Magnetic Forces
Magnetic Forces

Charged particles experience an electric force when in an electric field regardless of whether they are moving or not moving.

There is another force that charged particles can experience even in the absence of an electric field but only when they are motion.

A Magnetic Force

Magnetic Interactions are the result of relative motion.
Quick Note on Magnetic Fields

Like the electric field, the magnetic field is a *vector*, having both direction and magnitude.

We denote the magnetic field with the symbol \( B \).

The unit for the magnetic field is the *tesla*.

\[ 1 \text{ tesla} = 1 \text{T} = 1 \frac{N}{A \cdot m} \]

There is another unit that is also used and that is the *gauss*.

\[ 1 \text{ gauss} = 10^{-4} \text{T} \]

Unlike Electric Fields which begin and end on charges, Magnetic Fields have neither a beginning nor an end.
Magnetic Forces

Given a charge $q$ moving with a velocity $v$ in a magnetic field, it is found that there is a force on the charge $F$.

This force is
- proportional to the charge $q$
- proportional to the speed $v$
- perpendicular to both $v$ and $B$
- proportional to $\sin \phi$ where $\phi$ is the angle between $v$ and $B$

This can be summarized as $\vec{F} = q\vec{v} \times \vec{B}$

This is the cross product of the velocity vector of the charged particle and the magnetic field vector.
Right Hand Rule

To get the resultant direction for the force do the following:

1. Point your index finger (and your middle finger) along the direction of motion of the charge \( \mathbf{v} \)
2. Rotate your middle finger away from your index finger by the angle \( \theta \) between \( \mathbf{v} \) and \( \mathbf{B} \)
3. Hold your thumb perpendicular to the plane formed by both your index finger and middle finger
4. Your thumb will then point in the direction of the force \( \mathbf{F} \) if the charge \( q \) is positive
5. For \( q < 0 \), the direction of the force is opposite your thumb
Magnetic Forces

There is no force if $v$ and $B$ are either parallel or antiparallel

$$\sin(0) = \sin(180) = 0$$

The force is maximum when $v$ and $B$ are perpendicular to each other

$$\sin(90) = 1$$

The force on a negative charge is in the opposite direction
Example

Three points are arranged in a uniform magnetic field. The magnetic field points into the screen.

1) A positively charged particle is located at point A and is stationary. The direction of the magnetic force on the particle is:

a) Right  

b) Left  

c) Into the screen  

d) Out of the screen  
\(\text{e) Zero}\)

The magnetic force is given by  \(\vec{F} = q\vec{v} \times \vec{B}\)

But \(v\) is zero. Therefore the force is also zero.
Example

Three points are arranged in a uniform magnetic field. The magnetic field points into the screen.

2) The positive charge moves from point A toward B. The direction of the magnetic force on the particle is:

a) Right  b) Left  c) Into the screen  
d) Out of the screen  e) Zero

The magnetic force is given by \( \vec{F} = q\vec{v} \times \vec{B} \)

The cross product of the velocity with the magnetic field is to the left and since the charge is positive the force is then to the left
Three points are arranged in a uniform magnetic field. The magnetic field points into the screen.

3) The positive charge moves from point A toward C. The direction of the magnetic force on the particle is:
   a) up and right  b) up and left  c) down and right  
d) down and left

The magnetic force is given by \( \vec{F} = q\vec{v} \times \vec{B} \)

The cross product of the velocity with the magnetic field is to the upper left and since the charge is positive the force is then to the upper left
Motion due to a Magnetic Force

When a charged particle moves in a magnetic field it experiences a force that is perpendicular to the velocity.

Since the force is perpendicular to the velocity, the charged particle experiences an acceleration that is perpendicular to the velocity.

The magnitude of the velocity does not change, but the direction of the velocity does producing circular motion.

The magnetic force does no work on the particle.
Motion due to a Magnetic Force

The magnetic force produces circular motion with the centripetal acceleration being given by

\[ \frac{v^2}{R} \]

where \( R \) is the radius of the orbit

Using Newton’s second law we have

\[ F = qvB = m \frac{v^2}{R} \]

The radius of the orbit is then given by

\[ R = \frac{m v}{q B} \]

The angular speed \( \omega \) is given by

\[ \omega = \frac{v}{R} = \frac{q B}{m} \]
Motion due to a Magnetic Force

What is the motion like if the velocity is not perpendicular to B?

We break the velocity into components along the magnetic field and perpendicular to the magnetic field.

The component of the velocity perpendicular to the magnetic field will still produce circular motion.

The component of the velocity parallel to the field produces no force and this motion is unaffected.

The combination of these two motions results in a helical type motion.
Velocity Selector

An interesting device can be built that uses both magnetic and electric fields that are perpendicular to each other.

A charged particle entering this device with a velocity \( \vec{v} \) will experience both

- An electric force \( \vec{F}_E = q \vec{E} \)

- A magnetic force \( \vec{F}_B = q \vec{v} \times \vec{B} \)
If the particle is positively charged then the magnetic force on the particle will be downwards and the electric force will be upwards.

If the velocity of the charged particle is just right then the net force on the charged particle will be zero.

\[ qvB = qE \implies v = \frac{E}{B} \]
Magnetic Forces

We know that a single moving charge experiences a force when it moves in a magnetic field.

What is the net effect if we have multiple charges moving together, as a current in a wire?

We start with a wire of length $l$ and cross section area $A$ in a magnetic field of strength $B$ with the charges having a drift velocity of $v_d$.

The total number of charges in this section is then $nAl$ where $n$ is the charge density.

The force on a single charge moving with drift velocity $v_d$ is given by $F = qv_dB$.

So the total force on this segment is $F = nqv_dAlB$. 
Magnetic Force on a Current Carrying Wire

We have so far that \( F = n q v_d A l B \)

But we also have that \( J = n q v_d \) and \( I = J A \)

Combining these, we then have that \( F = I l B \)

The force on the wire is related to the current in the wire and the length of the wire in the magnetic field.

If the field and the wire are not perpendicular to each the full relationship is

\[
\vec{F} = I \vec{l} \times \vec{B}
\]

The direction of \( l \) is the direction of the current.
Current Loop in a Magnetic Field

Suppose that instead of a current element, we have a closed loop in a magnetic field

We ask what happens to this loop
Current Loop in a Magnetic Field

Each segment experiences a magnetic force since there is a current in each segment.

As with the velocity, it is only the component of the wire that is perpendicular to $B$ that matters.

Each of the two shorter sides experiences a force given by

$$F' = I b B \cos \phi$$

in the directions shown.

Since the magnitudes are the same, the net force in the $y$-direction is $\sum F_y = 0$.

No translational motion in the $y$-direction.
Current Loop in a Magnetic Field

Now for the two longer sides of length $a$

Each of these two sides experiences a force given by

$$ F = I a B $$

in the directions shown

But since the forces are of the same magnitude but in opposite directions we have

$$ \sum F_x = 0 $$

No translational motion in the $x$-direction
Current Loop in a Magnetic Field

There is no translational motion in either the x- or y-directions

While the two forces in the y-direction are colinear, the two forces in the x-direction are not

Therefore there is a torque about the y-axis

The lever arm for each force is

\[ \frac{b}{2} \sin \phi \]

The net torque about the y-axis is

\[ \tau = 2F \left( \frac{b}{2} \right) \sin \phi = Ibab \sin \phi \]
Current Loop in a Magnetic Field

This torque is along the positive y-axis and is given by

$$\tau = I B A \sin \phi$$

The product $IA$ is referred to as the magnetic moment

$$\mu = I A$$

We rewrite the torque as

$$\tau = \mu B \sin \phi$$
Magnetic Moment

We defined the magnetic moment to be \( \mu = I A \)

It also is a vector whose direction is given by the direction of the area of the loop

The direction of the area is defined by the sense of the current

We can now write the torque as \( \vec{\tau} = \vec{\mu} \times \vec{B} \)
Potential Energy of a Current Loop

As the loop rotates because of the torque, the magnetic field does work on the loop

We can talk about the potential energy of the loop and this potential energy is given by

\[ U = -\mu \cdot B \]

The potential energy is the least when \( \mu \) and \( B \) are parallel and largest when \( \mu \) and \( B \) are antiparallel
Example

Two current carrying loops are oriented in a uniform magnetic field. The loops are nearly identical, except the direction of current is reversed.

1) What direction is the torque on loop 1?
   a) clockwise   b) counter-clockwise   c) zero

The magnetic moment for Loop 1, $\mu_1$, points to the left, while that for Loop 2, $\mu_2$, points to the right.

The torque is given by $\vec{\tau} = \vec{\mu} \times \vec{B}$

But since $\mu_1$ and $B$ are antiparallel, the cross product is zero, therefore the torque is zero!
Example

Two current carrying loops are oriented in a uniform magnetic field. The loops are nearly identical, except the direction of current is reversed.

2) How does the magnitude of the torques on the two loops compare?

a) \( \tau_1 > \tau_2 \)  
   b) \( \tau_1 = \tau_2 \)  
   c) \( \tau_1 < \tau_2 \)

Loop 1: Since \( \mu_1 \) points to the left the angle between \( \mu_1 \) and \( B \) is equal to 180° therefore \( \tau_1 = 0 \).

Loop 2: Since \( \mu_2 \) points to the right the angle between \( \mu_2 \) and \( B \) is equal to 0° therefore \( \tau_2 = 0 \).

So the two torques are equal!
Example

Two current carrying loops are oriented in a uniform magnetic field. The loops are nearly identical, except the direction of current is reversed.

3) Which loop occupies a potential energy minimum, and is therefore stable?

a) Loop 1  b) Loop 2  c) the same

The potential energy is given by \( U = - \mu \cdot \vec{B} \)

For Loop 1 the potential energy is then \( U_1 = +\mu_1 B \)

While for Loop 2 the potential energy is then \( U_2 = -\mu B \)

The potential energy for Loop 2 is less than that for Loop 1
Motion of Current Loop

The current loop in its motion will oscillate about the point of minimum potential energy.

If the loop starts from the point of minimum potential energy and is then displaced slightly from its position, it will “return”, i.e. it will oscillate about this point.

This initial point is a point of **Stable equilibrium**.

If the loop starts from the point of maximum potential energy and is then displaced, it will not return, but will then oscillate about the point of minimum potential energy.

This initial point is a point of **Unstable equilibrium**.
More Than One Loop

If the current element has more than one loop, all that is necessary is to multiply the previous results by the number of loops that are in the current element.
Hall Effect

There is another effect that occurs when a wire carrying a current is immersed in a magnetic field.

Assume that it is the positive charges that are in motion.

These positive charges will experience a force that will cause them to also move in the direction of the force towards the edge of the conductor, leaving an apparent negative charge at the opposite edge.
The fact that there is an apparent charge separation produces an electric field across the conductor. Eventually the electric field will be strong enough so that subsequent charges feel an equivalent force in the opposite direction.

\[ q E_e = q v_d B \quad \text{or} \quad E_e = v_d B \]

Since there is an electric field, there is a potential difference across the conductor which is given by

\[ V = E_e d = v_d B d \]
Hall Effect

The Hall Effect allows us to determine the sign of the charges that actually make up the current.

If the positive charges in fact constitute the current, then potential will be higher at the upper edge.

If the negative charges in fact constitute the current, then potential will be higher at the lower edge.

Experiment shows that the second case is true.

The charge carriers are in fact the negative electrons.