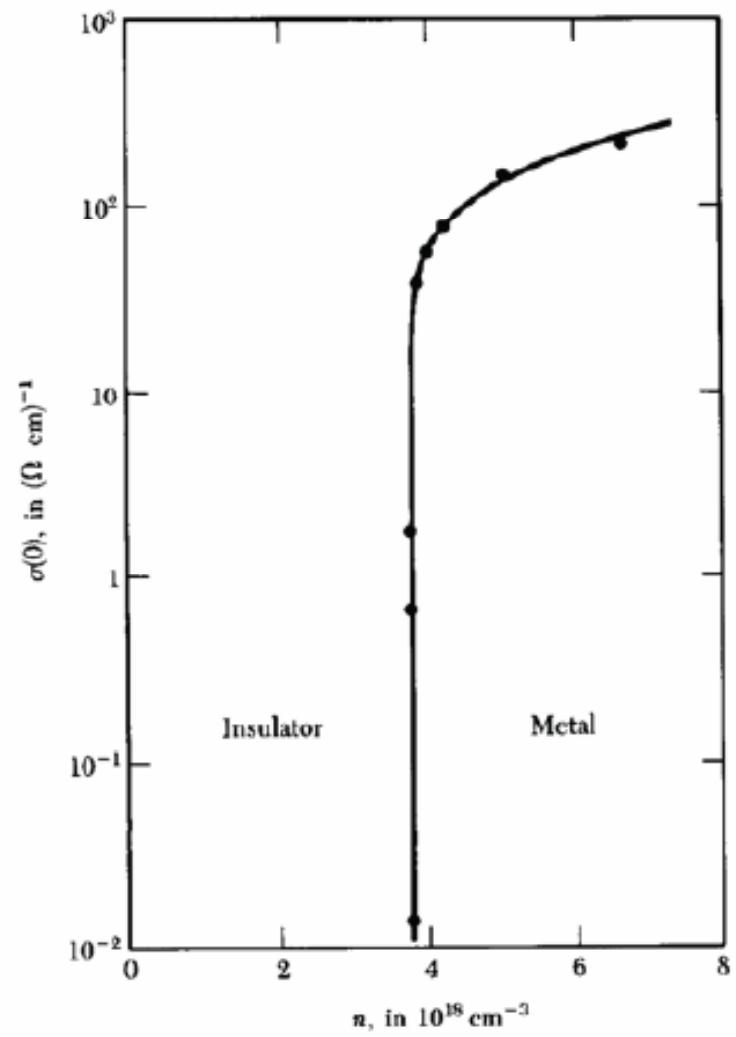
A crystal composed of one hydrogen atom per primitive cell should be a metal, following free electron theory. A crystal with one hydrogen molecule per primitive cell is different.

Question: Is a lattice of hydrogen atoms at zero temperature a metal or an insulator?

Assume metallic state where a conduction electron sees a screened coulomb interaction from each proton:

$$U(r) = -(e^2 / r) \exp(-k_s r),$$

where $k_s^2 = 3.939 n_0^{1/3} / a_0$. At large a_0 a bound state may condense about the protons to form an insulator.



Electron-electron interaction

The effects of electron-electron interactions are usually described within the framework of the Landau theory of a Fermi liquid. A Fermi gas is a system of noninteracting fermions; the same system with interactions is a Fermi liquid.

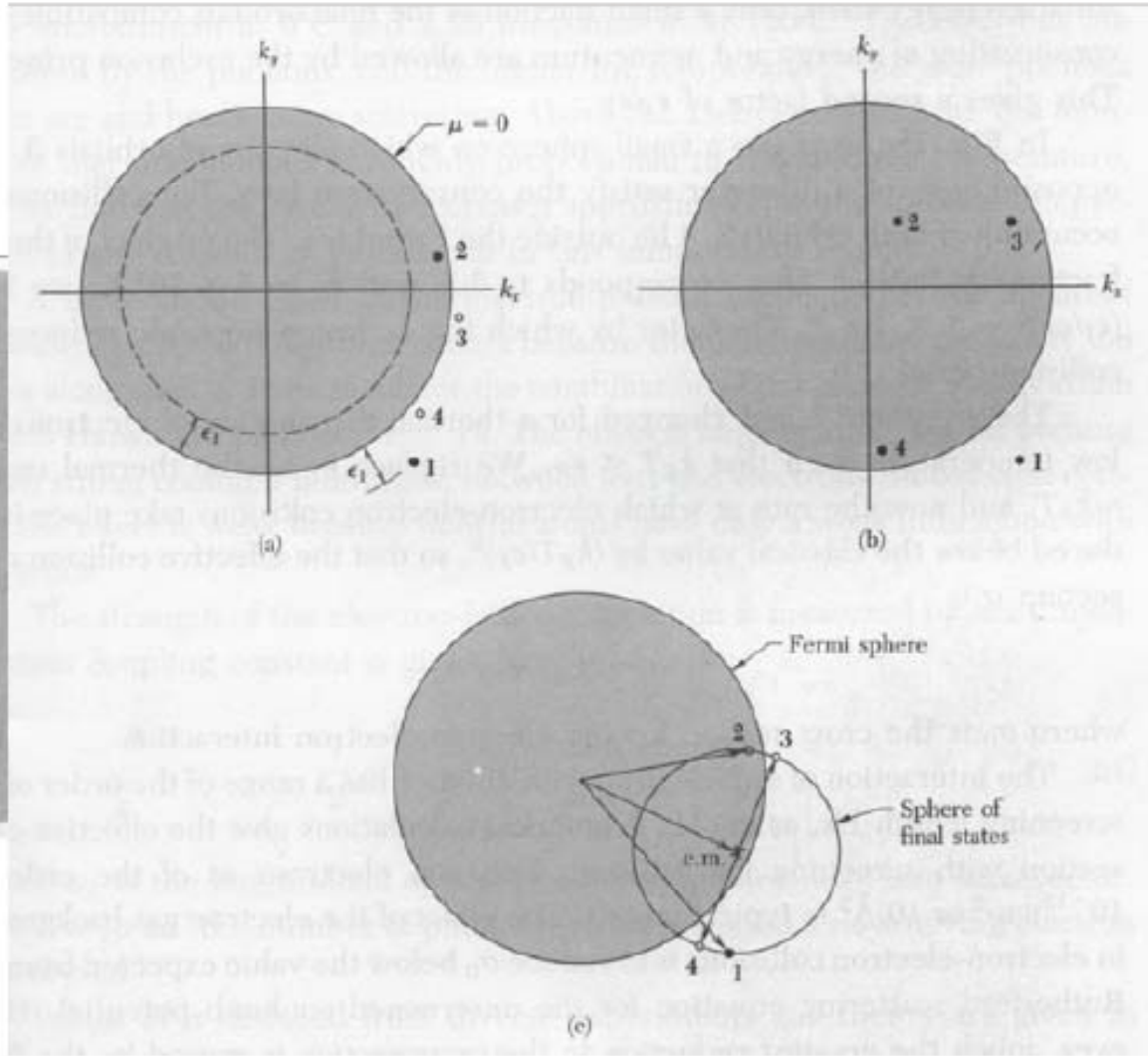
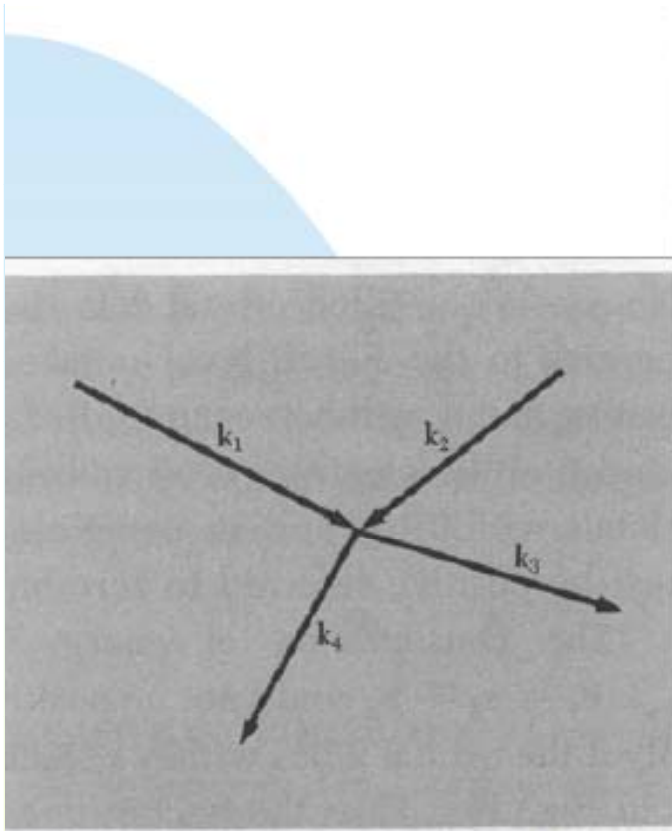
Consider that we gradually turning on the interaction between electrons, they will have two effects:

1. The energies of each on-electron level will be modified.

2. Electron will be scattered in and out of the single electron level, which are no longer stationary. Whether this scattering is serious enough to invalidate the independent electron picture depends on how rapid the rate of scattering is. If the scattering rate is low, electron-electron relaxation time is much larger than other relaxation time, then we can ignore it and use the independent electron theory with modified mass.

In metal, although conduction electrons are crowded together only 2 Å apart, they travel a long distance before colliding with each other due to

1. Exclusion principle.
2. The screening of the coulomb interaction between two electrons.



Suppose N electron state consists of a filled Fermi sphere (at $T = 0$) plus a single excited electron in a level with $\varepsilon_1 > \varepsilon_F$.

In order for this electron to be scattered, it must interact with an electron of energy ε_2 , which must be less than ε_F . The exclusion principle requires that these two electrons can only scatter into unoccupied levels, whose energies ε_3 and ε_4 must be greater than ε_F . Or $\varepsilon_2 < \varepsilon_F, \varepsilon_3 > \varepsilon_F, \varepsilon_4 > \varepsilon_F$.

In addition, energy conservation requires that

$$\varepsilon_1 + \varepsilon_2 = \varepsilon_3 + \varepsilon_4.$$

If ε_1 is exactly ε_F , $\varepsilon_2, \varepsilon_3, \varepsilon_4$ must also be ε_F . Thus the allowed wave vectors occupy a region of K space of zero volume. The life time of an electron at the Fermi surface at $T = 0$ is infinite.

When ε_1 is different from ε_F , some phase space becomes available since the other three energies can now vary within a shell of thickness of order $|\varepsilon_1 - \varepsilon_F|$ about the Fermi surface, leading to a scattering rate of order $(\varepsilon_1 - \varepsilon_F)^2$.

If the excited electron is superimposed not on a filled Fermi surface, but on a thermal equilibrium distribution of electrons at nonzero T . There will be partially occupied levels in a shell of width $k_B T$ about ε_F . This provides an additional range of choice of order $k_B T$, and therefore leads to a scattering rate as $(k_B T)^2$. At temperature T , an electron of energy ε_1 near the Fermi surface has a scattering rate

$$\frac{1}{\tau} = a(\varepsilon_1 - \varepsilon_F)^2 + b(k_B T)^2,$$

where the coefficients a and b are independent of ε_1 and T .

Thus the electron life time due to electron-electron scattering can be made as large as one wishes by reducing T and by considering electrons sufficiently close to the Fermi surface.

Assume that the temperature dependence of τ is taken into account by a factor $1/T^2$. We expect from lowest-order perturbation theory that τ will depend on the electron-electron interaction through the square of the Fourier transform of the interaction potential.

$$\frac{1}{\tau} \propto (k_B T)^2 \left(\frac{4\pi e^2}{k_0^2} \right)^2 \propto (k_B T)^2 \left(\frac{\pi^2 \hbar^2}{m k_F} \right)^2,$$