The Force Table

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Objective: The objectives of this experiment are: (1) to study and learn vector addition, (2) to learn how to resolve vectors into their x and y components, (3) to learn how to find the magnitude and direction of a vector from its components, (4) to learn how to find the balancing force for a body that has two or more vector forces exerted on it.

Theory

Physical quantities commonly are of two different mathematical types, scalars and vectors. Scalar quantities are quantities in which only the magnitude of the quantity is needed to express its function. Vector quantities require both a magnitude and direction to completely express their characteristics. Common scalar quantities are time, mass, energy, and temperature. Common vector quantities are displacement, velocity, acceleration, force, and momentum. Scalar quantities can be added and subtracted algebraically using only their values. However, vectors must be added vectorially so that both their magnitude and direction is taken into account.

A vector is specified by stating its magnitude and direction relative to some coordinate system. In a Cartesian coordinate system, the direction can be specified by the angle the vector makes with respect to the x axis. Vector quantities are distinguished from scalar quantities by placing an arrow above the symbol or by printing the symbol in bold type. Here we will use arrows, since it is much easier when using pen and paper. For example, two vectors may be written as $\vec{A}$ and $\vec{B}$. It is understood that these quantities have a direction associated with their magnitude. The symbols A and B without the arrow then means just the magnitute of the vectors $\vec{\mathrm{A}}$ and $\vec{\mathrm{B}}$.

In order to add vectors in a practical manner, vectors can be resolved into two orthogonal components which when added together will be equal the vector. For example, the vector $\vec{A}$ can be resolved into a component along the positive x axis and one component along the positive y axis. The direction of the positive x axis can be designated by $\hat{i}$ and the direction of the positive y axis can be designated by $\hat{j}$. These symbols are called "unit vectors" and have unit value and are used to specify the x and y directions. The vector $\vec{A}$ can then be resolved into its components and written as $\vec{A}=A_x\hat{i}+A_y\hat{j}$. The direction of $\vec{A}$ can be specified by the angle θ_A it makes with respect to the positive x axis. The components A_x and A_y can be found from the trigonometric relationships and are given by

$$A_x = A\times\cos\theta_A \quad (1)$$

and

$$A_y = A\times\sin\theta_A \quad (2)$$

where A represents just the magitude of the vector. Similarly for a vector $\vec{B}$

$$\vec{B}=B_x\hat{i}+B_y\hat{j} \quad (3)$$

and

$$B_x = B\times\cos\theta_B \quad (4)$$

and

$$B_y = B\times\sin\theta_B \quad (5)$$

Just as the vectors may be resolved into their orthogonal components, the components may be combined to reconstitute the vector as a magnitude and direction. The magnitudes of vectors $\vec{A}$ and $\vec{B}$, A and B, are given by

$$A=\sqrt{A_x^2+A_y^2} \quad (6)$$

and

$$B=\sqrt{B_x^2+B_y^2} \quad (7)$$

and their directions are specified by their angles with respect to the x axis given by

$$\theta_A = \mathrm{ArcTan}\left(\frac{A_y}{A_x}\right) \quad (8)$$

and

$$\theta_B = \mathrm{ArcTan}\left(\frac{B_y}{B_x}\right) \quad (9)$$

Vectors $\vec{A}$ and $\vec{B}$ may be added together to produce a sum vector, $\vec{C}$,called the "resultant," and

$$\vec{C}=\vec{A}+\vec{B}. \qquad (10)$$

Just as $\vec{A}$ and $\vec{B}$ can be resolved into x and y components, $\vec{C}$ can also be resolved into x and y components, and as a result,

$$\vec{C}=C_x\hat{i}+C_y\hat{j} \qquad (11)$$

and

$$C_x=A_x+B_x, \qquad (12)$$

$$C_y=A_y+B_y \qquad (13)$$

and

$$\vec{C}=\left(A_x+B_x\right)\hat{i}+\left(A_y+B_y\right)\hat{j}. \qquad (14)$$

Equations 11-14 simply state that the x and y components of $\vec{C}$ are the algebraic sum of the x and y components of $\vec{A}$ and $\vec{B}$. As was the case for $\vec{A}$ and $\vec{B}$, the magnitude and direction of the vector $\vec{C}$ is C and θ_C and are given by the equations

$$C=\sqrt{C_x^2+C_y^2}=\sqrt{\left(A_x+B_x\right)^2+\left(A_y+B_y\right)^2} \qquad (15)$$

and

$$\theta_C=\mathrm{ArcTan}\left(\frac{C_y}{C_x}\right)=\mathrm{ArcTan}\left(\frac{A_y+B_y}{A_x+B_x}\right). \qquad (16)$$

For N vectors $\vec{A}_i$, $(i=1 \text{ to } N)$ added together the resultant vector, $\vec{A_R}$, can be found by generalizing Equations 15 and 16, so that

$$A_R=\sqrt{A_{Rx}^2+A_{Ry}^2}=\sqrt{\left(\sum_{i=1}^{N}A_{ix}\right)+\left(\sum_{i=1}^{N}A_{iy}\right)^2} \qquad (17)$$

and

$$\theta_R=\mathrm{ArcTan}\left(\frac{A_{Ry}}{A_{Rx}}\right)=\mathrm{ArcTan}\left(\frac{\sum_{i=1}^{N}A_{iy}}{\sum_{i=1}^{N}A_{ix}}\right). \qquad (18)$$

Newton's Second Law states that if a body or mass is in equilibrium, then the sum of all the forces acting on the body must be zero, or in equation form,

$$\sum_{i=1}^{N} \vec{F_i} = 0 \qquad (19)$$

Since force is a vector quantity, this means that the sum must be the vector sum of the forces. When a body is in equilibrium with several forces acting upon it, any one of the forces must be equal in magnitude and opposite in direction to a resultant vector that is the sum of the remaining forces. Any one force can balance out the remaining forces which can be summed as one resultant force, $\vec{F_R}$. This balancing force is called the equilibrant force, $\vec{F_E}$, and

$$\vec{F_R} + \vec{F_E} = 0 . \qquad (20)$$

so that

$$\vec{F_E} = -\vec{F_R} \qquad (21)$$

In this experiment 3 forces will be specified to be applied to a ring on a force table. These forces will be summed analytically to find their resultant force, $\vec{F_R}$. Then the equilibrant force, $\vec{F_E}$, will be found using Equation 21. The equilibrant force will be determined experimentally by finding the force that is required to balance the ring when the 3 assigned forces are applied to it. This measured equilibrant force is then compared with the calculated equilibrant force.

If $\vec{F_R}$ is the resultant force from the sum of the three assigned forces, then

$$\vec{F}_R = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 , \qquad (22)$$

$$\vec{F}_R = (F_{1x} + F_{2x} + F_{3x})\hat{i} + (F_{1y} + F_{2y} + F_{3y})\hat{j} , \qquad (23)$$

$$F_R = \sqrt{\left(F_{1x} + F_{2x} + F_{3x}\right)^2 + \left(F_{1y} + F_{2y} + F_{3y}\right)^2} , \qquad (24)$$

and

$$\theta_R = \text{ArcTan}\left(\frac{F_{Ry}}{F_{Rx}}\right) = \text{ArcTan}\left(\frac{F_{1y} + F_{2y} + F_{3y}}{F_{1x} + F_{2x} + F_{3x}}\right). \qquad (25)$$

The equilibrant force, , $\vec{F_E}$, then will be given by

$$\vec{F}_E = -(F_{1x} + F_{2x} + F_{3x})\hat{i} - (F_{1y} + F_{2y} + F_{3y})\hat{j} \qquad (26)$$

$$F_E = F_R = \sqrt{\left(F_{1x} + F_{2x} + F_{3x}\right)^2 + \left(F_{1y} + F_{2y} + F_{3y}\right)^2} \qquad (27)$$

$$\theta_E = \text{ArcTan}\left(\frac{F_{Ry}}{F_{Rx}}\right) + 180^\circ = \text{ArcTan}\left(\frac{F_{1y} + F_{2y} + F_{3y}}{F_{1x} + F_{2x} + F_{3x}}\right) + 180^\circ. \tag{28}$$

and is the force to balance the given forces and to compare with the measured force.

Apparatus

The apparatus for the force table experiment is shown in Figure 1. The apparatus consists of a force table, level, and masses. The force table is circular with a graduated circular scale to conveniently determine the angles and directions of the forces that are applied to a center ring. The center ring is positioned at a post at the center of the table to help determine when the forces on the ring are balanced and to prevent the ring from moving off the table before it is balanced. Forces are applied to the center ring by attaching one end of a string to the ring and passing the string over a pulley and attaching masses to the other end. The table should be level so that there is no unbalanced gravitational force acting on the ring.

Figure 1. Force Table with weight set.

Procedure

1. Begin the experiment by opening up an Excel spreadsheet. Type the row and column headings in column A and row 1 as shown in the example shown in Table 1. Use the copy and paste operations to enter repeated text. The cells with "####" are cells that

you will make calculations or will enter measured values. The shaded cells will have no information and should be left blank and shaded.

Table 1

	A	B	C	D	E
1	**Forces**	**Magnitude**	**Angle**	**X Component**	**Y Component**
2	**Force #1**	400	30	=B2*COS((C2/180)*PI())	=B2*SIN(C2/180*PI())
3	**Force #2**	200	-60	####	####
4	**Force #3**	300	135	####	####
5	**Sum of Components**			####	####
6	**Resultant Force** $\vec{R}$	####	=ATAN(E5/D5)/PI()*180		
7	**Equilibrant Force** $\vec{E}$	####	####		
8	**Measured Force**				
9					
10	**Force #1**	400	45	####	####
11	**Force #2**	200	-45	####	####
12	**Force #3**	500	270	####	####
13	**Sum of Components**			####	####
14	**Resultant Force** $\vec{R}$	####	####		
15	**Equilibrant Force** $\vec{E}$	####	####		
16	**Measured Force**				
17					
18	**Force #1**	350	20	####	####
19	**Force #2**	200	315	####	####
20	**Force #3**	500	250	####	####
21	**Sum of Components**			####	####
22	**Resultant Force** $\vec{R}$	####	####		
23	**Equilibrant Force** $\vec{E}$	####	####		
24	**Measured Force**				
25					
26	**Force #1**	300	25	####	####
27	**Force #2**	200	310	####	####
28	**Force #3**	250	200	####	####
29	**Sum of Components**			####	####
30	**Resultant Force** $\vec{R}$	####	####		
31	**Equilibrant Force** $\vec{E}$	####	####		
32	**Measured Force**				
33					
34					
35					

2. Four sets of vectors are given, with each set having 3 forces to apply to the center ring. With each force, its magnitude and direction is given. Type these values shown in Table 1 into your spreadsheet.

3. Set up the force table with these 3 forces being applied to the ring. Once this is done you should apply a fourth force, the measured equilibrant force, to balance the ring. The easiest way to do this is to start with the 3 forces being applied to the ring and then sliding the pulley clamp around the circumference of the table while using one hand to pull on the string and observing the position of the pulley where the ring will be centered. After the direction of the equilibrant force is found, you can then load the weight hanger with enough weights to completely balance the ring around the center post. Record this force, the magnitude and direction, as the measured equilibrant force in cells B8 and C8.

4. Using the relationships given in Equations 1 and 2, compute the x and y components of force #1 from its magnitude and direction and record the results in cells D2 and E2.

   This can be done automatically in the Excel spreadsheet in the following way: In cell D2 type "=B2*" and then click on Insert on the top menu bar. Choose function and a Paste Function window should appear. Choose Math & Trig from the Function category list and COS from the Function name list. Then click on OK. An enter Number box will appear with a button at the far right hand end. Click on this button and another input line box will appear. Click on cell C2 and "C2" should appear in this box. Add "*PI()/180" to the line so that it reads "C2*PI()/180" and then click on the button at the far right of the input box. This will return to the Function palette. Click on "OK." This procedure multiplies the magnitude of the first vector times the cosine of the angle to find the x component. The angle recorded in degrees in cell C2 has to be changed to radians to be a valid argument for the Excel cosine function and that is the reason that the angle is multiplied by π and then divided by 180. The y component can be found and recorded in cell E2 similarly by choosing the sine (SIN) function.

5. Repeat procedure 4 for forces #2 and #3.

6. In cells D5 and E5, record the sums of the x components and the y components.

7. Using Equation 24, record the magnitude of the resultant vector in cell B6. Do this by typing in a formula and use functions from the insert menu where appropriate.

8. Use Equation 25 to find the angle in degrees of the resultant vector and record this value in cell C6. Again type in a formula and use functions from the insert menu where appropriate. Excel finds the angle in radians using the arctan function, ATAN() and should be converted to degrees.

9. In cells B7 and C7 record the values of the magnitude and angle of the equilibrant force as determined from the resultant vector in step 8 above. Use Equations 27 and 28 to find these results.

10. Compare the magnitude of the measured equilibrant force with its calculated value. Compute the percent difference.

11. Compare the measured value of the angle of the equilibrant force with its calculated value. In this case, it doesn't make sense to compute a percent difference. If the angle of the calculated angle is more than 360°, substract 360° from the angle to reduce it to a normal angular value.

12. Repeat these procedures for the remaining sets of forces.