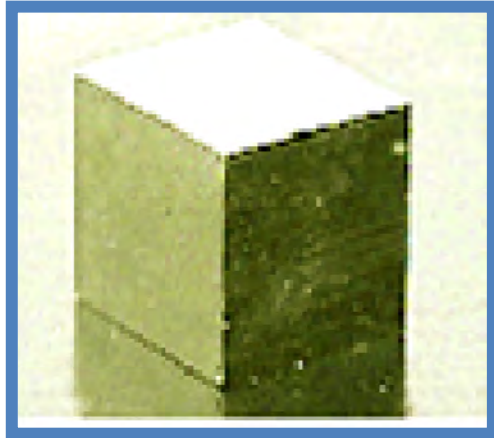


Lecture 3 Jan 14 2013

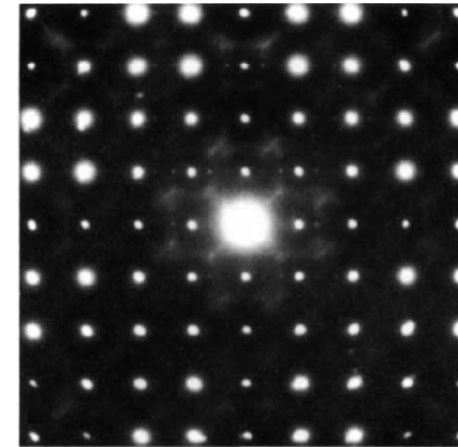


$$\vec{R} = n_1 \vec{a}_1 + n_2 \vec{a}_2 + n_3 \vec{a}_3$$

Primitive cell  
Wigner-Seitz cell (WS)

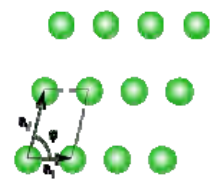
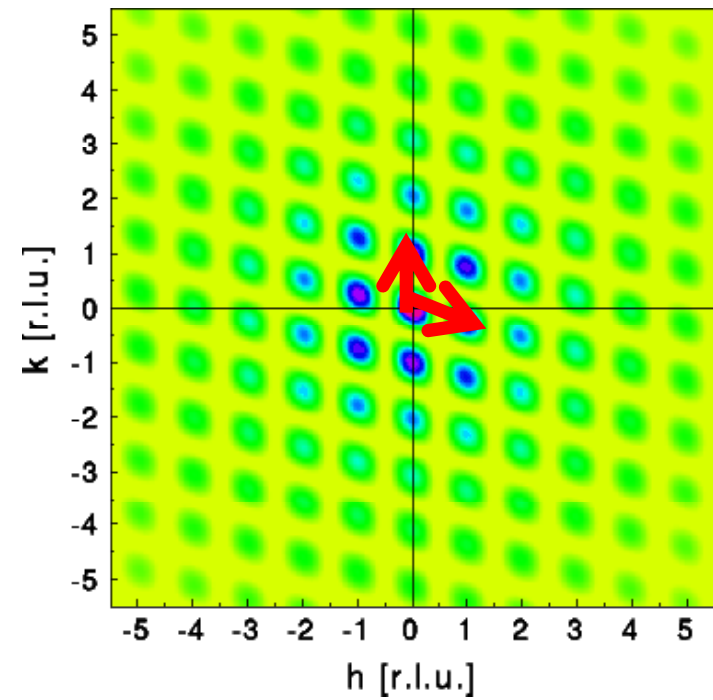
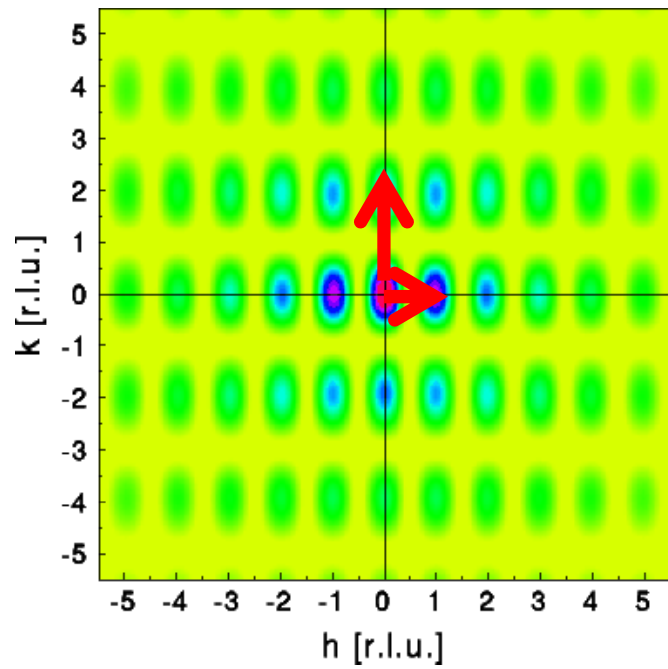
$$\mathbf{K} \cdot \mathbf{R} = 2\pi(k_1 n_1 + k_2 n_2 + k_3 n_3)$$

$$[\mathbf{b}_1 \mathbf{b}_2 \mathbf{b}_3]^T = 2\pi [\mathbf{a}_1 \mathbf{a}_2 \mathbf{a}_3]^{-1}$$



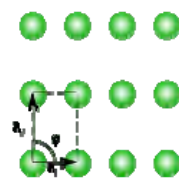
$$\mathbf{K} = k_1 \mathbf{b}_1 + k_2 \mathbf{b}_2 + k_3 \mathbf{b}_3$$

Primitive cell



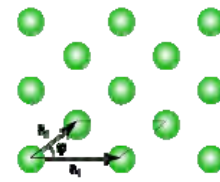
$$|a_1| = |a_2|, \varphi = 90^\circ$$

1



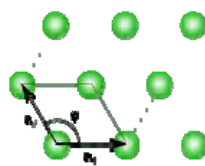
$$|a_1| = |a_2|, \varphi = 90^\circ$$

2



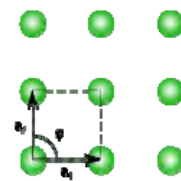
$$|a_1| \neq |a_2|, \varphi = 90^\circ$$

3



$$|a_1| = |a_2|, \varphi = 120^\circ$$

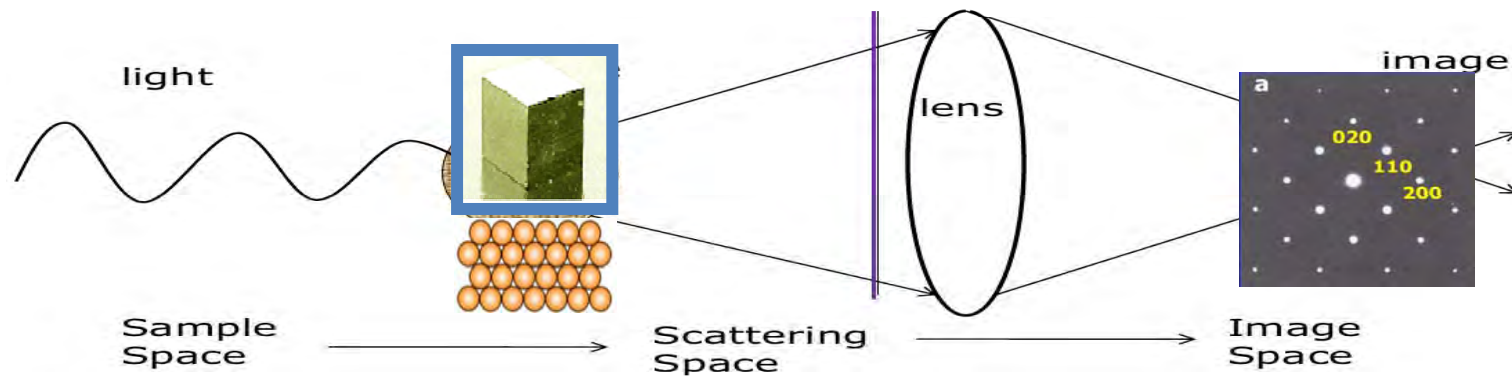
4



$$|a_1| = |a_2|, \varphi = 90^\circ$$

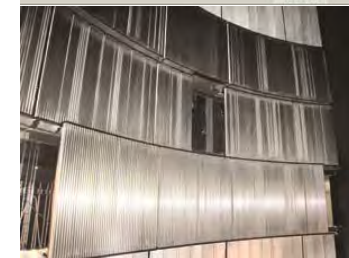
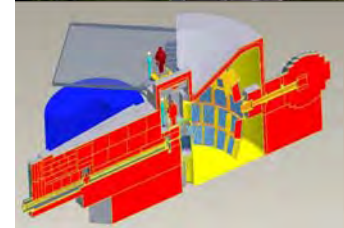
5

# Thinking in Reciprocal space

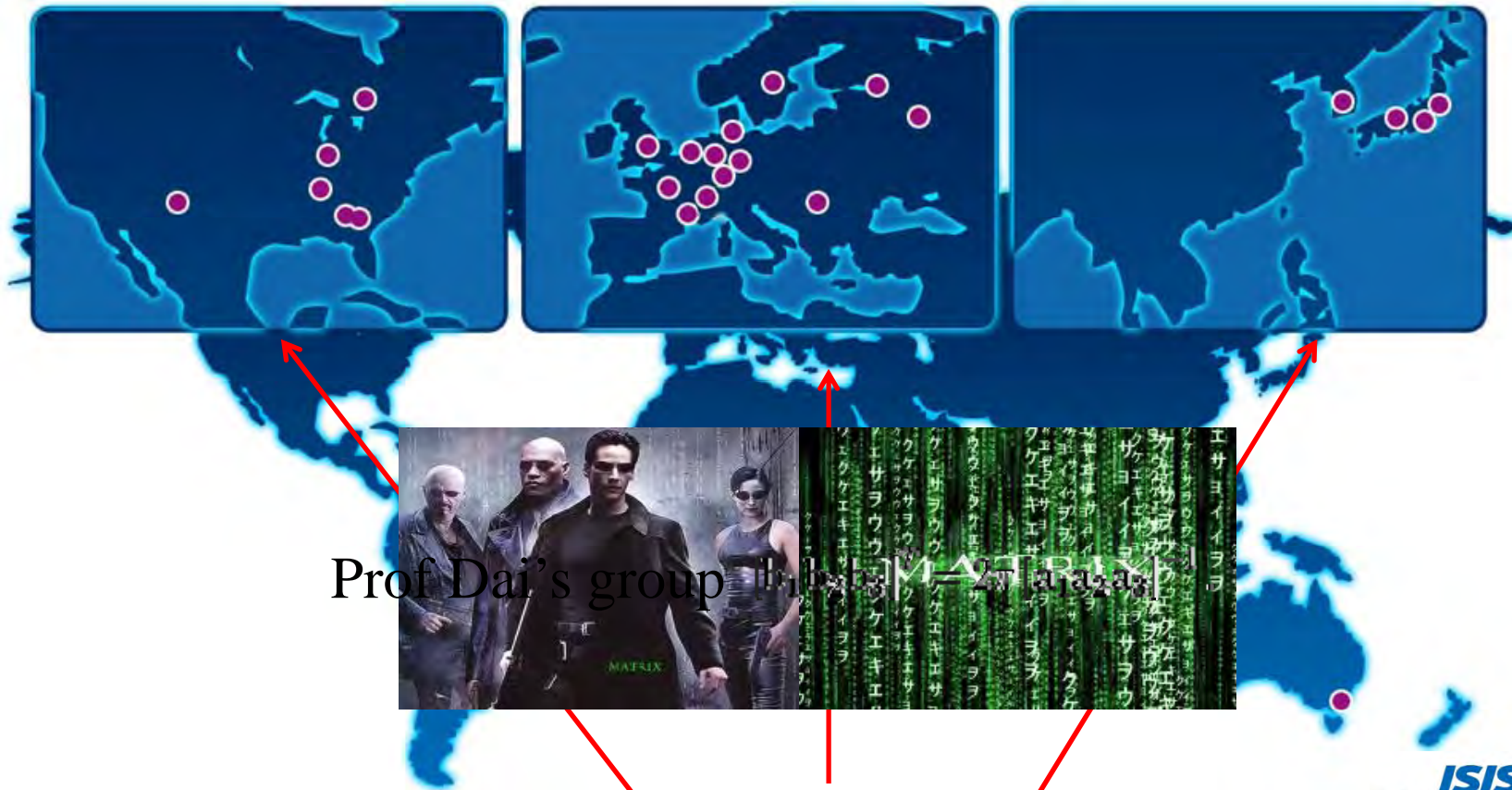


MATRIX

$$\mathbf{K} \cdot \mathbf{R} = 2\pi(k_1 n_1 + k_2 n_2 + k_3 n_3)$$
$$[b_1 \ b_2 \ b_3]^T = 2\pi [a_1 \ a_2 \ a_3]^{-1}$$



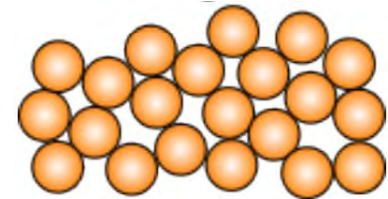
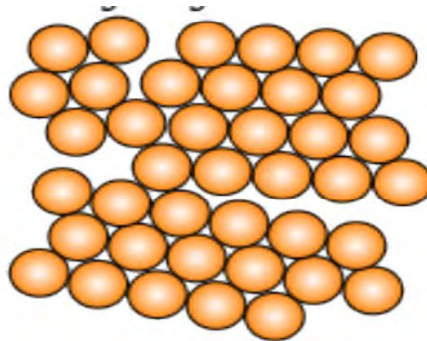
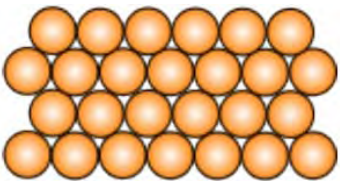
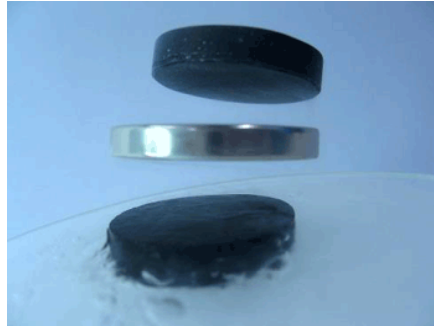
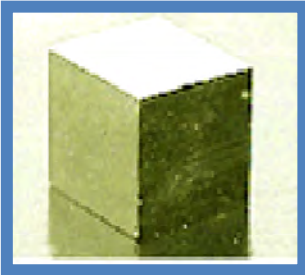
# Where are the reciprocal space stations on earth?



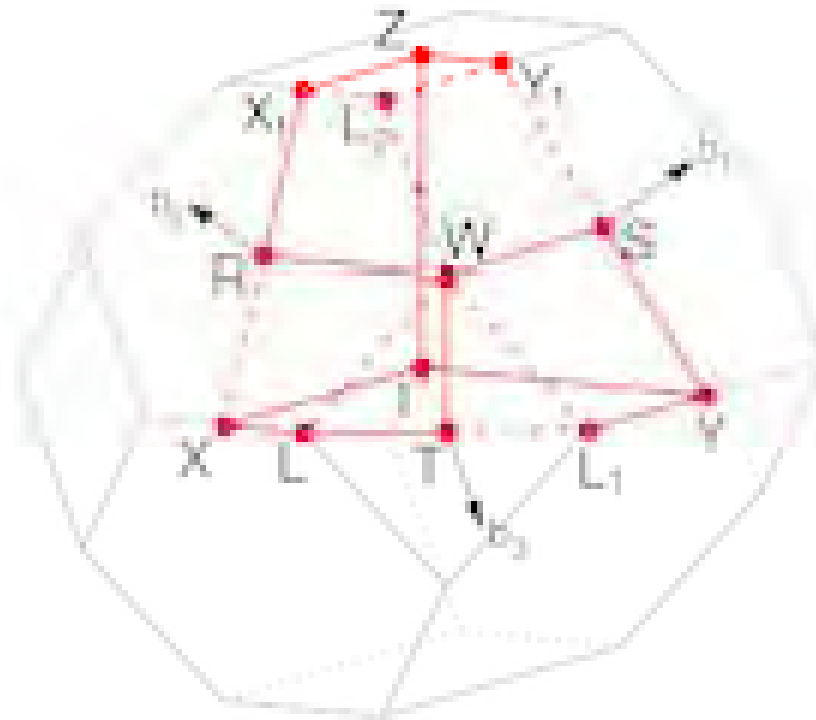
Prof Dai's group



# Which one is physicists' favorite?



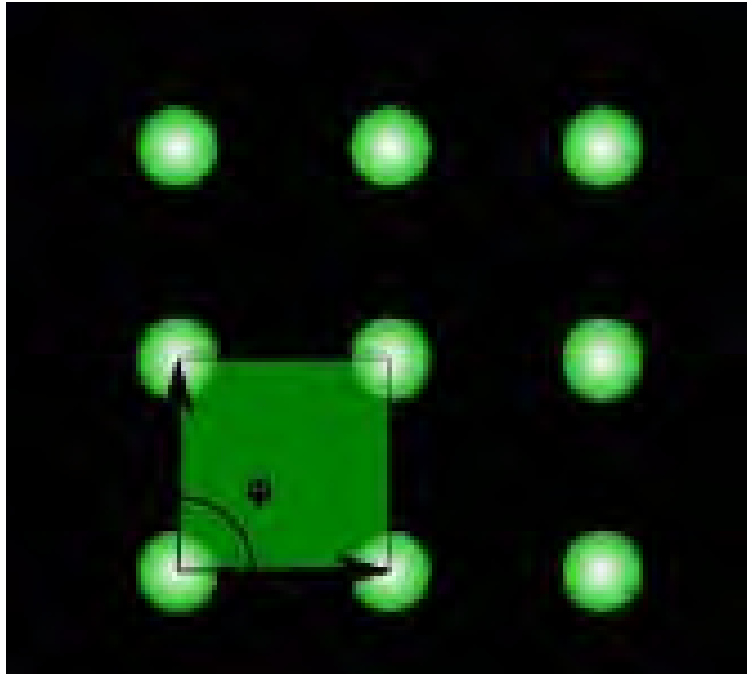
# Brillouin zone



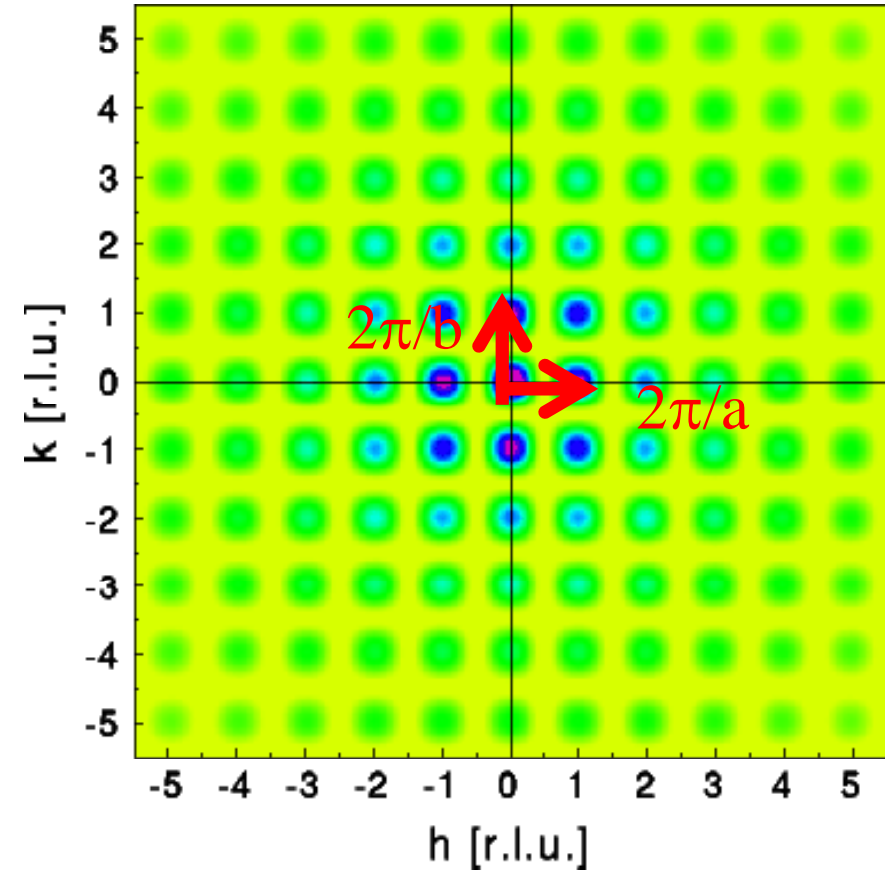
ORCI path:  $\Gamma$ - $X$ - $L$ - $T$ - $W$ - $R$ - $X_1$ - $Z$ - $\Gamma$ - $Y$ - $S$ - $W_1$ - $L_1$ - $Y_1$ - $Z_1$

[Selyessy & Curtarolo, DOI: 10.1016/j.commatsci.2010.08.010]

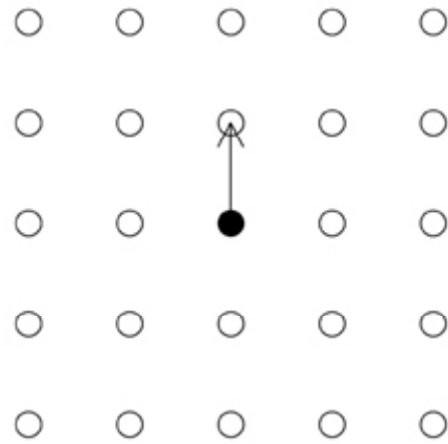
R space



K space

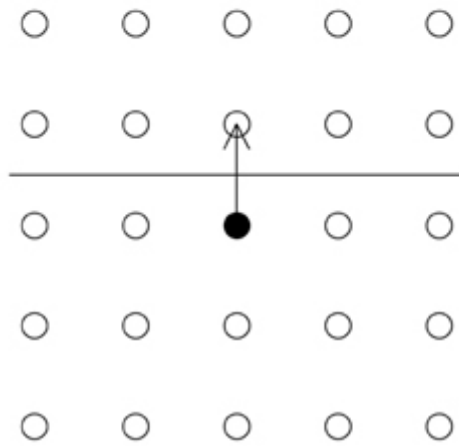


# Square Lattice

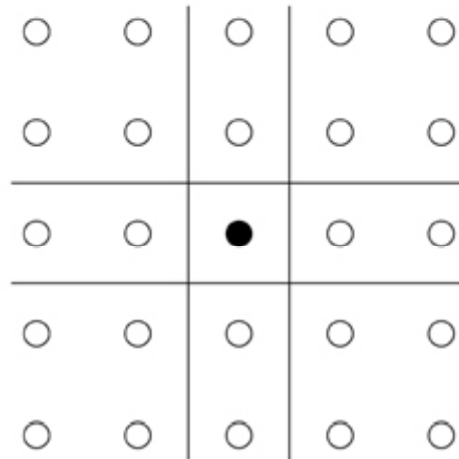




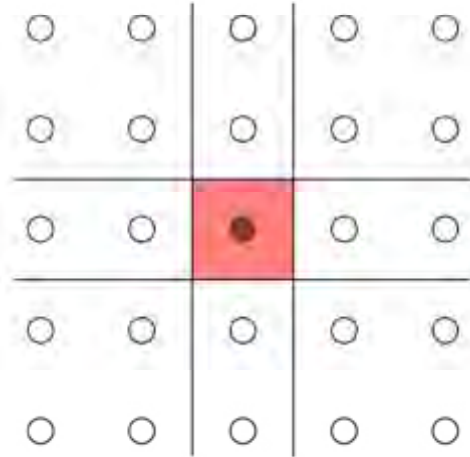
# Square Lattice



# Square Lattice

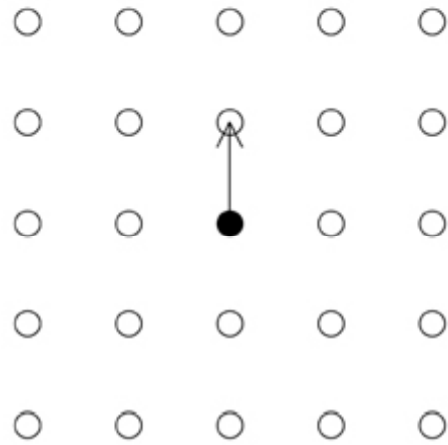


## Square Lattice: 1<sup>st</sup> brillouin zone

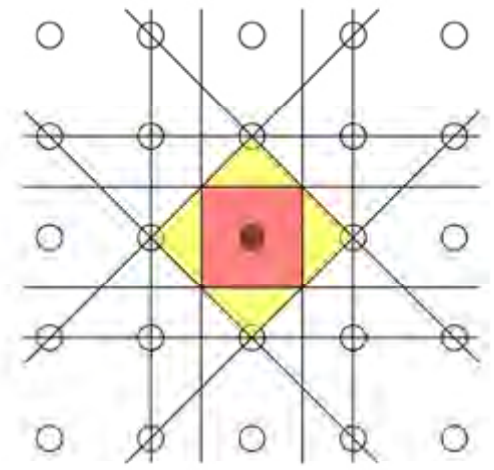
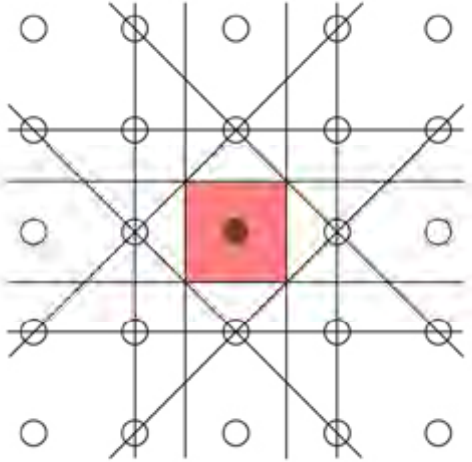


The locus of points in reciprocal space that have no **Bragg Planes** between them and the origin defines the first Brillouin Zone.

How to built the 2<sup>nd</sup> BZ?

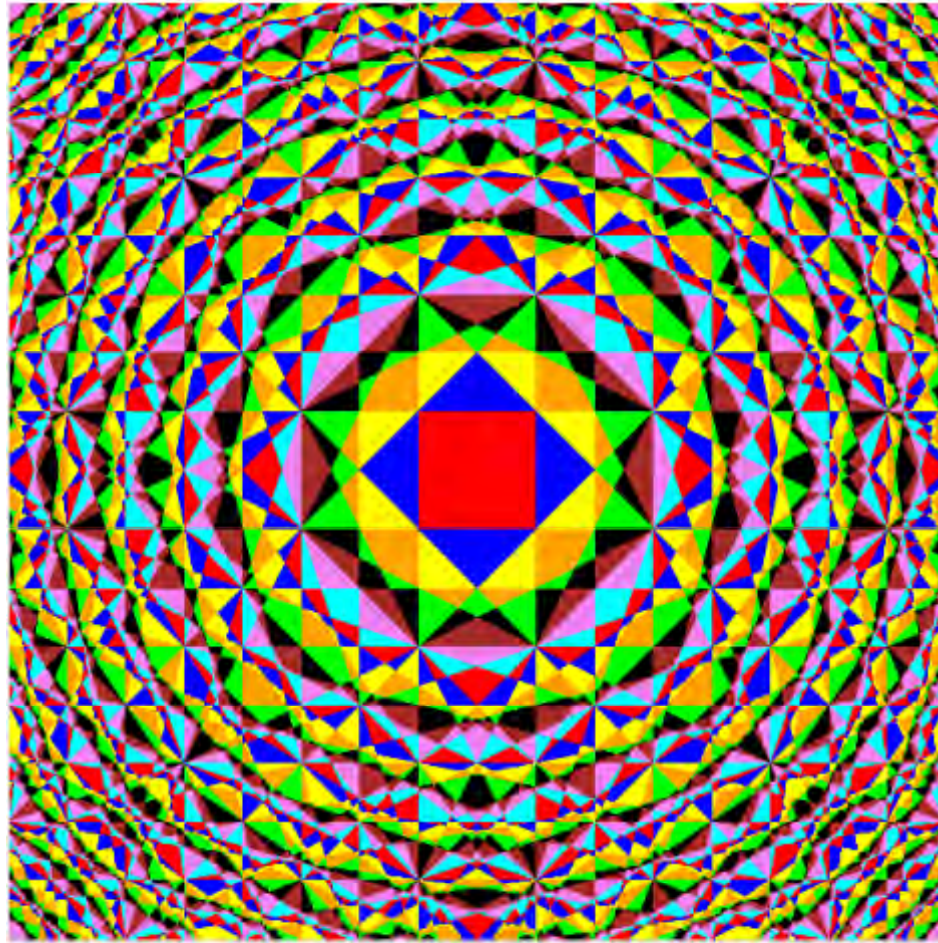


# Square Lattice: 2<sup>nd</sup> brillouin zone



<http://nptel.iitm.ac.in/courses/113106040/Lecture30.pdf>

# All Brillouin Zones: Square Lattice

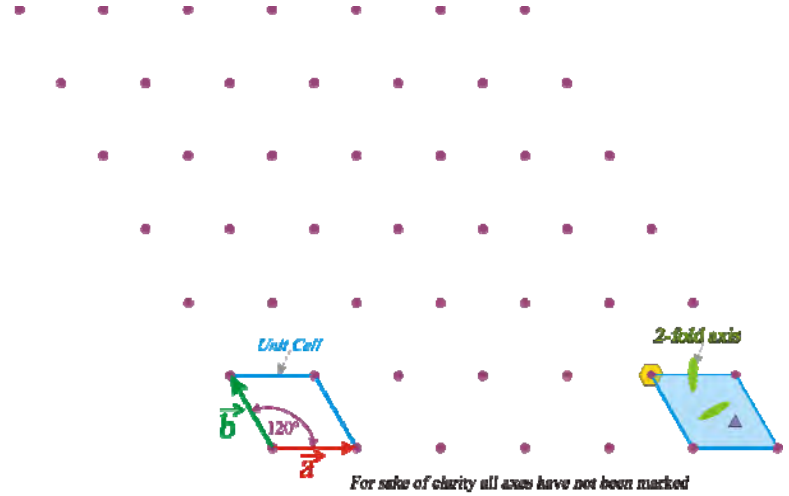


# Zone folding

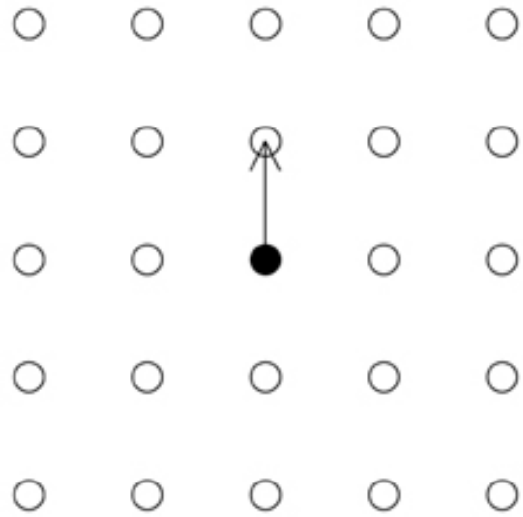
- [http://www.doitpoms.ac.uk/tlplib/brillouin\\_zones/folding.php](http://www.doitpoms.ac.uk/tlplib/brillouin_zones/folding.php)



1<sup>st</sup> BZ



2<sup>nd</sup> BZ



1<sup>st</sup> BZ

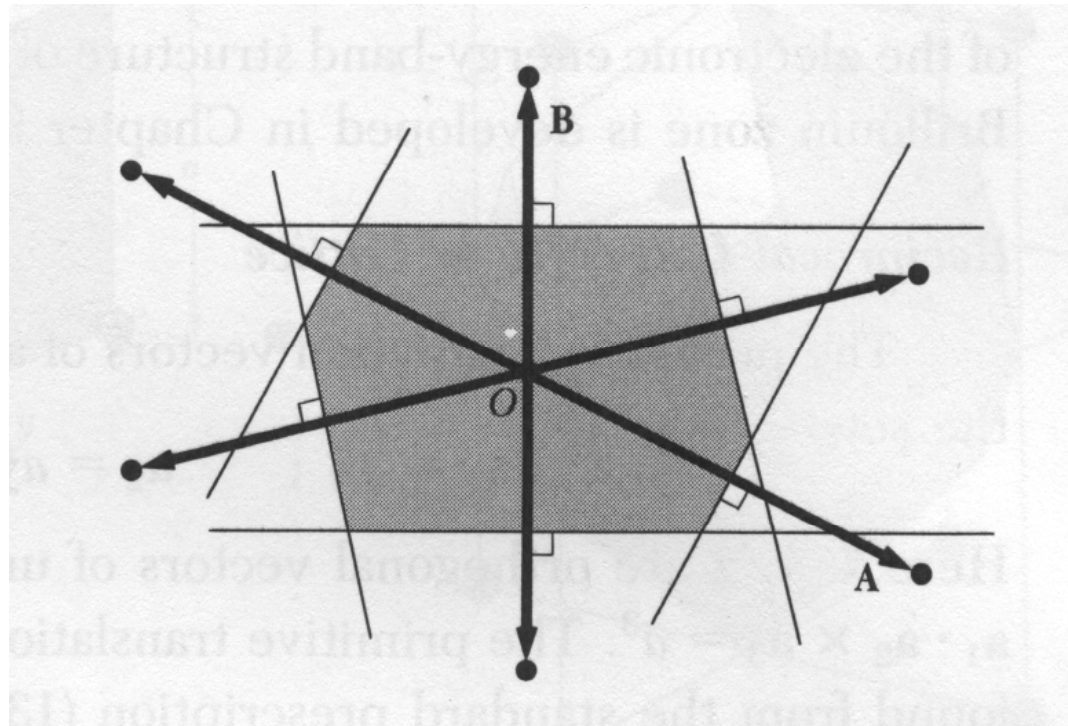


# Zone folding

- [http://www.doitpoms.ac.uk/tlplib/brillouin\\_zones/folding.php](http://www.doitpoms.ac.uk/tlplib/brillouin_zones/folding.php)

## First Brillouin zone

The Wigner-Seitz primitive cell of the reciprocal lattice is known as the first Brillouin zone.  
(Wigner-Seitz is real space concept while Brillouin zone is a reciprocal space idea).



# 3D Lattices (14)

# 3D BZ

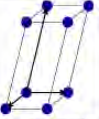
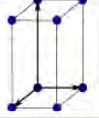
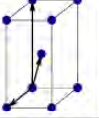
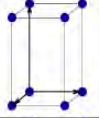
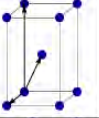
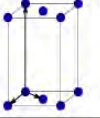
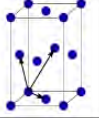
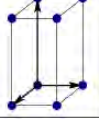
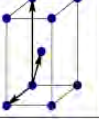
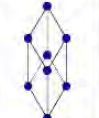
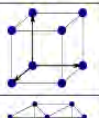
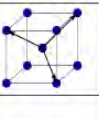
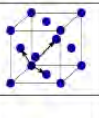
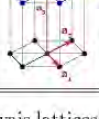
Bravais lattice	Parameters	Simple (P)	Volume centered (I)	Base centered (C)	Face centered (F)
Triclinic	$a_1 \neq a_2 \neq a_3$ $\alpha_{12} \neq \alpha_{23} \neq \alpha_{31}$				
Monoclinic	$a_1 \neq a_2 \neq a_3$ $\alpha_{23} = \alpha_{31} = 90^\circ$ $\alpha_{12} \neq 90^\circ$				
Orthorhombic	$a_1 \neq a_2 \neq a_3$ $\alpha_{12} = \alpha_{23} = \alpha_{31} = 90^\circ$				
Tetragonal	$a_1 = a_2 \neq a_3$ $\alpha_{12} = \alpha_{23} = \alpha_{31} = 90^\circ$				
Trigonal	$a_1 = a_2 = a_3$ $\alpha_{12} = \alpha_{23} = \alpha_{31} < 120^\circ$				
Cubic	$a_1 = a_2 = a_3$ $\alpha_{12} = \alpha_{23} = \alpha_{31} = 90^\circ$				
Hexagonal	$a_1 = a_2 \neq a_3$ $\alpha_{12} = 120^\circ$ $\alpha_{23} = \alpha_{31} = 90^\circ$				

Table 1.1: Bravais lattices in three-dimensions.

<http://lamp.tu-graz.ac.at/~hadley/ss1/bzones/>



# Acknowledge

- Pengcheng's slides
- Profs in condensed matter physics
- internet