## **Poisson Statistics**

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## **Objectives**

The objectives of this experiment are: (1) to study the Poisson probability distribution as it is applied to counting statistics, (2) to study the Gaussian probability distribution as a special case of the Poisson distribution applied to measurements with a large mean value, and (3) to learn some statistical analysis techniques using a spreadsheet to analyze, graph, and compare results.

#### Theory

Radioactive decay is a random process in which the emission of radiation depends on the number of atoms that can decay and a probability function that is characteristic their natural lifetimes. The detection of particles is random and any two measurements of the particles over equal periods of time will most likely be different. For large numbers, the difference between the measurements will be a small percentage. The probability of detecting a specific number of events for a given measurement is given by the standard normal or Gaussian distribution as was studied in the experiment on statistical analysis. However, for the measurement of a small number of events, the probability distribution for detecting a specific number of events is different and is given by a Poisson distribution. For example, in a small period of time, the number of particles that are detected may only be about 5 on the average. A change of 1 or 2 detected events is a significant percentage of the average number.

For rare events, the average number detected might also be much less than one, The Poisson distribution applies to these measurements and is useful for determining the probability of detecting a single event or more than one event in the same period. The Poisson distribution is a special case of the binomial distribution, similar to the Gaussian distribution being a special case.

The Poisson distribution is given by

$$P(n) = \frac{\overline{n}^n}{n!} e^{-\overline{n}} \tag{1}$$

where P(n) is the normalized probability that in a given time interval n events will be observed and  $\overline{n}$  is the average number of events that are observed when many samples are taken. P(n) is said to be a normalized distribution so that when it is summed over all possible values of n, it is equal to unity. P(n) is normalized to one sample and is therefore given by

$$\sum_{n=0}^{N} P(n) = 1 \tag{2}$$

If M samples are measured with each yielding some number  $m_j$ , (j=1 to M), the average number for a sample will be  $\overline{n}$  given by

$$\overline{n} = \frac{\sum_{j=1}^{M} m_j}{M} \tag{3}$$

If some fraction of the values of  $m_j$  have the same value  $n_i$  and there are N possible values of  $n_i$ , (i=1 to N), then

$$\sum_{i=1}^{N} f_i n_i = \sum_{j=1}^{M} m_j \tag{4}$$

where  $f_i$  is the frequency that the same number  $n_i$  occurs. By making this substitution into Equation (3),

$$\overline{n} = \frac{\sum_{i=1}^{N} f_i n_i}{M} \tag{5}$$

and

$$\overline{n} = \sum_{i=1}^{N} \frac{f_i}{M} n_i \tag{6}$$

The term  $\frac{f_i}{M}$  is just the probability of measuring a value  $n_i$ ,  $P(n_i)$ , so that

$$\overline{n} = \sum_{i=1}^{N} n_i P(n_i) \tag{7}$$

By dropping the subscripts

$$\overline{n} = \sum nP(n) . (8)$$

When the average number of events is a large number, Poisson's equation is very difficult to deal with because of the large values of n! For large values of n, the binomial distribution is better characterized with a Gaussian distribution. Recall from the previous experiment on statistical analysis that the Gaussian distribution is given by

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-\overline{x})^2}{2\sigma^2}\right]$$
 (9)

where  $\bar{x}$  is the mean value about which the distribution is centered,  $\sigma$  is the standard deviation from the mean, and P(x) is the normalized probability of measuring a value x.  $P(x_i)$  is a continuous function and must be multiplied by an increment in x,  $\Delta x$ , in order for it to represent a finite number. Therefore, the mean value is given by

$$\overline{\mathbf{x}} = \sum_{i=1}^{N} \mathbf{x}_{i} \mathbf{P}(\mathbf{x}_{i}) \Delta x. \tag{10}$$

The value of  $\sigma$  is given by the square root of the variance,  $\sigma^2$ ,

$$\sigma^2 = \sum_{i=1}^{N} \frac{(x_i - \bar{x})^2}{N - 1}.$$
 (11)

Since, the distribution is normalized,

$$\sum_{i=1}^{N} P(x_i) \Delta x = 1.$$
 (12)

For integers, the smallest increment in x,  $\Delta x = \Delta n$ , is 1, so when the continuous function P(x) is changed to an integer form P(n), Equation (9) becomes

$$P(n) \Delta n = \frac{1}{\sigma \sqrt{2\pi}} \exp \left[ -\frac{\left( n - \overline{n} \right)^2}{2\sigma^2} \right] \Delta n.$$
 (13)

and

$$P(\mathbf{n}) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{\left(\mathbf{n}-\overline{\mathbf{n}}\right)^2}{2\sigma^2}\right]. \tag{14}$$

since  $\Delta n = 1$ .

The standard deviation can be approximatedy by the square root of the mean,

$$\sigma = \sqrt{\overline{n}} \,\,\,\,(15)$$

and Equation (14) can be approximated by

$$P(\mathbf{n}) = \frac{1}{\sqrt{2\pi\overline{\mathbf{n}}}} \exp\left[-\frac{(\mathbf{n}-\overline{\mathbf{n}})^2}{2\overline{\mathbf{n}}}\right]. \tag{16}$$

The standard deviation can be computed with Equation (11). The standard deviation computation is provided as a statistical function in the Excel functional operators and the Gaussian distribution can be computed easily and more correctly as will be shown.

#### **Apparatus**

The apparatus is shown in Figure 1 and consists of: (1) a radioactive source, (2) a Geiger-Mueller tube with stand and source holder, (3) a scaler to count the radioactive decays, (4) a Pasco 750 Science Workshop data acquisition interface, (5) Pasco DataStudio software, and (6) computer with Excel spreadheet program.



Figure 1. Apparatus and setup for performing the Poisson Statistics experiment.

#### **Procedure**

#### **General Considerations**

The procedure has two parts: (1) measurements where the average number of counts is low, approximately 2-7 counts per period, and (2) measurements where the average number of counts is 100 to 200 counts per period. In the first case Poisson statistics apply, but in the second case, a Gaussian distribution is applied.

The performance of the Geiger counter depends on the high voltage supply and that has been pre-set for this experiment. Normally, an experiment would be run to determine the operating voltage, but that process has been eliminated for this experiment to concentrate on the study of the counting statistics, the Poisson distribution, and Gaussian distribution. Unless instructed otherwise, the high voltage for the counter should be set at 900 volts.

The number of radioactive events that are detected by the Geiger counter depends on the geometrical relation that the source has to the counter. The closer the source is to the counter, the more events will be detected. The source can be located at various distances from the entrance window of the Geiger tube, and this distance will need to be adjusted to give the desired counting rate for the study. Also, because of the location of the radioactive source material embedded in the plastic disc,the orientation of the source will affect the count rate. There will be a difference in detected events depending on which side of the disc is closest to the Geiger window. In Part I, the source will need to be placed near the bottom of the source holder and will need to be moved closer to the tube for Part II. The number of counts that are detected in a 0.2 second time interval in Part I needs to be approximately 2-7, while for Part II the number needs to be approximately 100-200.

#### Part I – Poisson Distribution

## **Data Acquisition Setup**

- 1. From the desktop open the DataStudio program by clicking on the *DataStudio* icon.
- 2. Select *Create an Experiment* from the choices in the *Welcome to DataStudio* pop-up window.
- 3. In the *Experiment Setup* window scroll through the list of *Sensors* in the pull-down menu to the *Geiger Counter* sensor icon, Geiger Counter. Double click on the *Geiger Counter* sensor icon to add it to *Digital Channel 1*. Check and make sure that the cable connected to the Nucleus scaler is plugged into Digital Channel 1 on you 750 interface.
- 4. Double click on the *Geiger Counter* sensor icon connected to the 750 interface in the *Experiment Setup* window. This brings up the *Sensor Properties* window. Under the general tab, click on the *Fast* [> 1 Hz] button to select it (if it isn't already selected), and change the sample rate to 5 Hz using the +/- buttons, will set the data acquisition system to count signal pulses during 0.2 second periods. Click on the *Measurement* tab in this window. Make sure that the *Geiger Counts*, *Ch1* (*counts/sample* check box is checked, and then click *OK*.
- 5. Click on the *Options* button on the *Setup Toolbar* and this will bring up the *Sampling Options* window. This window can be used to control the starting and stopping of data collection. In the *Delay Start* section, click the radio button to select *None*.

- 6. In the automatic stop section, select the radio button *Time* and set the time to 500.2 seconds. This will allow 2500 measurements of the number of counts that occur in 0.2 second increments.
- 7. Under the *Displays* window double click both on the graph icon and the table icon. This will display the data collected from the Geiger counter in a table and graph.
- 8. Arrange the graph and table windows to best display your results. You can hide the *Displays* and *Data* windows by clicking on the small up and down arrow beside the *Summary* button on the DataStudio toolbar.
- 9. When ready, click the *Start* button to start collecting the data. The data will be recorded in the table and graphed on the graph. If for some reason data collection doesn't stop, you can stop data collection by clicking on the *Stop* button.
- 10. On each of the window's *Graph* and *Table* there is a drop down menu beside the toolbar button labeled *Data*. Select *Clear Data* from the drop down menu. This will allow you to run the experiment again. Each data set is saved separately, so you can run as many trials as you want to run so that you can then go back and compare your results.

## Acquisition of Data for Part I

- 1. Place the source in the source holder near the bottom. Turn on the scaler with the *Power* rocker switch. Place the scaler in the stopped mode by depressing the *Stop* rocker switch and then depressing the *Reset* rocker switch.
- 2. Click on the *Start* button of DataStudio and observe the number of events detected in each of the 0.2 second time intervals. Move the source to a position that yields an average number equal to 2-7 counts.
- 3. Click on the *Stop* button to stop the data acquisition. Select *Experiment* from the DataStudio main menu bar and then *Delete All Data Runs* from the pull down menu. Click on *OK* and the experiment is set to take data.
- 4. Click on the *Start* button and collect data for the next 500 seconds. Observe the data as it is being accumulated in the table. If the data collection isn't meeting the criteria that is desired, stop the experiment, clear out the results, readjust the parameters, and begin again. The accumulation of data should stop automatically.

#### Data Analysis for Part I

1. Start the Excel spreadsheet program either through the start menu or with a shortcut on the desktop.

- 2. Return to the DataStudio program, and using the left mouse button, click on the area of Table 1 that has the title *Geiger Counts, Ch1* and *Run #x*. This will highlight in yellow all the numbers in the table. (As an alternative, click on the uppermost of the cells in the left column of time data and while holding the left mouse button down, drag the cursor to the lowermost cell containing data in the right, counts, column to select the entire data set of Geiger counts versus time. Then select the *Edit* menu option from the top menu bar of the DataStudio program and then select *Copy*.
- 3. Return to Sheet 1 of the Excel program and locate your cursor in cell A1 and click on this cell with the left button of your mouse. Choose the *Edit* option from the top menu bar and then select *Paste*. These operations should place your data from your run into the spreadsheet. Examine the copied data to make sure that this has been done. Your spreadsheet should look something like the one in Figure 2.

| B Elle Edit View Insert Format Iools Data Window Help |                |                 |         |    |              |              |   |  |  |  |  |  |
|---|----------------|-----------------|---------|----|--------------|--------------|---|--|--|--|--|--|
|   | А              | В               | С       | D  | Е            | F            | G |  |  |  |  |  |
|   | Geiger Counts, |                 |         |    |              |              |   |  |  |  |  |  |
| 1   | Ch1, Run #3    |                 |         |    |              |              |   |  |  |  |  |  |
|   |                | Geiger Counts   |         |    | Frequency    | Poisson      |   |  |  |  |  |  |
| 2   | Time (s)       | (counts/sample) | Average | n  | Distribution | Distribution |   |  |  |  |  |  |
| 3   | 0.2            | 6               | 3.2084  | 0  | 94           | 101.0530888  |   |  |  |  |  |  |
| 4   | 0.4            | 4               |         | 1  | 315          | 324.2187302  |   |  |  |  |  |  |
| 5   | 0.6            | 3               |         | 2  | 549          | 520.1116871  |   |  |  |  |  |  |
| 6   | 0.8            | 6               |         | 3  | 558          | 556.2421123  |   |  |  |  |  |  |
| 7   | 1              | 4               |         | 4  | 448          | 446.1617982  |   |  |  |  |  |  |
| 8   | 1.2            | 3               | 3.2084  | 5  | 274          | 286.2931027  |   |  |  |  |  |  |
| 9   | 1.4            | 2               |         | 6  | 143          | 153.0904651  |   |  |  |  |  |  |
| 10  | 1.6            | 2               | 3.2084  | 7  | 68           | 70.16792118  |   |  |  |  |  |  |
| 11  | 1.8            | 2               |         | 8  | 33           | 28.14084479  |   |  |  |  |  |  |
| 12  | 2              | 9               |         | 9  | 9            | 10.03189849  |   |  |  |  |  |  |
| 13  | 2.2            | 3               |         | 10 | 6            | 3.218634312  |   |  |  |  |  |  |
| 14  | 2.4            | 1               |         | 11 | 3            | 0.938787848  |   |  |  |  |  |  |
| 15  | 2.6            | 2               |         | 12 | 0            | 0.251000578  |   |  |  |  |  |  |

Figure 2. Example Excel spreadsheet for the recording and analysis of Poisson statistics data.

4. Add the column headings shown in the example for columns C, D, E, and F. These should be labeled, *Average*, *n*, *Frequency Distribution*, and *Poisson Distribution*.

Format the column widths by selecting columns A through F and choosing *Format* from the Excel main menu bar. Choose *Column* and *Width*... from the pull down menus and type 15 in the *Column Width*'s popup window input box.

Select the block of cells A1:F2 and choose *Format* from the Excel main menu bar. Choose *Cells*... from the pull down menu and click on the *Alignment* tab in the *Format Cells* popup window. Click on the check box to choose the *Wrap text* option.

5. In cell C3 find the average count for the 2500 measurements. To do this, choose *Insert* from the main menu bar and then choose *Function*. From the *Paste Function* window choose *Statistics* from the *Function category* list and *AVERAGE* from the *Function name* list. Then click on *OK*. Input *B3:B2502* into the *Number 1* input box and click on *OK*. You can check this result by summing all the numbers in the B

- column and dividing by the number of numbers. The average should be less than 5 in order to provide a good test for the Poisson distribution.
- 6. In column D, under the n label, insert numbers 0, 1, 2, 3, ..., 20. You can enter 0 in cell D3 and insert an equation in cell D4, and then use the copy and fill functions to enter the rest of the numbers.
- 7. In cells E3 to E23 find the frequency (the number of times) each of the numbers n appear in 2500 measurements. In other words, how many of the measurements resulted with a 0 count, how many resulted in 1 count, how many resulted in 2 counts, etc.

To do this it is helpful to use a subtle feature of the Excel spreadsheet that uses arrays to enter and extract data. Arrays must be entered in a different manner than other entries. To begin, click on cell E3 and while holding the left mouse button down, drag the cursor down to cell E23 so that cells E3 to E23 are selected. Choose *Insert* from the main menu bar and then choose *Function*. From the *Paste Function* window choose *Statistics* from the *Function category* list and *FREQUENCY* from the *Function name* list. Then click on *OK*. Input B3:B2502 into the *Data array* input box and D3:D23 in the *Bins array* input box. While holding the *Ctrl* and *Shift* keys down, click on *OK*. Numbers should appear in all of cells D3 through D23. Check to see if these numbers seem reasonable.

- 8. Column E should now contain the number of times the measurements produced the value of n listed in column D, i.e. columns D and E should contain the data for the frequency distribution. A graph of the frequency distribution should now be made.
- 9. Select the data to be graphed in columns D and E. Choose *Insert* and then *Chart*. Choose *XY (Scatter)* as the *Chart type* and the option of *Scatter*. *Compares pairs of values*. for the *Chart sub-type*, You can click on *Next* and supply the information to format your graph in a meaningful way or just press *Finish* and go back later to format your graph. You should make sure the *Series* data assigns the values in the D column as the n values and the values in the E column as the Frequency values.
- 10. Check your graph to make sure it is correct. It should look something like the one shown in Figure 3. The frequency of numbers n should be on the y-axis versus the numbers n on the x-axis. Click on the graph and hold the mouse button down to drag it to the desired location on your spreadsheet.
- 11. Add a title to the graph and label the axes This can be done by clicking on *Chart* on the top menu bar and then choosing the *Chart Options* . . . selection and supplying the information.

Save your Excel workbook frequently to prevent loss of your work.

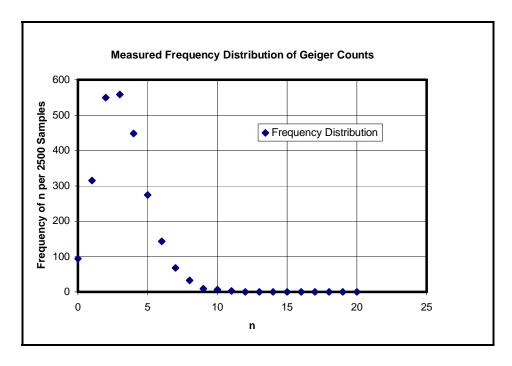


Figure 3. Example of graph of Poisson statistic data.

12. Test the data to check that it follows the Poisson distribution given by Equation (1),

$$P(n) = \frac{\overline{n}^n}{n!} e^{-\overline{n}}.$$

- 13. In cell F3, enter the formula,  $= C\$3^D3/FACT(D3)*EXP(-\$C\$3)*2500$ . Recall that P(n) is a normalized distribution and that  $\sum_{n=0}^{N} P(n) = 1$ . Therefore each value of P(n) that is computed must be multiplied by 2500 to correspond to the frequency distribution of 2500 measurements.
- 14. Copy the formula in cell F3 and copy it to the other cells, F4 through F23.
- 15. Compare the values of the frequency distribution in column E with those of the Poisson distribution computed in column F. The values should be about the same.
- 16. Add the data for the Poisson distribution to your graph. To do this click on your chart near its edges to select it and then select *Chart* from the main menu bar. Choose *Add Data*... from the drop down menu and input =*Sheet1!F3:F23* into the *Range* input box. Click on *OK* and this should add the calculated values for the Poisson distribution to your graph. Your graph should look something like the one in Figure 4.
- 17. Examine the Frequency and Poisson distributions and determine how well they agree.

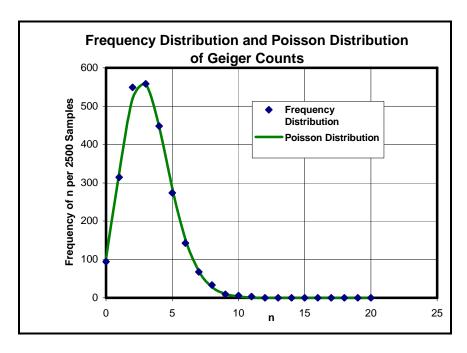


Figure 4. Graph showing both the Frequency distribution and the Poisson distribution.

#### Part II - Gaussian Distribution

#### Acquisition of Data for Part II

- 1. Move the source closer to the Geiger tube to increase the count rate. Click on the *Start* button of DataStudio and observe the number of events detected in each of the 0.2 second time intervals. Adjust the source to a position that yields an average number equal to 100-200 counts.
- 2. Click on the *Stop* button to stop the data acquisition. Select *Experiment* from the DataStudio main menu bar and then *Delete All Data Runs* from the pull down menu. Click on *OK* and the experiment is again set to take data.
- 3. Click on the *Start* button and collect data for the next 500 seconds. Observe the data as it is being accumulated in the table. If the data collection isn't meeting the criteria that is desired, stop the experiment, clear out the results, readjust the parameters, and begin again. The accumulation of data should stop automatically.

#### Data Analysis

1. Open the Excel workbook used in Part I and go to a new worksheet, Sheet 2.

2. Return to the DataStudio program, copy the data in Table 1, and paste it into the Excel spreadsheet. Examine the copied data to make sure that this has been done. Your spreadsheet should look something like the one in Figure 5.

| Bi Elle Edit View Insert Format Iools Data Window Help |                               |                                  |                 |    |           |   |  |   |  |  |  |  |
|--|-------------------------------|----------------------------------|-----------------|----|-----------|---|--|---|--|--|--|--|
|  | А                             | В                                | С               | D  | E         | F   | G  | Н |  |  |  |  |
| 1  | Geiger Counts,<br>Ch1, Run #1 |                                  |                 |    |           |   |  |   |  |  |  |  |
| 2  | Time (s)                      | Geiger Counts<br>(counts/sample) | Average         | n  | Frequency | Gaussian Distribution using Approximation | Gaussian Distribution using Standard Deviation |   |  |  |  |  |
| 3  | 0.2                           | 142                              | 130.9204        | 80 | 0         | 0.004362456                               | 2.42916E-05                                    |   |  |  |  |  |
| 4  | 0.4                           | 132                              |                 | 81 | 0         | 0.006411913                               | 4.40621E-05                                    |   |  |  |  |  |
| 5  | 0.6                           | 142                              | Minimum         | 82 | 0         | 0.009352484                               | 7.89851E-05                                    |   |  |  |  |  |
| 6  | 0.8                           | 118                              | 102             | 83 | 0         | 0.013537832                               | 0.000139925                                    |   |  |  |  |  |
| 7  | 1                             | 127                              |                 | 84 | 0         | 0.019447062                               | 0.000244972                                    |   |  |  |  |  |
| 8  | 1.2                           | 112                              | Maximum         | 85 | 0         | 0.02772309                                | 0.000423847                                    |   |  |  |  |  |
| 9  | 1.4                           | 148                              | 158             | 86 | 0         | 0.039220401                               | 0.000724724                                    |   |  |  |  |  |
| 10   | 1.6                           | 131                              |                 | 87 | 0         | 0.055063675                               | 0.001224636                                    |   |  |  |  |  |
| 11   | 1.8                           | 125                              | Standard Dev    | 88 | 0         | 0.076718678                               | 0.002045089                                    |   |  |  |  |  |
| 12   | 2                             | 140                              | 9.201822279     | 89 | 0         | 0.106076648                               | 0.003375114                                    |   |  |  |  |  |
| 13   | 2.2                           | 129                              |                 | 90 | 0         | 0.145553022                               | 0.005504725                                    |   |  |  |  |  |
| 14   | 2.4                           | 118                              |                 | 91 | 0         | 0.198200809                               | 0.008872658                                    |   |  |  |  |  |
| 15   | 2.6                           | 121                              | Square Rt. Ave. | 92 | 0         | 0.267838114                               | 0.014133275                                    |   |  |  |  |  |
| 16   | 2.8                           | 143                              | 11.44204527     | 93 | 0         | 0.359188227                               | 0.022248606                                    |   |  |  |  |  |
| 17   | 3                             | 121                              |                 | 94 | 0         | 0.478029357                               | 0.034612565                                    |   |  |  |  |  |
| 18   | 3.2                           | 142                              |                 | 95 | 0         | 0.631349444                               | 0.053215201                                    |   |  |  |  |  |
| 19   | 3.4                           | 141                              |                 | 96 | 0         | 0.827499605                               | 0.080855316                                    |   |  |  |  |  |

Figure 5. Example Excel spreadsheet for Gaussian distribution data.

- 3. Add the column headings shown in the example for columns C, D, E, F, and G. These should be labeled, *Average*, *n*, *Frequency Distribution*, *Gaussian Distribution using Approximation*, and *Gaussian Distribution using Standard Deviation*. Format the column widths to a width of 15. Center the labels and entries in the cells and use the wrap text option for the labels.
- 4. In cell C3 find the average count for the 2500 measurements. The average should be 100-200 in order to provide a good test for the Gaussian distribution.
- 5. Type the text *Minimum* and *Maximum* in cells C5 and C8 respectively. In cell C6 enter the function for finding the minimum count for all 2500 measurements, =*MIN(B3:B2502)*, and in cell C9 enter the function for finding the maximum count for all 2500 measurements, =*MAX(B3:B2502)*. This will give lower and upper limits on the range of values measured and will eliminate graphing a large number of zeros.
- 6. In cell D3, under the n heading, insert an integer that is approximately 50 less than an integer close to the nearest integer value to the average count for the 2500 measurements. Enter the formula =D3+1 in cell D4 and use the fill function to copy this formula in cells D5 to D103.

Check to see if the range of values between the minimum and maximum values for n is covered by these numbers in column D. If not, add numbers to the beginning and end to cover the range.

- 7. In cells E3 to E103 find the frequency (the number of times) each of the numbers n appear in 2500 samples.
  - Remember that arrays must be entered in a different manner than other entries. Click on cell E3 and select the cells down to cell E103. Type =FREQUENCY(B3:B2502,D3:D103) and while holding the Ctrl and Shift keys down, hit the Enter key.
- 8. Column E should now contain the number of times the measurements produced the value of n listed in column D, i.e. colums D and E should contain the data for the frequency distribution. A graph of the frequency distribution should now be made.
- 9. Select the data to be graphed in columns D and E. Choose *Insert* and then *Chart*. Choose *XY* (*Scatter*) as the *Chart type* and the option of *Scatter*. *Compares pairs* of values. for the *Chart sub-type*, You can click on *Next* and supply the information to format your graph in a meaningful way or just press *Finish* and go back later to format your graph. You should make sure the *Series* data assigns the values in the D column as the n values and the values in the E column as the Frequency values.
- 10. Check your graph to make sure it is correct. It should look something like the plot of data points shown in Figure 6. The frequency of numbers n should be on the y-axis versus the numbers n on the x-axis. Click on the graph and hold the mouse button down to drag it to the desired location on your spreadsheet.
- 11. Add a title to the graph and label the axes This can be done by clicking on *Chart* on the top menu bar and then choosing the *Chart Options* . . . selection and supplying the information.

Save your Excel workbook frequently to prevent loss of your work.

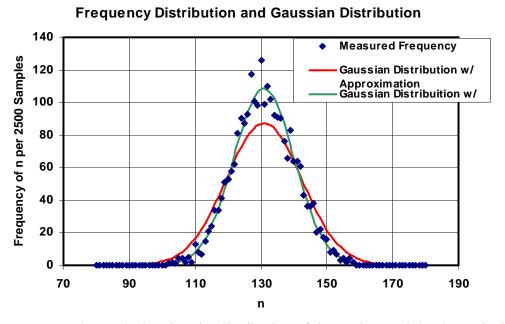
12. Check the data to see if it follows the Gaussian distribution given by Equation (16),

$$P(n) = \frac{1}{\sqrt{2\pi\overline{n}}} \exp \left[ -\frac{(n-\overline{n})^2}{2\overline{n}} \right]$$

where  $\sigma$  has been approximated by  $\sqrt{n}$ . Recall that P(n) is a normalized distribution and that  $\sum_{n=0}^{N} P(n) = 1$ . Therefore each value of P(n) that is computed must be multiplied by 2500 to correspond to the frequency distribution of 2500 measurements. In cell F3, enter the formula,

=(1/(SQRT(2\*PI()\*\$C\$3)))\*EXP(-((D3-\$C\$3)^2)/(2\*\$C\$3))\*2500

and copy it to the other cells, F4 through F103.



# Figure 6. Example graph showing the distribution of data points and the theoretical fits of the the Gaussian distribution to the data.

- 13. Compare the values of the frequency distribution in column E with those of the Gaussian distribution computed in column F. The values should be about the same.
- 14. Add the data for the Gaussian distribution to your graph. To do this click on your chart near its edges to select it and then select *Chart* from the main menu bar. Choose *Add Data*... from the drop down menu and input =*Sheet2!F3:F103* into the *Range* input box. Click on *OK* and this should add the calculated values for the Gaussian distribution to your graph.
- 15. Examine the measured frequency distribution and the calculated Gaussian distributions with the  $\sigma = \sqrt{\overline{n}}$  approximation and determine how well they agree.
- 16. In cell C12 compute the standard deviation of the 2500 samples. To do this, insert the formula **=STDEV(B3:B2502)**. **STDEV** is a statistical function that can be inserted from the **Insert** option on the main menu bar followed by choosing the **Function** selection. The standard deviation function is available from the Function name list associated with the Statistical selection under the Function category list.
- 17. Calculate the proper Gaussian distribution using the equation for the Gaussian distribution using the standard deviation of the samples,

$$P(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{\left(n-\overline{n}\right)^2}{2\sigma^2}\right]. \tag{5}$$

In cell G3, enter the formula,

$$= (1/(\$C\$12*SQRT(2*PI())))*EXP(-((D3-\$C\$3)^2)/(2*\$C\$12^2))*2500$$

and copy it to the other cells, G4 through G103. Remember that P(n) is multiplied by 2500 since P(n) is a normalized distribution where  $\sum_{i=0}^{N} P(n_i) = 1$ .

- 18. Add the data for this Gaussian distribution to your graph. To do this click on your chart near its edges to select it and then select *Chart* from the main menu bar. Choose *Add Data*... from the drop down menu and input =*Sheet2!G3:G103* into the *Range* input box. Click on *OK* and this should add this Gaussian distribution to your graph.
- 19. Compare this distribution with the measured frequency distribution and the Gaussian distribution using the  $\sigma = \sqrt{n}$  approximation. Which one is the best fit to the measurements?

CAUTION: READ THE FOLLOWING INSTRUCTIONS VERY CAREFULLY BEFORE ATTEMPTING TO PRINT YOUR SPREADSHEETS AND GRAPHS. ONLY A PORTION OF THE SPREADSHEETS WILL NEED TO BE PRINTED. OTHERWISE, THE SPREADSHEETS REQUIRE MORE THAN 50 PAGES TO BE PRINTED.

20. Print the 2 spreadsheets and the 2 charts in your Excel workbook using the following instructions:

For each spreadsheet that you need to print, you will only need to print a portion of the spreadsheet. Printing the entire spreadsheet will take a very long time and will consume more than 50 sheets of paper. You should only print the first page. To print only the first page, select *File* from the main menu bar, and *Print*... from the pull down menu. In the *Print Range* section, select the *Page(s)* option and enter 1 in the *From:* input box and 1 in the *To:* input box.

Click on *Preview* to check to see if more than one page is set for printing. If only one page shows up in the preview, then hit *Print* and then *Close* the preview. If more than one page shows up in the preview, then hit *Close* to close the preview to readjust the parameters.

Charts may be printed by selecting the chart and then choosing **Print**... from the pull down menu under **File** from the main menu.

## Questions

- 1. The Stingy Salt Shaker Problem: Suppose after 1000 shakes of a stingy salt shaker, the average number of grains of salt that comes out is 0.1 grain per shake. What is the probability that 2 grains of salt will come out for a given single shake?
- 2. On the average, there are about 2 major airline crashes per year. What is the probability that in any month, 2 crashes will occur?