Simple Pendulum and Hooke's Law Prelab

by

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Simple harmonic motion is a sinusoidal periodic motion that can be described by the equation

$$y = A \sin \omega t$$

where *y* is the displacement, *A* is the amplitude of the motion, ω is the angular frequency, and *t* is the time. Recall that $\omega = 2\pi f$ where *f* is frequency, and the period *T* is related to *f* by $T = \frac{1}{f}$. In today's lab you will study simple harmonic motion with both the simple pendulum and a mass on a spring.

Simple harmonic motion is used as a model to study bipolar disorder.[1] Patients with bipolar disorder will have mood changes between hypomania and major depression which may be modelled with a harmonic oscillator where mood stabilizers can limit the oscillations by effectively damping the oscillations.[2-4] In nature there are many things that obey simple harmonic motion.

In the Simple Pendulum lab, you will measure the period of a pendulum (ball attached to a string) by timing the number of oscillations.[5] You will divide the total time by the number of cycles to get the period of the pendulum. You will show that the period only depends on the length of the pendulum (measure to center of mass of ball) and the acceleration of gravity for small angles. You will also see that the pendulum obeys simple harmonic motion at small angles.

When the pendulum swings to the endpoints, it changes direction so the magnitude of the velocity is zero, but the acceleration is at a maximum since the velocity is changing in direction. The potential energy is at a maximum at the endpoints. When the pendulum swings to the center, its velocity is at a maximum, its acceleration is at a minimum, and its potential energy is zero. Its kinetic energy is at a maximum here since its velocity is at a maximum. Go to https://phet.colorado.edu/sims/html/pendulum-lab/latest/pendulum-lab_en.html and doubleclick on "Lab."[6] Click on the Velocity and Acceleration radio buttons and open up the Energy Graph by clicking on the + sign. Pull the pendulum to an angle and release. Watch the velocity and acceleration vectors and the energy graph to see how these entities change.

Many people trust physics and that the pendulum will not swing out farther than where it is released unless you give it a push. Watch the video at https://www.youtube.com/watch?v=i2GdY1OlDpA or at https://www.youtube.com/watch?v=EZNpnCd4ZBo to see that the ball does not come back and hit the person in the face.[7, 8] It is important to not lean forward after you release the ball and to not give the ball a push. There are other videos where the people did not stay still and got a surprise whack!

During the Hooke's Law lab you will hang different masses from a spring and measure the displacement of the spring from equilibrium. By taking several measurements, you will extract the spring constant of the spring by plotting force on the spring vs. displacement. You will also count the oscillations of the mass when disturbed and calculate the period. The period of the spring will depend on the mass and the spring constant.

When you hang a mass from a spring, the forces acting on the spring are the force due to gravity (or weight) (called the applied force in today's lab) and the restoring force of the spring as shown in Fig. 1. If the mass is at rest, then the net force is zero, and these two forces are equal and opposite. The restoring force is always in the direction going back to equilibrium. If you stretch a spring and let go, it will want to coil back up. If you compress a spring and let go, it will want to stretch back out to its equilibrium state. The net force equation can be written as

$$F_{net} = 0 = F_{restoring} - F_g = kx - mg$$

where k is the spring constant of the spring and m is the hanging mass. Thus

$$kx = mg.$$

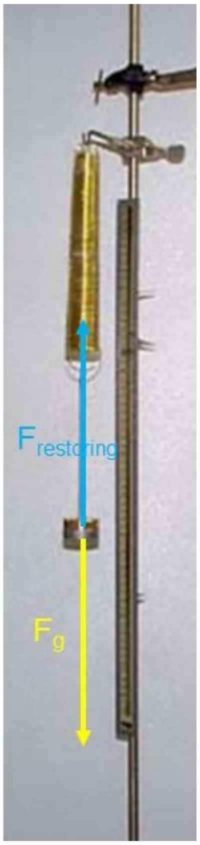

Figure 1. The mass on the spring exerts the force of gravity on the spring that balances the restoring force of the spring.

The motion of a mass on a spring will also obey simple harmonic motion when there are no outside forces acting on it.[9]

$$y = A \sin \omega t = A \sin \sqrt{\frac{k}{m}}\, t$$

Here you see that

$$\omega = \sqrt{\frac{k}{m}}$$

and

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

Questions

Simple Pendulum

1. Go to https://phet.colorado.edu/sims/html/pendulum-lab/latest/pendulum-lab_en.html and doubleclick on "Intro." Make sure that the Stopwatch is checked. Adjust the length of the string to 0.70 m and the mass to 1.00 kg. The gravity should be that of Earth, and no friction should be applied. Move the mass to 5^o. Start the stopwatch when the mass comes back to 5^o and count ten oscillations and stop the stopwatch. What is the period of the pendulum (take your number and divide by 10)?
2. Repeat this for a few angles. About what angle does the period start to change (when the small angle approximation no longer holds)?
3. Change the length of the pendulum to 0.4 m. What is your period at 5^o?
4. Keep the length at 0.4 m, and change the mass to 1.3 kg. What is your period at 5^o?

Hooke's Law

5. What is the spring constant of a spring if the hanging mass is 0.100 kg and the displacement from equilibrium is 0.065 m?

Things to consider when doing the lab:

1. The length of the pendulum is from the pivot point to the center of mass of the ball.
2. During the Hooke's Law lab, attach the small diameter coil part of the spring to the hook. Remove the spring when you are done so that it does not get needlessly stretched out where it is no longer within its elastic limit.

References:

[1] Koutsoukos, Elias and Elias Angelopoulos, "Mood regulation in bipolar disorders viewed through pendulum dynamics concept." International Journal of Bipolar Disorders, **2**:9 (2014).

[2] https://people.ok.ubc.ca/rtyson/Teaching/Math225/W09/assignments/a3_article_sample.pdf

[3] https://mtbi.asu.edu/sites/default/files/models_of_negatively_damped_harmonic_oscillators_the_case_of_bipolar_disorder.pdf

[4] Daugherty, Darryl, *et al*. "Mathematical models of bipolar disorder." Commun Nonlinear Sci Numer Simulat **14**, 2897 (2009).

[5] Parks, James E. *Contemporary Introductory Physics Experiments, 2nd Edition*, Hayden-McNeil Publishing: Plymouth 2014.

[6] https://phet.colorado.edu/sims/html/pendulum-lab/latest/pendulum-lab_en.html

[7] https://www.youtube.com/watch?v=i2GdY1OlDpA

[8] https://www.youtube.com/watch?v=EZNpnCd4ZBo

[9] http://hyperphysics.phy-astr.gsu.edu/hbase/shm.html