Two identical violin strings

with same tension
\[ f_a = f_b = 440.0 \text{ Hz} \]

When one string gets 1.5 beats/second

\( f_a \neq f_b = f_a - f_b \)

+1.5 Hz = 440 Hz - f_b

\[ f_b = 440 \text{ Hz} - 1.5 \text{ Hz} \]

\[ f_b = 438.5 \text{ Hz} \]

By what fractional amount was string tension changed if it was increased

\[ f = \frac{1}{2} \left( \frac{1}{\sqrt{v}} \right) \]

slightly to both ends

\[ f = \frac{1}{2} \]

We are interested in change of tension related to change in freq.

Then in derivative (differential) of above relation

\[ \Delta f \propto \frac{\Delta T}{T} \]

Dividing both sides, we get

\[ \frac{\Delta T}{T} \propto \frac{2\Delta f}{f} \]

For increase of 1.5 Hz, then

\[ \Delta T = \frac{2\times 1.5 \text{ Hz}}{440 \text{ Hz}} = \frac{3}{440} \text{ Hz} = \frac{6.82 \times 10^{-3}}{440} \approx 0.000153 \text{ Hz} \]

\[ \Delta T = \frac{2(-1.5 \text{ Hz})}{440 \text{ Hz}} = \frac{-3}{440} = \frac{-6.82 \times 10^{-3}}{440} \approx -0.000153 \text{ Hz} \]