

Hooke's Law

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Objective

The objectives of this experiment are: (1) to study simple harmonic motion, (2) to learn the requirements for simple harmonic motion, (3) to learn Hooke's Law, (4) to verify Hooke's Law for a simple spring, (5) to measure the force constant of a spiral spring, (6) to learn the definitions of period and frequency and the relationships between them, (7) to learn the definition of amplitude, (8) to learn the relationship between the period, mass, and force constant of a vibrating mass on a spring undergoing simple harmonic motion, (9) to determine the period of a vibrating spring with different masses, and (10) to compare the measured periods of vibration with those calculated from theory.

Theory

The motion of a body that oscillates back and forth is defined as simple harmonic motion if there exists a restoring force F that is opposite and directly proportional to the distance x that the body is displaced from its equilibrium position. This relationship between the restoring force F and the displacement x may be written as

$$F = -kx \quad (1)$$

where k is a constant of proportionality. The minus sign indicates that the force is oppositely directed to the displacement and is always towards the equilibrium position. Whenever a mass m is subject to such a restoring force and is caused to oscillate back and forth, the time for one complete oscillation is defined as the period T and will be given by

$$T = 2\pi\sqrt{\frac{m}{k}}. \quad (2)$$

The frequency f of oscillations is the number of oscillations per unit time and is the reciprocal of the period, $f = 1/T$, and is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \quad (3)$$

The maximum distance that the mass is displaced from its equilibrium position is called the amplitude of the oscillation. In simple harmonic motion the period and frequency are independent of the amplitudes of the oscillations.

There are a number of examples of bodies that undergo simple harmonic motion, and one of the simplest is a mass suspended and set to vibrating on a spring. When the mass is hung on the spring and then displaced from its equilibrium position, it will oscillate up and down. The motion of the mass will be simple harmonic motion because the spring supplies a force that is directly proportional to the displacement. The restoring force supplied by the spring is always directed back towards the equilibrium position regardless of whether the mass is above or below the equilibrium position. The constant of proportionality k is called the spring constant and can be found by subjecting the spring to an applied force and measuring the amount that the spring stretches. The general property of matter, and springs in particular, whereby an applied force F_a , causes a displacement x from the equilibrium position that is directly proportional to the force, is known as Hooke's Law. Hooke's Law may be stated as

$$F_a = kx \quad (4)$$

and may be used to calculate the spring constant k . For equal displacements, the applied force and the restoring force are equal and opposite. Therefore, Hooke's Law provides the conditions for simple harmonic motion and the spring constant can be used with Equation (2) to measure the period of a mass vibrating on a spring.

Ideally, it is assumed that the spring has no mass and that the restoring force of the spring is only used to move the attached mass, but in fact, part of the restoring force is used to move the spring back to its equilibrium position. As a result, the mass of the spring cannot be neglected and must be considered in using Equation (2) to calculate the period of vibration. At the point of suspension, the spring doesn't move at all, and at the place where the mass is attached, the spring moves as much as the mass. Therefore, each part of the spring moves with a different amplitude. It can be shown that 1/3 of the mass of the spring must be included as the mass in the equations for calculating the period and frequency. Therefore, in practice, Equation (2) must be modified to take into account the mass of the spring m_s , so that the period of an oscillating mass m is given by

$$T = 2\pi\sqrt{\frac{\left(m + \frac{m_s}{3}\right)}{k}}. \quad (5)$$

Apparatus

The equipment needed for this experiment includes (1) a helical spring (cylindrical and slightly tapered), (2) a support stand with mirror scale attached, (3) a mass holder with 50 gram masses, and (4) a timer. See Figure 1. An analytic beam balance should be available for measuring the mass of the spring.



Figure 1. Apparatus for Hooke's Law experiment.

Procedure

Part I.

1. Hooke's Law will be verified for a spring and the force constant will be calculated by measuring the amount the spring is stretched when various masses are hung on it. The forces supplied by the masses are due to their weight which can be found by multiplying the mass by the acceleration due to gravity.
2. Hang the small end of the spiral spring from the support stand with mirror and centimeter scale. Choose a convenient reference point near the bottom of the spring with which to measure the elongation of the spring. The distance that the spring is

stretched is to be measured by taking the difference in the position of the reference point when the spring is stretched and relaxed. This is equivalent to taking the difference of the unstretched length of the spring and its extended length.

- With no masses attached to the spring, measure the position s_0 of the reference point by sighting into the mirror and aligning the reference point with its reflection. Open an Excel spreadsheet and label the columns as shown in Figure 2. Then record this position as the position with zero mass.

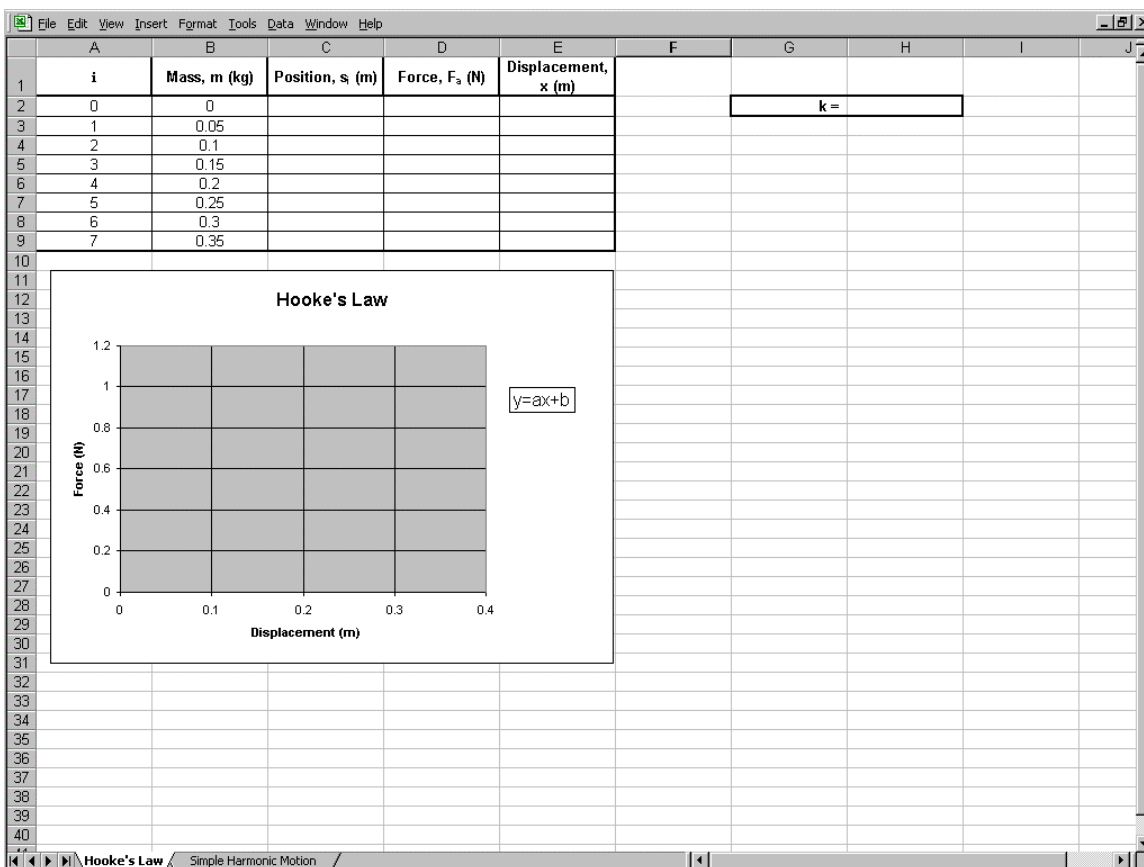


Figure 2. Example Excel spreadsheet for Hooke's Law experiment.

- Attach a 50 gram (0.050 kg) mass holder to the spring and measure the new position s_1 to which the reference point on the spring is extended. Record the position with the 50 gram mass holder in the spreadsheet.
- Add additional 50 gram masses to the mass holder and record the position of extension s_i each time the mass is increased by 50 grams until the total mass reaches 350 grams.
- Compute the applied force F_a the masses exert on the spring by calculating the weights of the masses. Remember that the weight W is given by $W = mg$ and is

measured in newtons if m is the mass measured in kilograms and g is the acceleration due to gravity, 9.80 m/s^2 . Record these forces F_a in your spreadsheet.

7. The displacement x of the reference position is the amount the spring is stretched and is calculated by taking the difference in the relaxed position s_0 and the extended position s_i , i.e., $x = s_i - s_0$. Compute the displacement x in meters for each of the applied forces and record your results in the data table.
8. Graph the applied force F_a to the spring as a function of its displacement x using the chart feature of Excel. In order to adhere to the form of Hooke's Law as stated by Equation (4), plot the displacement x on the horizontal axis (x axis) and the applied force on the vertical axis (y axis).
9. If Hooke's Law, $F_a = kx$, holds for the spring, the data points should lie along a straight line. Examine your data to see if it is linear and lies along a straight line.
10. Use the trendline feature to perform a linear regression to find a straight line that best fits your data points. To do this, from the main menu, choose *Chart* and then *Add Trendline . . .* from the dropdown menu. This will bring up a *Add Trendline* dialog window. From the *Trend* tab, choose *Linear* from the *Trend/Regression type* selections. Click on the *Options* tab and select the *Display equations on chart* option. This should draw a straight line that best fits the data and should display the equation for this straight line. The equation for the trendline should be of the form $y = ax + b$ where a is the slope of the line and b is the y -axis intercept.
11. The spring constant k will be the slope of the straight line. Record the value of the spring constant, based on Hooke's Law, as determined from the slope of the trendline.

Part II.

1. If Hooke's Law holds for the spring and the data fits a straight line, then the motion of masses vibrating up and down on the spring should be simple harmonic motion.
2. Open or add a new Excel worksheet and prepare it to record your new data by labeling the columns as shown in Figure 3.
3. With only the mass holder attached to the spring, measure the period of vibration by first displacing the holder about 2 cm and then measuring the total time t for 50 complete oscillations. Record your data in the new worksheet.
4. Add an additional 50 grams to the mass holder and repeat step 2 until the total mass reaches 350 grams (0.350 kg).

- Calculate the period of oscillation T for each oscillating mass by dividing the total time t by 50. Do this by entering a formula into the proper cell of your worksheet and copying it to the other cells.

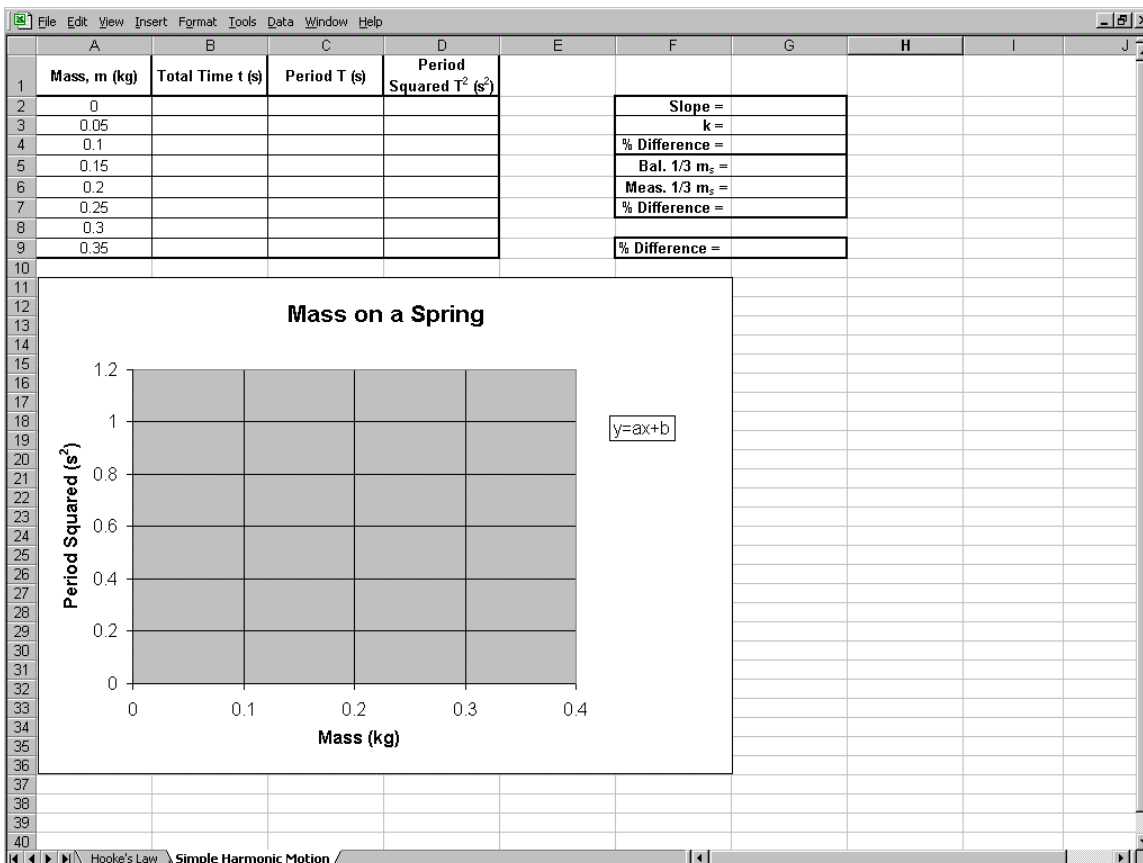


Figure 3. Example Excel spreadsheet for analysis of simple harmonic motion of a vibrating mass on a spring data.

- If Hooke's Law applies to the spring and the motion is simple harmonic, Equation (5) should apply. If both sides of Equation (5) are squared, then

$$T^2 = \left(\frac{4\pi^2}{k} \right) \left(m + \frac{m_s}{3} \right). \quad (6)$$

The period squared T^2 depends linearly on the mass m and the equation for T^2 is in the form of an equation for a straight line, $y = ax + b$, where $y = T^2$, $a = 4\pi^2/k$, and $b = 4\pi^2 m_s/3k$. If T^2 (y) is plotted as a function of m (x), the data should be along a straight line whose slope equals $4\pi^2/k$ (a). Moreover, if the straight line is extrapolated back until the period is zero, then the value of the mass should be one third the mass of the spring. This can be understood by setting T^2 equal to zero and solving for m so that

$$T^2 = 0 = \left(\frac{4\pi^2}{k} \right) \left(m + \frac{1}{3} m_s \right) \quad (7)$$

and

$$\left(m + \frac{1}{3} m_s \right) = 0 \quad (8)$$

and

$$m = -\frac{1}{3} m_s \quad (9)$$

Since the slope should be $4\pi^2/k$, the spring constant k can be found independently from Hooke's Law by dividing the slope into $4\pi^2$. This will be done in the following steps.

7. Square each value of the period T and record those values in your worksheet under the column labeled T^2 .
8. Use the chart feature to graph the period squared, T^2 , as a function of the mass on the vibrating spring, m . The period squared is the dependent variable and should be plotted on the y axis. The mass is the independent variable and should be plotted on the x axis.
9. If the motion is simple harmonic and Equation (5) is true, then the points should lie along a straight line. Examine the graph of your data to see if it indicates a linear relationship between T^2 and m .
10. With the same procedure employed in Part I above, use the trendline feature to perform a linear regression to find a straight line that best fits your data points. This procedure should draw a straight line that best fits the data and should display the equation for this straight line. The equation is of the form $y = ax + b$, where a is the slope and b is the y-intercept value when x (or m) is equal to zero.
11. The slope a is equal to $4\pi^2/k$ and k is equal to $4\pi^2/a$. Calculate the spring constant k from your value of the slope and record your value.

$$k = 4\pi^2/a$$

12. Recall your value for the spring constant found in Part I using Hooke's Law, k_1 , and compare it with the value found in this part, Part II, by measuring the period, k_2 . Compute the percent difference in these values.

$$\% \text{ Difference} = [(k_1 - k_2)/k_1] \times 100\%$$

13. Measure the mass of the spring m_s using an analytic beam balance and take one third of its value for $1/3 m_s$. Record this value as the accepted value of $1/3 m_s$.
14. Determine from your graph and the equation for the trendline, the value of the mass at which the period squared extrapolates to zero. This is the value for T^2 (or y) being

equal to zero, and the value of m (or x) for this condition will be $1/3$ the mass of the spring, $1/3 m_s$, and will be equal to b/a from the trendline parameters. Determine the ratio of b/a and record this value as the measured value of $1/3 m_s$.

15. Compare your results for $1/3 m_s$ by computing the percent difference in the results.

$$\% \text{ Difference} = [(\text{measured value} - \text{balance value})/\text{balance value}] \times 100\%$$

16. Verify directly that Equation (5) holds by calculating the period of a 250 gram mass. Use the value of the spring constant k found in Part I using Hooke's Law and the value of the mass of the spring found by using the analytic beam balance. Compare your results with the measured value of the period and calculate the percent difference.

$$\% \text{ Difference} = [(\text{calculated value} - \text{measured value})/\text{measured value}] \times 100\%$$

Questions

1.
 - a. At which point(s) does the mass on a vibrating spring have the greatest acceleration?
 - b. At which point(s) does it have the least acceleration?
 - c. At which point(s) does the mass have the largest force exerted on it?
 - d. At which point(s) does the mass have the smallest force exerted on it?
2.
 - a. At which point(s) does the mass on a vibrating spring have its largest velocity?
 - b. At which point(s) does the mass have its smallest velocity?
3. If the amplitude of the vibration for a spring is doubled, what happens to the period? Answer this question by measuring the period of an oscillating 250 gram mass whose amplitudes of vibration are 2 cm, 4 cm, and 8 cm.
4. If a stiffer spring were used so that the spring constant were 4 times as great, how would the frequency and period be affected?