RL Circuits

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A circuit consisting of a resistor, an inductor, and a power supply (or Emf source) is called a RL circuit. An inductor is especially helpful in circuits where you do not want a rapidly changing current. Inductors are usually made up of coils of wire. A ferromagnetic material may be inserted in the coil to increase inductance. If you turn on current in a circuit, then the current through the coil goes from 0 to some value and hence the magnetic field due the current goes from 0 to some value along the axis of the coil. This changing magnetic field induces a back Emf in the coil, which opposes the change in current per time. We have

$$V = -L\frac{di}{dt} \tag{1}$$

where *V* is the back Emf across the inductor, *L* is its inductance, and $\frac{di}{dt}$ is the change in current flowing in the current per unit time.

Just as in the case of RC circuits, we can put a switch in the RL circuit to turn the current on and off as shown in Fig. 1. As soon as the switch is closed (at *t*=0) at connections *b* and *c*, there is no current through the resistor. The inductor resists the flow of current. Once a steady-state is reached (at *t*=∞), there is no back Emf since the current is constant and the potential drop across the resistor is *E*. When the switch is moved to connect point *c* with *f*, the Emf source is removed and the circuit is grounded so that the flow of current goes to 0 at *t*=∞.

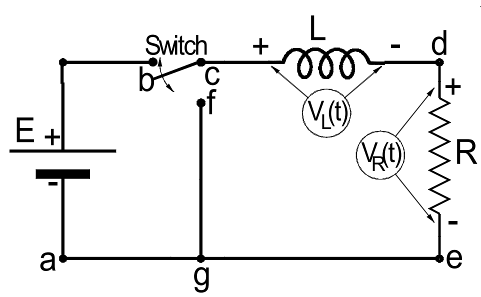

Figure 1. A simple RL circuit. When the switch is closed so that contact *b* is connected to contact *c*, the inductor resists the flow of current. The Emf source is removed by switching the switch so that contacts *c* and *f* are connected. Taken from Ref. [1].

Using Kirchoff's Laws, the potential increases and decreases will sum to zero when we go around a closed loop. Applying Kirchoff's Laws to the circuit in Fig. 1, we get

$$E - V_R(t) - V_L(t) = 0 \tag{2}$$

where E is the voltage of the Emf source, $V_R(t)$ is the voltage drop across the resistor and $V_L(t)$ is the voltage drop across the inductor. The current is related to charge by

$$i(t) = \frac{dq}{dt} \tag{3}$$

The voltage drop across the resistor is determined with Ohm's Law

$$-V_R(t) = -i(t)R \tag{4}$$

and the potential drop across the inductor is given by Eqn. 1

$$-V_L(t) = -L\frac{di(t)}{dt} \tag{5}$$

Inserting Eqn. (3) and (5) into Eqn. (2)

$$E - i(t)R - L\frac{di(t)}{dt} = 0. \tag{6}$$

Rearranging Eqn. (6) gives

$$\frac{di(t)}{i(t)-\frac{E}{R}} = -\frac{1}{\frac{L}{R}}dt. \tag{7}$$

Solving this equation yields

$$i(t) = \frac{E}{R}\left(1 - e^{-\frac{t}{\frac{L}{R}}}\right). \tag{8}$$

The voltage across the resistor using Ohm's Law is just

$$V_R(t) = E\left(1 - e^{-\frac{t}{\frac{L}{R}}}\right). \tag{9}$$

Using Eqns. (1) and (8) we find that the voltage across the inductor is

$$V_L(t) = Ee^{-\frac{t}{\frac{L}{R}}}. \tag{10}$$

The time constant, τ, is the time that it takes the voltage across the resistor to be $(1 - \frac{1}{e})$ of its maximum change, *E*. Thus, the time constant, τ, is equal to *L/R*.

Let's revisit our two cases at *t=0* and *t=∞*. As soon as the switch is closed, then there is no current flowing through the resistor. This observation is verified with Eqn. (9)

that shows $V_R(t)=0$ at $t=0$. The voltage across the inductor is E as seen with Eqn. (10). At $t=\infty$, the voltage across the inductor is 0 and the voltage across the resistor is E.

Now let's look at the case where we close the switch across the contact points c and f as seen in Fig. 1 where we ground out the circuit and remove the voltage source. Current is initially flowing in the circuit. In our experiment, this will correspond to where the voltage is 0 in the square wave outputted from the function generator. When E=0 in Eqn. (7), we have

$$\frac{di(t)}{i(t)} = -\frac{1}{\frac{L}{R}}dt, \tag{11}$$

which can be solved to give

$$i(t) = \frac{E}{L}e^{-\frac{t}{\frac{L}{R}}}. \tag{12}$$

The voltage across the resistor and inductor is then found to be

$$V_R(t) = Ee^{-\frac{t}{\frac{L}{R}}} \tag{13}$$

$$V_L(t) = -Ee^{-\frac{t}{\frac{L}{R}}}. \tag{14}$$

We can see the waveforms for these two corresponding scenarios in Fig. 2.

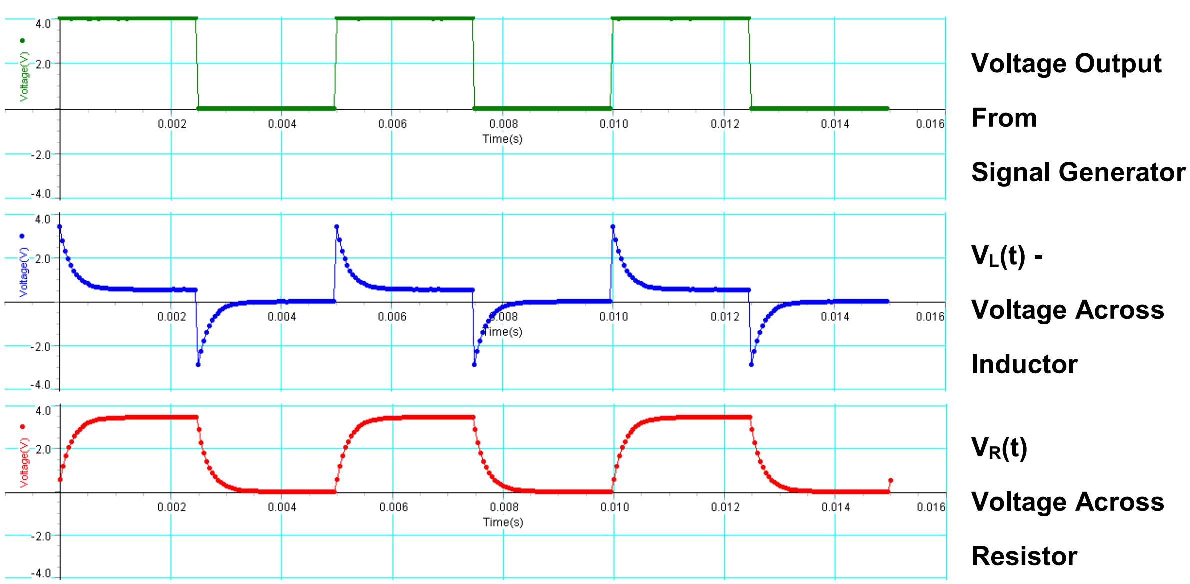

Figure. 2. The voltage is turned on in the circuit and then turned off (and grounded) in the circuit with a square-wave voltage waveform. The waveform response of the voltage across the inductor and resistor are shown. Taken from Ref [1].

Here we are going to study RL circuits with a RL circuit, a function generator and an oscilloscope. The circuit board is a Pasco Scientific RLC circuit board shown in Fig. 3 where three resistor values and one inductor values can be chosen to create the circuit. The voltage is supplied by the Pasco Scientific PI-9587C function generator where a square wave is generated. The signal turns on and then turns off, which is equivalent to closing the switch to charge the circuit and then opening the switch to discharge the circuit. The Tektronix TBS 1072B-EDU oscilloscope displays the waveform and is connected to the computer by a USB cable. The waveform is collected using the Tektronix Open Choice software program.

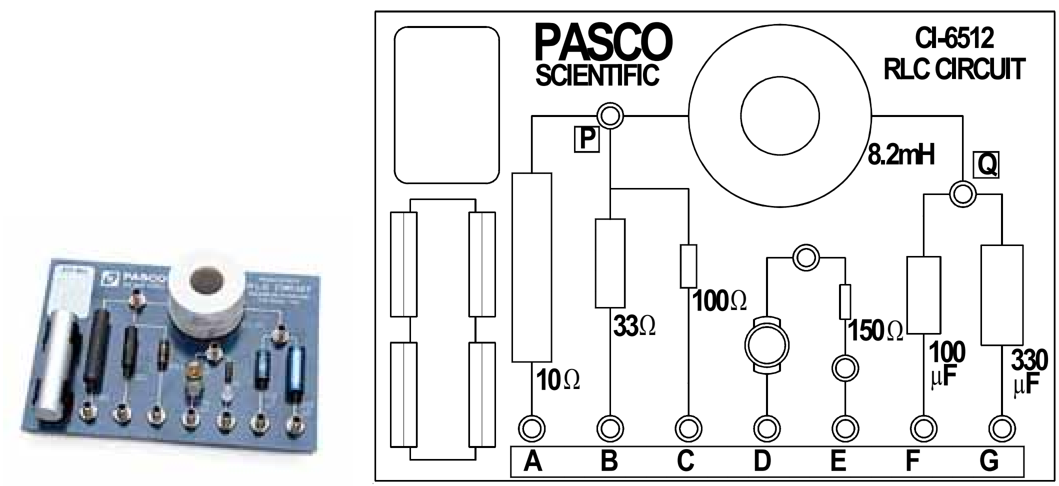

Figure 3. Pasco RLC (resistor-inductor-capacitor) circuit board (CI-6512) with Pasco 850 Universal Interface data acquisition system. Taken from Ref. [1].

The frequency of the function generator can be adjusted. It should be chosen so that the current in the RL circuit has enough time to reach a steady-state condition. A good guide is to adjust the frequency to *1/20τ*. Calculate the time constant *τ*, *20τ*, and signal frequency, *f* (*1/20τ*), for an 8.2 mH inductor and 100 Ω resistor in an Excel spreadsheet. Set the function generator to output the signal frequency that you calculated in your Excel worksheet and set the output waveform to a square wave on the function generator. Next completer the circuit as shown here in Figs. 4 and 5.

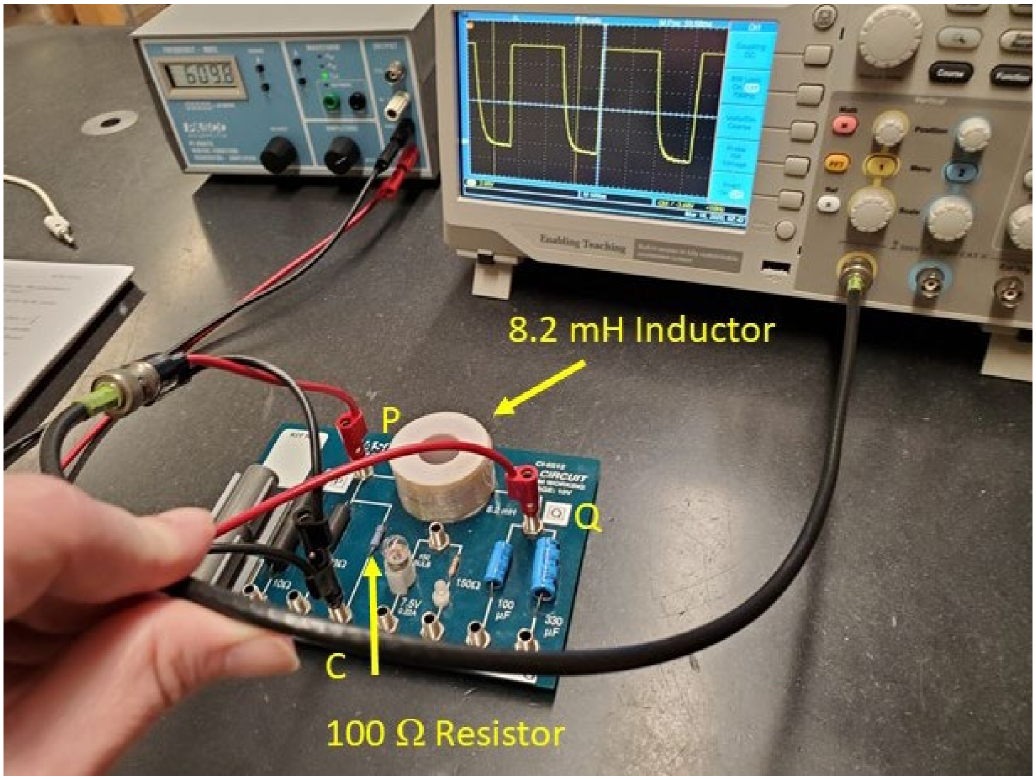

Figure 4. The function generator connects to points Q (red lead) and C (black lead) on the RL circuit board. Points P (red lead) and C (black lead) are connected to an oscilloscope to measure the voltage drop across the 100 Ω resistor.

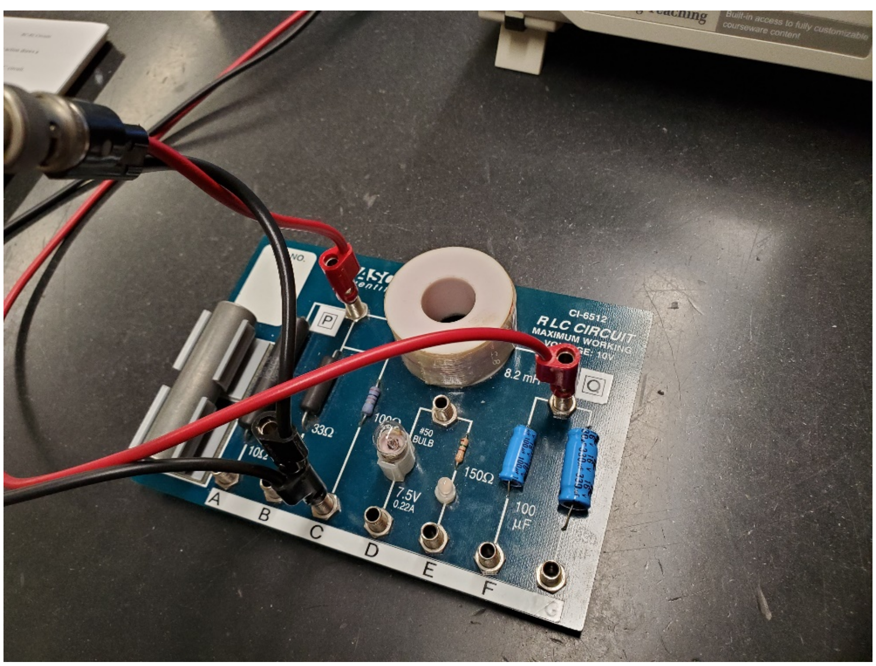

Figure 5. The function generator connects to points Q (red lead) and C (black lead) on the RL circuit board. Points P (red lead) and C (black lead) are connected to an oscilloscope to measure the voltage drop across the 100 Ω resistor.

Set the oscilloscope up so that you can see the voltage drop across the resistor. The waveform should look like that shown in Figs. 6 and 7. The waveform is offset so that it can be seen on the screen. You can see the yellow arrow with a 1 on it on the left of the screen. The yellow arrow shows the offset value and is normally set at the zero line. Therefore, the voltage signal is all positive above the zero line when you remove the offset. Compare this waveform to the one in Fig. 2.

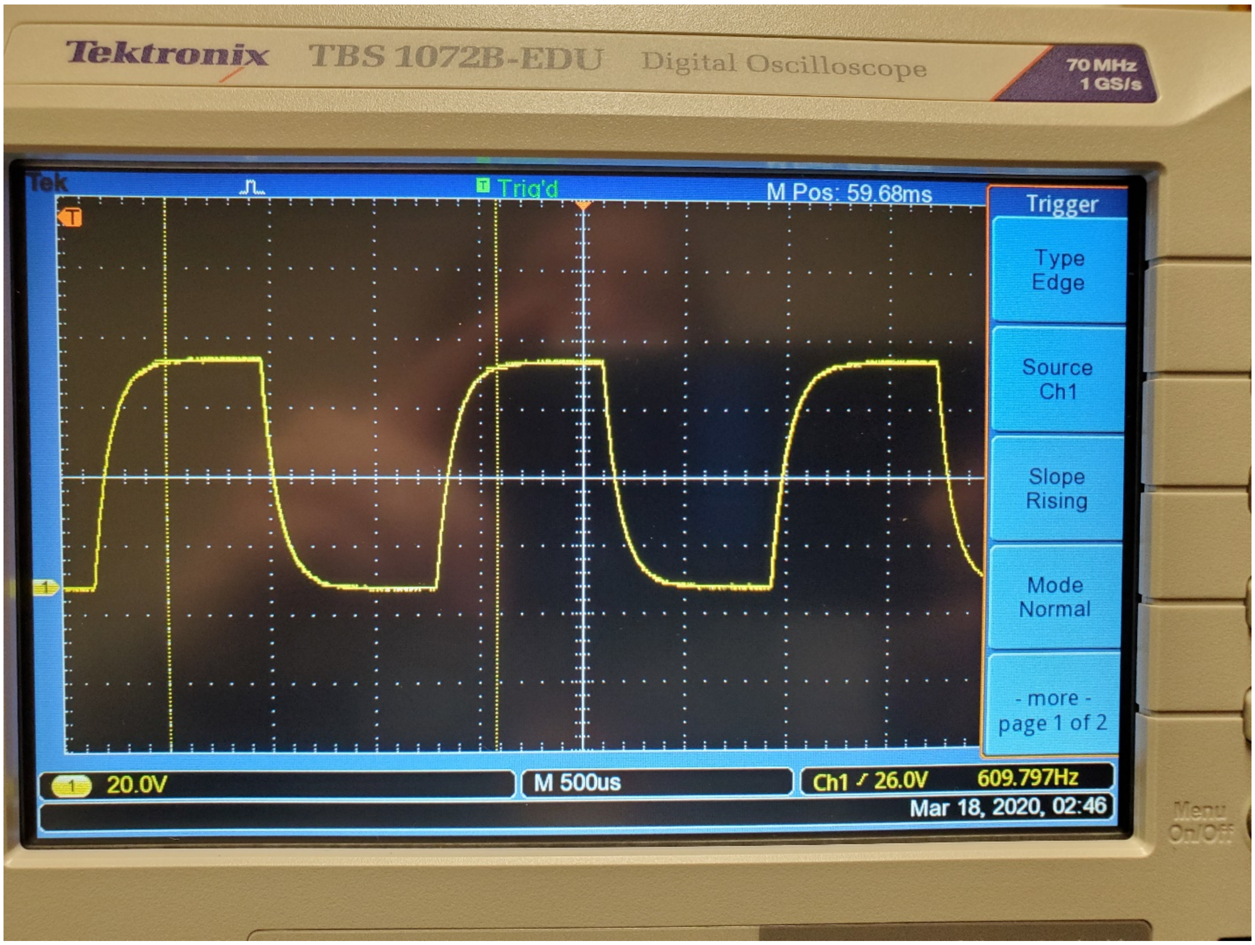

Figure 6. The voltage signal across the 100 Ω resistor in the RL circuit. Notice the little yellow arrow with the 1 on the left of the screen in the picture. That marks the offset of the signal. If there is no offset, then that yellow arrow with the 1 would be in the middle of the screen on the zero line. So this means that all of the signal is positive.

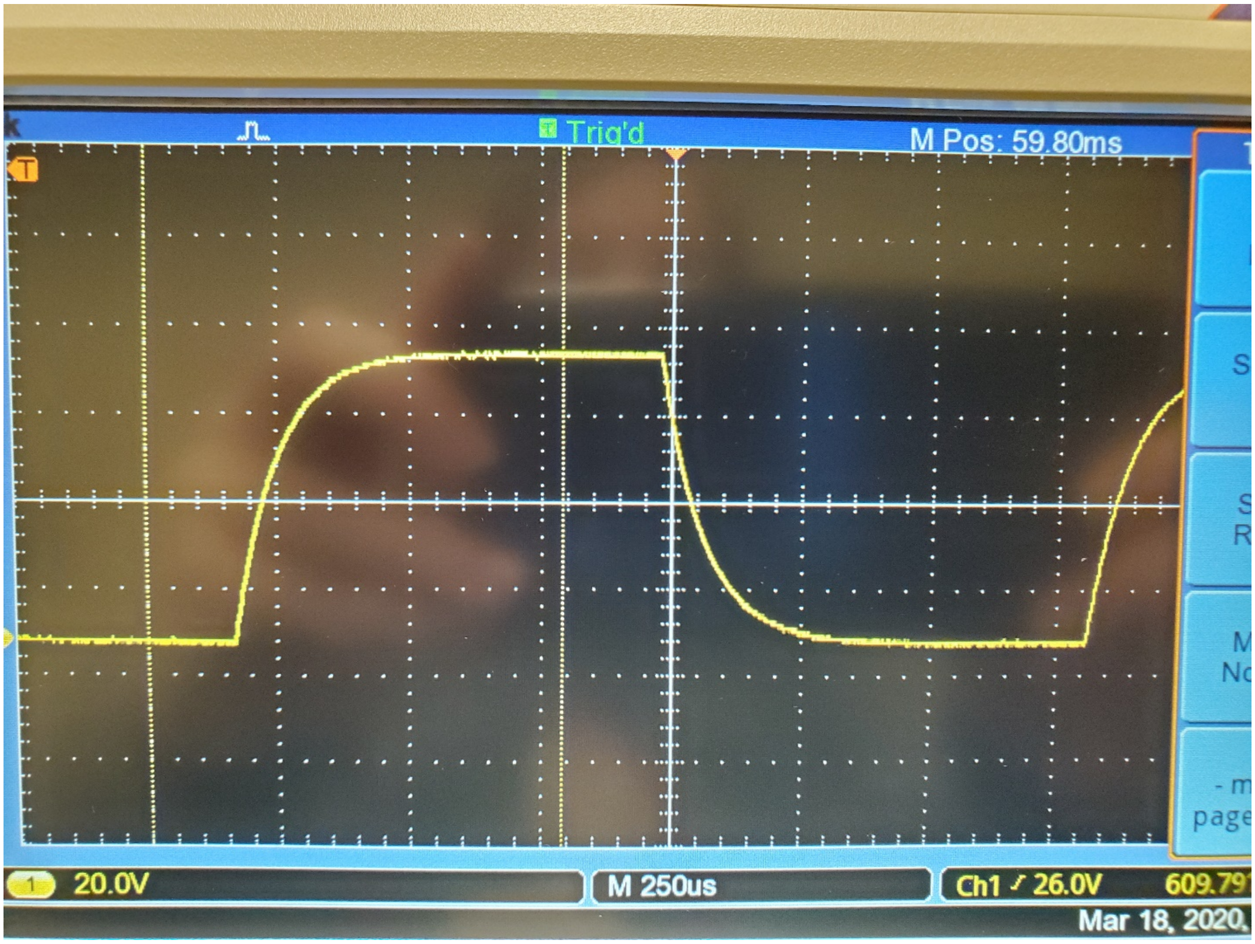

Figure 7. A zoomed in version of the voltage drop over the resistor in the RL circuit.

You will want to zoom in as much as possible on the signal on the oscilloscope and capture the decay of the signal. If your signal takes up less of the screen of the oscilloscope, then the resolution of the data will not be very good since the data is binned (less bins for the same data give less resolution). Collect the data using the Tektronix OpenChoice software located on the desktop. (The data will be all positive – the offset just helps visually see the data on the screen of the oscilloscope better.)

To collect the data, make sure that the USB connection is highlighted under "Select Instrument" under the "Screen Capture" tab. Click on the "Waveform Data Capture" tab and select Channel 1. Under the "Screen Capture" tab, select "Get Data" and then "Save As" and save the data as a .csv file. Open the data in Excel.

Now you will need to analyze the data. Copy the data from Excel and paste it into a table in the Pasco Capstone software. Graph the data. Now use the highlighting tool in the Pasco Capstone software to select the decay of the signal. Then use the exponential fitting tool to fit the data and determine the time constant. Is this value close to the value that you expected? How would the tolerance in the resistor and inductor values affect your time constant?

The voltage drop across the inductor is measured with the oscilloscope by connecting the red lead to point P and the black lead to point Q as seen in Fig. 8. An example waveform is shown in Fig. 9. The voltage source is measured with the oscilloscope by connecting the red lead to point C and the black lead to point Q as seen in Fig. 10. Compare your waveform to Fig. 2.

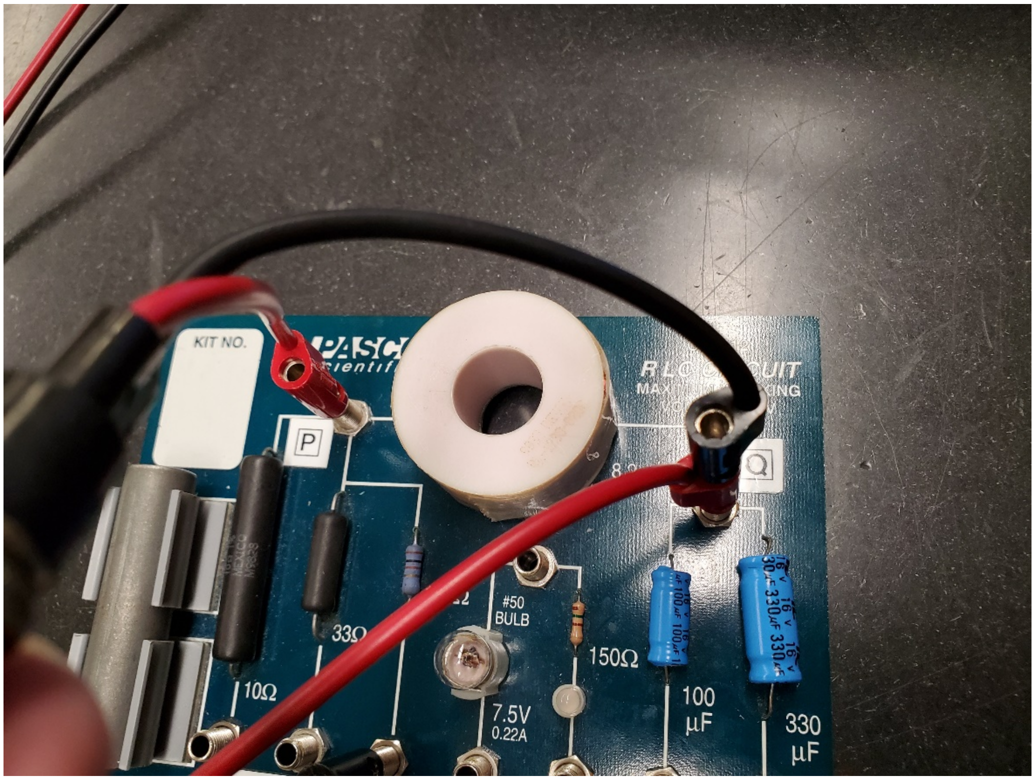

Figure 8. The oscilloscope measures the voltage drop across the 82 mH inductor.

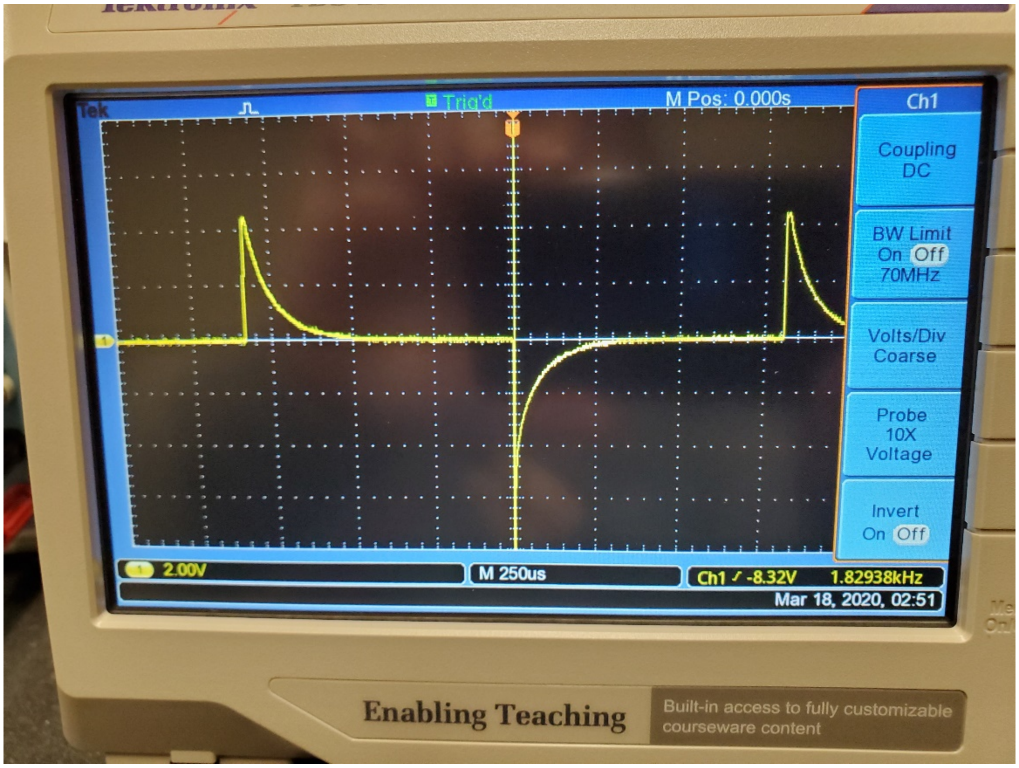

Figure 9. The waveform of the voltage drop across the 8.2 mH inductor in the RL circuit.

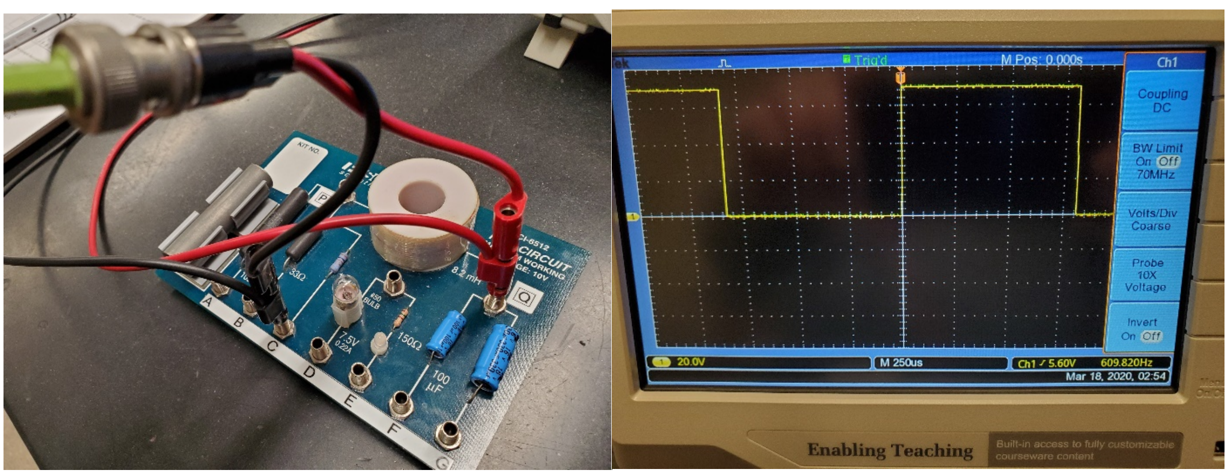

Figure 10. The voltage is measured across point Q (red lead) and point C (black lead) which are the input lead points from the function generator. The voltage measured is from the voltage source (from the function generator).

Repeat the experiment by adding an iron core inside the inductor as seen in Fig. 11. In Fig. 11 the voltage drop is measured across the 100 Ω resistor, but you can see that the inductance has changed based on the shape of the waveform. To get a waveform that completes the cycle to the maximum and a minimum values, the frequency was adjusted to 165.9 Hz as seen in Fig. 12. You should adjust your signal frequency to whatever works best with your iron core. Using your value for the resistor as R=100 Ω, calculate the value of the inductance using your measured value of the time constant. How does it compare to the inductance value without the iron core (8.2 mH)?

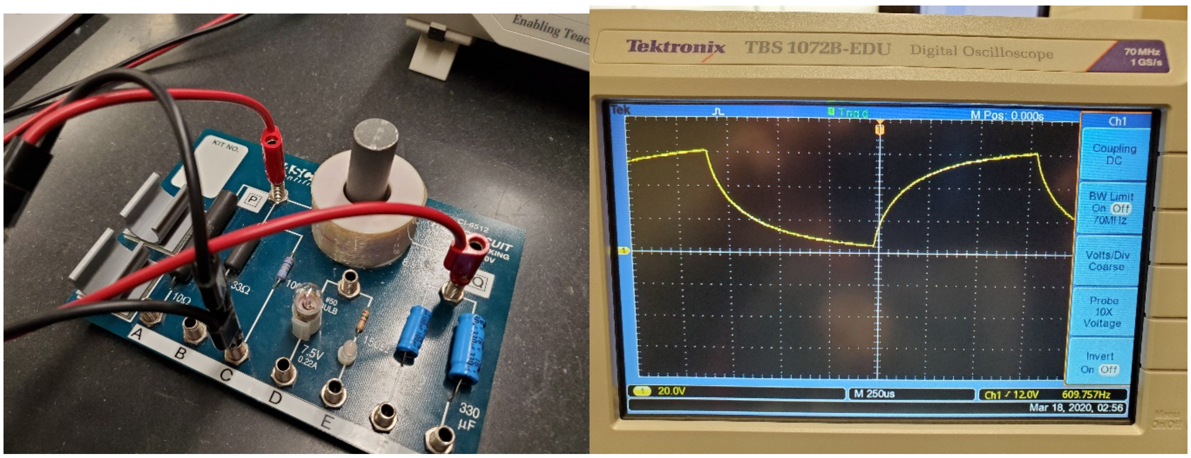

Figure 11. An iron core is added inside the inductor, and the voltage drop is measured across the 100 Ω resistor.

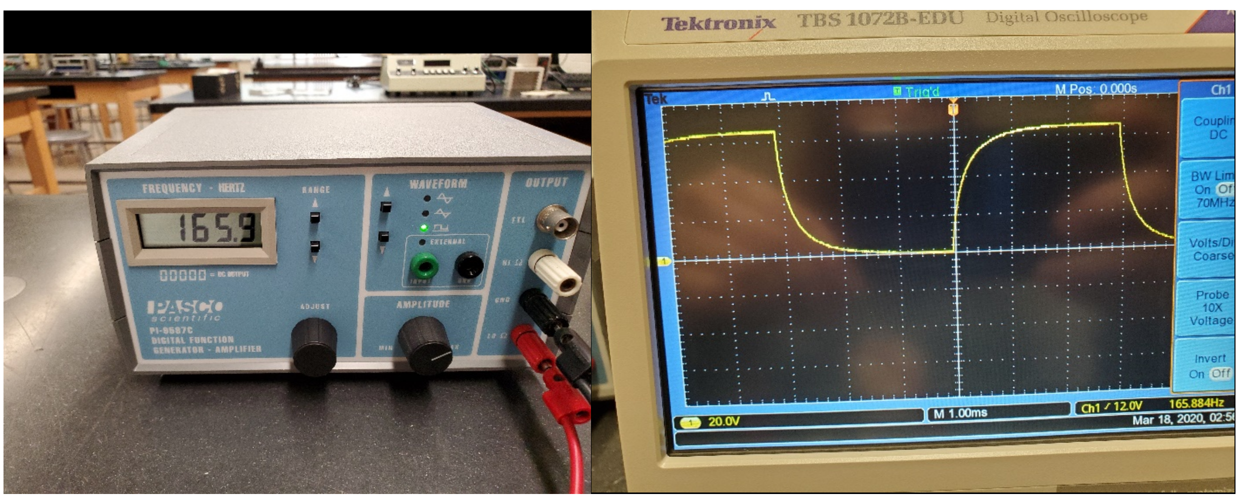

Figure 12. An iron core is added inside the inductor, and the frequency of the function generator is adjusted to achieve a waveform that reaches a maximum and minimum.