Weak Interactions – I

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Lecture 1: A survey of basic principles and features with an emphasis on low-energy probes

Lecture 2: The electroweak Standard Model — and interpreting it as an effective field theory

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How "Weak" is the Weak Interaction?

We know of four fundamental interactions: electromagnetic, strong, weak, and gravitational. Let's set gravity aside and focus on the others. Particles of comparable mass can have very different lifetimes.

$$\pi^+ \to \mu^+\nu_\mu \quad [99.98\% \text{ of all } \pi^+ \text{ decays}]; \quad \tau_{\pi^+} \sim 2.6 \cdot 10^{-8}\,\text{s}$$
$$\pi^0 \to 2\gamma \quad [98.8\% \text{ of all } \pi^0 \text{ decays}]; \quad \tau_{\pi^0} \sim 8.4 \cdot 10^{-17}\,\text{s}.$$

$$\Gamma \propto \tau^{-1} \Longrightarrow \frac{|g_{\text{eff}}^{em}|^2}{|g_{\text{eff}}^{weak}|^2} \sim 10^8 \Longrightarrow |g_{\text{eff}}^{em}| \sim 10^4 |g_{\text{eff}}^{weak}|$$

whereas

$$\rho^0 \to \pi^+\pi^- \quad [\sim 100\% \text{ of all } \rho^0 \text{ decays}]$$
$$\rho^0 \to \mu^+\mu^- \quad [\sim 4.6 \cdot 10^{-5} \text{ of all } \rho^0 \text{ decays}]$$

$$\Longrightarrow \frac{|g_{\text{eff}}^{em}|^2}{|g_{\text{eff}}^{str}|^2} \sim 4 \cdot 10^{-5} \Longrightarrow |g_{\text{eff}}^{str}| \sim 10^2 |g_{\text{eff}}^{em}|$$

Conclude weak interaction is $\sim 10^6$ times weaker than the strong interaction!

Building a "Standard Model"

A first thought: perhaps we can describe all 3 interactions independently?

What precepts should we impose?

A list circa 1980:

- Particle Content (i.e., 3 generations of quarks and leptons)
- Symmetries
  - CPT (and Lorentz) Symmetry
  - Gauge Symmetry (SU(3)$\times$...$\times$...)
  - Unitarity
- Renormalizability

  This makes our theory predictive even as $E \to \infty$! It is "UV complete"!

This line of thinking ultimately yields a $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ gauge theory we call the Standard Model (SM). It is predictive — and successful — once all its parameters are fixed.

In 2015 the existence of known unknowns (e.g., dark matter) is now definite.

Perhaps we can describe these new features within the context of a theory with a SM-like gauge symmetry? Let's take just "baby steps" beyond the SM!

To explain phenomena at some fixed energy scale, we need only include the degrees of freedom operative at that energy scale.

E.g., we can predict the outcome of chemical reactions without understanding how the electron gets it mass!

A simple application of effective field theory: "Why is the sky blue?"

Here we consider low-energy scattering of photons from neutral atoms

$E_\gamma \ll \Delta E \sim \alpha^2 m_e \ll a_0^{-1} \sim m_e \alpha \ll M_{\text{atom}}$

The low-energy interactions of the atom (ψ) are fixed by symmetry: gauge and P and C invariance....

$$\mathcal{L}_{\text{int}} = c_1 a_0^3 \psi^\dagger \psi F_{\mu\nu} F^{\mu\nu} + \dots$$

with the a_0^3 factor fixed by common sense and "power counting" so that $c_1 \sim \mathcal{O}(1)$. ($\dim[\psi] = 3/2$ and $\dim[F^{\mu\nu}] = 2$)

Thus $\mathcal{A} \sim c_1 a_0^3 E_\gamma^2 \Longrightarrow \sigma \sim a_0^6 E_\gamma^4$ $[\hbar = c = 1]$ and we conclude that blue light is scattered more strongly than red light!

Thus a theory need not be "UV complete" to be predictive. Through identifying traces of new physics at low energy we hope to identify the nature of $E \to \infty$ physics!

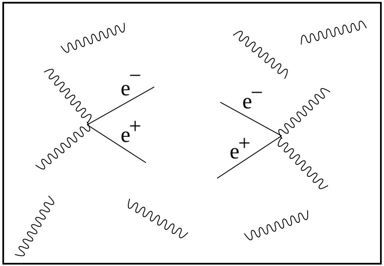
We return now to the path that led to the rise of the SM....

The Discrete Symmetries – C, P, and T

In particle interactions, can we tell...

- Left from Right? (P)
- Positive Charge from Negative Charge? (C)
- Forward in Time from Backward in Time? (T)
- Matter from Antimatter? (CP)

If we "observed" a box of photons at constant temperature $T \sim m_e$, interacting via electromagnetic forces, the answer would be No.



However, ...

On the Possibility of Parity Violation

Context: Dirac – the existence of a magnetic monopole can explain the quantization of electric charge! [Dirac, Proc. Roy. Soc. London A 133, 60 (1931)]

$$\nabla \cdot E = 4\pi\rho \quad ; \quad \nabla \cdot B = 0 \Longrightarrow 4\pi\rho_M$$

Dirac also showed that the circulation of opposite magnetic monopoles in the nucleon could give rise to a nonzero *electric dipole moment*.

[Dirac, Phys. Rev. 74, 817 (1948).]

The electric dipole moment d of a nonrelativistic particle with spin S is defined via $\mathcal{H} = -d\frac{\mathbf{S}}{S} \cdot \mathbf{E}$

But both quantities violate P and T!

E. M. Purcell and N. F. Ramsey, "On the Possibility of Electric Dipole Moments for Elementary Particles and Nuclei," Phys. Rev. 78, 807 (1950):

The argument against electric dipoles, in another form, raises the question of parity.... But there is no compelling reason for excluding this possibility....

Discrete Symmetries — P, T, and C

Parity P:

Parity reverses the momentum of a particle without flipping its spin.

$$P a_p^s P^\dagger = a_{-p}^s \quad , \quad P b_p^s P^\dagger = -b_{-p}^s \qquad \Longrightarrow P\psi(t,x)P^\dagger = \gamma^0\psi(t,-x)$$

Time-Reversal T:

Time-reversal reverses the momentum of a particle and flips its spin. It is also antiunitary; note $[x,p] = i\hbar$.

$$T a_p^s T^\dagger = a_{-p}^{-s} \qquad T b_p^s T^\dagger = b_{-p}^{-s} \qquad \Longrightarrow T\psi(t,x)T^\dagger = -\gamma^1\gamma^3\psi(-t,x)$$

Charge-Conjugation C:

Charge conjugation converts a fermion with a given spin into an antifermion with the same spin.

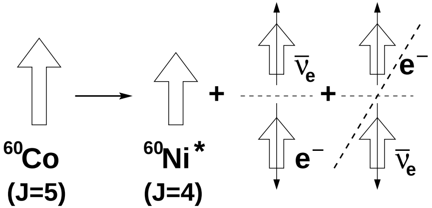
$$C a_p^s C^\dagger = b_p^s \quad , \quad C b_p^s C^\dagger = a_p^s \qquad \Longrightarrow C\psi(t,x)C^\dagger = -i\gamma^2\psi^*(t,x)$$

The Weak Interactions Violate Parity

There is a "fore-aft" asymmetry in the e^- intensity in $^{60}\vec{Co}$ β-decay....
[Wu, Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. 105, 1413 (1957);

note also Garwin, Lederman, and Weinrich, Phys. Rev. 105, 1415 (1957); http://focus.aps.org/story/v22/st19 .]

Schematically



$$I_e(\theta) = 1 - \frac{\vec{J}\cdot\vec{p}_e}{E_e}$$

P is violated in the weak interactions!

Both P and C are violated "maximally"

$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^+ \to \mu^+ \nu_R) = 0 \quad ; \text{ P violation}$$

$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^- \to \mu^- \overline{\nu}_L) = 0 \quad ; \text{ C violation}$$

The "Two-Component" Neutrino

A Dirac spinor can be formed from two 2-dimensional representations:

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

In the Weyl representation for γ^μ,

$$(i\gamma^\mu \partial_\mu - m)\psi = \begin{pmatrix} -m & i(\partial_0 + \sigma \cdot \nabla) \\ i(\partial_0 - \sigma \cdot \nabla) & -m \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

If m=0, ψ_L and ψ_R decouple and are of definite helicity for all p.
Thus, e.g.,

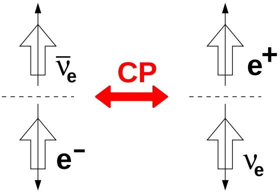
$$i(\partial_0 - \sigma \cdot \nabla)\psi_L(x) \Longrightarrow E\psi_L = -\sigma \cdot p\, \psi_L$$

$$\sigma \cdot \hat{p}\, \psi_L = -\psi_L$$

Note $\bar{\psi}_L \equiv \psi_L^\dagger \gamma^0$ transforms as a right-handed field.
Experiments $\Longrightarrow$ No "mirror image states": neither $\overline{\nu}_L$ nor ν_R exist.
Possible only if the neutrino is of zero mass.

The Weak Interactions Can Also Violate CP

CP could be a good symmetry even if P and C were violated.
Schematically



$$\Gamma(\pi^+ \to \mu^+ \nu_L) = \Gamma(\pi^- \to \mu^- \overline{\nu}_R) \quad ; \text{ CP invariance!}$$

Weak decays into hadrons, though, can violate *CP*.
There are “short-lived” and “long-lived” K states:

$$K_S \sim \frac{1}{\sqrt{2}}(K^0 - \overline{K}^0) \to \pi^+\pi^- \quad \text{(CP even)}$$

$$K_L \sim \frac{1}{\sqrt{2}}(K^0 + \overline{K}^0) \to \pi^+\pi^-\pi^0 \quad \text{(CP odd)}$$

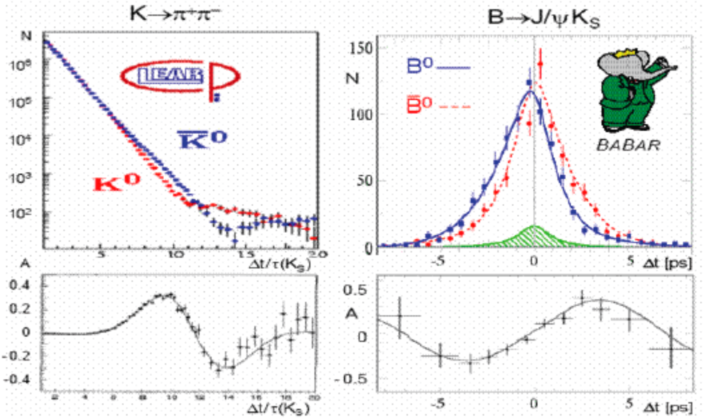
However, $K_L \to 2\pi$ as well! K_S and K_L do not have definite CP!

[Christenson, Cronin, Fitch, Turlay, PRL 13, 138 (1964).]

Matter and Antimatter are Distinguishable

The decay rates for $K^0, \bar{K}^0 \to \pi^+\pi^-$ and $B^0, \bar{B}^0 \to J/\psi K_S$ are appreciably different.

[I.I. Bigi, arXiv:0703132v2 and references therein.]



All Observed Interactions Conserve CPT

The CPT Theorem

Any **Lorentz-invariant**, **local** quantum field theory in which the observables are represented by Hermitian operators must respect CPT. [Pauli, 1955; Lüders, 1954]

Coda: CPT violation implies Lorentz violation. [Greenberg, PRL 89, 231602 (2002)]

CPT $\Longrightarrow$ the lifetimes, masses, and the absolute values of the magnetic moments of particles and anti-particles are the same!

Note, e.g.,

$$\frac{|M_{K^0} - M_{\bar{K}_0}|}{M_{\rm avg}} < 6 \times 10^{-19} \, @90\% \, \rm CL$$

$$\frac{|M_p - M_{\bar{p}}|}{M_p} < 7 \times 10^{-10} \, @90\% \, \rm CL$$

Thus CP $\leftrightarrow$ T violation. Tests of CPT and Lorentz invariance are ongoing.

e.g., ATRAP Collaboration, arXiv:1301.6310, "... For the first time a single trapped $\bar{p}$ is used to measure the $\bar{p}$ magnetic moment $\mu_{\bar{p}}$. ... The 4.4 parts per million (ppm) uncertainty is 680 times smaller than previously realized. Comparing to the proton moment measured using the same method and trap electrodes gives $\mu_{\bar{p}}/\mu_p = -1.000\,000 \pm 0.000\,005$ to 5 ppm, for a proton moment $\mu_p = \mu_p S/(\hbar/2)$, consistent with the prediction of the CPT theorem."

Transformations of Lorentz Bilinears under P, T, and C

Notation: $\xi^\mu = 1$ for $\mu = 0$ and $\xi^\mu = -1$ for $\mu \neq 0$.
$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$; $\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$

	$\bar{\psi}\psi$	$i\bar{\psi}\gamma_5\psi$	$\bar{\psi}\gamma^\mu\psi$	$\bar{\psi}\gamma^\mu\gamma_5\psi$	$\bar{\psi}\sigma^{\mu\nu}\psi$	∂_μ
	S	P	V	A	T	
P	$+1$	-1	ξ^μ	$-\xi^\mu$	$\xi^\mu\xi^\nu$	ξ^μ
T	$+1$	-1	ξ^μ	ξ^μ	$-\xi^\mu\xi^\nu$	$-\xi^\mu$
C	$+1$	$+1$	-1	$+1$	-1	$+1$
CPT	$+1$	$+1$	-1	-1	$+1$	-1

S is for Scalar
P is for Pseudoscalar
V is for Vector
A is for Axial-Vector
T is for Tensor

All scalar fermion bilinears are invariant under CPT.

Symmetries of a Dirac Theory

A Lagrangian must be a Lorentz scalar to guarantee Lorentz-invariant equations of motion. E.g., applying the Euler-Lagrange eqns to

$$\mathcal{L}_{\rm Dirac} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$$

yield Dirac equations for ψ and $\bar{\psi}$.
We can form two currents

$$j^\mu(x) = \bar{\psi}(x)\gamma^\mu\psi(x) \quad ; \quad j^{\mu 5}(x) = \bar{\psi}(x)\gamma^\mu\gamma^5\psi(x)$$

j^μ is always conserved if $\psi(x)$ satisfies the Dirac equation:

$$\partial_\mu j^\mu = (\partial_\mu\bar{\psi})\gamma^\mu\psi + \bar{\psi}\gamma^\mu\partial_\mu\psi = (im\bar{\psi})\psi + \bar{\psi}(-im\psi) = 0\,,$$

whereas $\partial_\mu j^{\mu 5} = 2im\bar{\psi}\gamma^5\psi$ — it is conserved only if $m = 0$.
By Noether's theorem a conserved current follows from an invariance in $\mathcal{L}_{Dirac}$:

$$\psi(x) \to e^{i\alpha}\psi(x) \quad ; \quad \psi(x) \to e^{i\alpha\gamma^5}\psi(x)$$

The last is a chiral invariance; it only emerges if $m = 0$.

Symmetries of a Dirac Theory

To understand why it is a chiral invariance, we note in the $m = 0$ limit that

$$j_L^\mu = \bar{\psi}\gamma^\mu \left(\frac{1-\gamma^5}{2}\right)\psi \quad , \quad j_R^\mu = \bar{\psi}\gamma^\mu \left(\frac{1+\gamma^5}{2}\right)\psi \,.$$

The vector currents of left- and right-handed particles are separately conserved.

Note in Weyl representation

$$\gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

The factor $(1 \pm \gamma^5)$ acts to project out states of definite handedness.

$$\psi_L \equiv \left(\frac{1-\gamma^5}{2}\right)\psi \quad , \quad \psi_R \equiv \left(\frac{1+\gamma^5}{2}\right)\psi \,.$$

so that $\mathcal{L} = \bar{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R i\gamma^\mu \partial_\mu \psi_R = \mathcal{L}_L + \mathcal{L}_R$

Electromagnetism

We assert that if we couple a Dirac field ψ to an electromagnetic field A^μ
j^μ is the electric current density. ψ can describe a free electron.

$$\psi(x)|p,s\rangle = u(p)e^{-ip\cdot x} \Longrightarrow (\gamma_\mu p^\mu - m)u(p) = 0\,.$$

By "canonical substitution" $p^\mu \to p^\mu + eA^\mu$

$$(\gamma_\mu p^\mu - m)u = \gamma^0 Vu \quad ; \quad \gamma^0 V = -e\gamma_\mu A^\mu$$

In $\mathcal{O}(e)$ the amplitude for an electron scattering from state $i \to f$ is

$$T_{fi} = -i\int u_f^\dagger V(x)u_i(x)\,d^4x = -i\int j_\mu^{fi}A^\mu\,d^4x \quad \text{with} \quad j_\mu^{fi} = -e\bar{u}_f\gamma_\mu u_i$$

For $e-p$ scattering, e.g., we have

$$T_{fi} = -i\int j_\mu^e(x)\left(-\frac{1}{q^2}\right)j_\mu^p(x)\,d^4x = -i\mathcal{M}(2\pi)^4\delta^{(4)}(p+k-p'-k')$$

$$\mathcal{M} \equiv -\frac{e^2}{q^2}\left(j_\mu^{em}\right)_p\left(j^{em\,\mu}\right)_e = (e\bar{u}_p(p')\gamma_\mu u_p(p))\left(-\frac{e^2}{q^2}\right)(-e\bar{u}_e(k')\gamma^\mu u_e(k))$$

A current-current interaction.

Fermi Theory

Now consider $n \to p e^- \bar{\nu}_e$.

Fermi's crucial insight was to realize that the weak currents could be modelled after electromagnetism:

$$\mathcal{M} = G(\bar{u}_p(p')\gamma_\mu u_n(p))(\bar{u}_e(k')\gamma^\mu u_\nu(k))$$

The observation of $e - p$ capture suggests

$$\mathcal{L}_{\rm Fermi} = -\frac{G_F}{\sqrt{2}} \left\{ (\bar{\psi}_p \gamma_\mu \psi_n)(\bar{\psi}_e \gamma^\mu \psi_\nu) + h.c. \right\}$$

An interaction with charged weak currents.

A weak neutral current was discovered in 1973.

G_F is the Fermi constant, though $G_F \sim 10^{-5}(\text{GeV})^{-2}$. **(N.B. not UV complete!!)**

Suggests the interaction is mediated by massive, spin-one particles.

Fermi's interaction cannot explain the observation of parity violation.

Nor can it explain the $|\Delta J| = 1$ ("Gamow-Teller") transitions observed in nuclear β-decay.

Some $A \times A$ or $T \times T$ interaction has to be present.

Enter the $V - A$ Law....

[Feynman, Gell-Mann, 1958; Sudarshan and Marshak, 1958]

Fermi vs. Gamow-Teller Transitions

Nuclear β-decay spin-isospin selection rules are dictated by the form of the nonrelativistic transition operator.

In allowed approximation (note $\pi_i \pi_f = +1$) ...

$$\sum_{j=1}^{A} \tau_\pm(j) = T_\pm \quad \text{"Fermi"} \Longrightarrow J_f = J_i, T_f = T_i$$

Here both parent and daughter are in isotopic analogue states, e.g., $^{10}\mathrm{C} \rightarrow ^{10}\mathrm{B}$.

$$\sum_{j=1}^{A} \sigma(j) \cdot \tau_\pm(j) \quad \text{"Gamow-Teller"} \Longrightarrow J_i = J_f, J_f \pm 1 \; (J_i = 0 \not\rightarrow J_f = 0)\,,$$

$$|\Delta T| = 0, 1$$

N.B. $0^+ \rightarrow 0^+$ "superallowed" decays are pure Fermi transitions, whereas neutron β decay is "mixed," containing both types.

The V-A Law

A "universal" charged, weak current:

$$\mathcal{L} = -\frac{1}{2}\frac{G_F}{\sqrt{2}}\left\{\mathcal{J}^\lambda \mathcal{J}^\dagger_\lambda + \mathcal{J}^\dagger_\lambda \mathcal{J}^\lambda\right\} \quad \text{with} \quad \mathcal{J}_\lambda = {j^l}_\lambda + {j^h}_\lambda$$

For the leptons...

$$j^{l\,\lambda} = \bar{\psi}_e \gamma^\lambda (1-\gamma_5)\psi_{\nu_e} + \bar{\psi}_\mu (k')\gamma^\lambda (1-\gamma_5)\psi_{\nu_\mu} + \bar{\psi}_\tau (k')\gamma^\lambda (1-\gamma_5)\psi_{\nu_\tau}$$

which describes $\nu_l \to l^-$ and $l^+ \to \bar{\nu}_l$ and asserts the leptons do not mix under the weak interactions.
Here the "V-A" law is equivalent to a "two-component" neutrino picture.
The interactions of the hadrons (quarks) are (and can be) much richer.

- The strong interaction is strong!
- The quarks *mix* under the weak interactions. E.g., $K^+ \to \mu^+\nu$ is observed. Recall K^+ is $(u\bar{s})$.

Let us continue to focus on neutron β-decay. Recall n is *ddu* and p is *uud*.
Isospin is an approximate symmetry:
$M_n = 939.565\,\text{MeV}$ $M_p = 938.272\,\text{MeV}$ $(M_n - M_p)/M_n \ll 1$.
$n \to pe^-\bar{n}u_e$ occurs because isospin is broken $\Longrightarrow$ large τ_n.

Polarized Neutron β-decay in a V-A Theory

$$d^3\Gamma = \frac{1}{(2\pi)^5 2m_B}\left(\frac{d^3\boldsymbol{p}_p}{2E_p}\frac{d^3\boldsymbol{p}_e}{2E_e}\frac{d^3\boldsymbol{p}_\nu}{2E_\nu}\right)\delta^4(p_n - p_p - p_e - p_\nu)\frac{1}{2}\sum_{spins}|\mathcal{M}|^2$$

$$\mathcal{M} = \frac{G_F V_{ud}}{\sqrt{2}}\langle p(p_p)|J^\mu(0)|\vec{n}(p_n,P)\rangle[\bar{u}_e(p_e)\gamma_\mu(1-\gamma_5)u_\nu(p_\nu)]$$

$$\langle p(p_p)|J^\mu(0)|\vec{n}(p_n,P)\rangle = \bar{u}_p(p_p)(f_1\gamma^\mu - i\frac{f_2}{M_n}\sigma^{\mu\nu}q_\nu + \frac{f_3}{M_n}q^\mu$$
$$-g_1\gamma^\mu\gamma_5 + i\frac{g_2}{M_n}\sigma^{\mu\nu}\gamma_5 q_\nu - \frac{g_3}{M_n}\gamma_5 q^\mu)u_{\vec{n}}(p_n,P)$$

Note $q = p_n - p_p$ and for baryons with polarization P,
$u_{\vec{n}}(p_n,P) \equiv (\frac{1+\gamma_5 \not{P}}{2})u_n(p_n)$

f_1 (g_V)	Fermi or Vector	g_1 (g_A)	Gamow-Teller or Axial Vector
f_2 (g_M)	Weak Magnetism	g_2 (g_T)	Induced Tensor or Weak Electricity
f_3 (g_S)	Induced Scalar	g_3 (g_P)	Induced Pseudoscalar

Since $(M_n - M_p)/M_n \ll 1$, a “recoil” expansion is very useful.
Also in nuclear decays!

Correlation Coefficients

$$d^3\Gamma \propto E_e|\boldsymbol{p}_e|(E_e^{\max} - E_e)^2 \times$$
$$[1 + a\frac{\boldsymbol{p}_e \cdot \boldsymbol{p}_\nu}{E_e E_\nu} + \boldsymbol{P} \cdot (A\frac{\boldsymbol{p}_e}{E_e} + B\frac{\boldsymbol{p}_\nu}{E_\nu} + D\frac{\boldsymbol{p}_e \times \boldsymbol{p}_\nu}{E_e E_\nu})]dE_e d\Omega_e d\Omega_\nu$$

A and B are P odd, T even, whereas D is (pseudo)T odd, P even.
$\lambda \equiv g_1/f_1 \equiv g_A/g_V > 0$ and predictions:

$$a = \frac{1-\lambda^2}{1+3\lambda^2} \qquad A = 2\frac{\lambda(1-\lambda)}{1+3\lambda^2} \qquad B = 2\frac{\lambda(1+\lambda)}{1+3\lambda^2} \qquad [+\mathcal{O}(R)]$$

implying $1 + A - B - a = 0$ and $aB - A - A^2 = 0$ [Mostovoy and Frank, 1976], testing the V-A structure of the SM to recoil order, $\mathcal{O}(R)$ with $R \sim E_e^{\max}/M_n \sim 0.0014$. Currently

$$a = -0.103 \pm 0.004 \quad A = -0.1184 \pm 0.0010 \; (S = 2.4) \quad B = 0.9807 \pm 0.0030$$

so that the relations are satisfied. [$\lambda = 1.2723 \pm 0.0023$ ($S = 2.2$)]
With $\tau_n = 880.3 \pm 1.1$ sec ($S = 1.9$) and $\tau_n \propto f_1^2 + 3g_1^2$ more tests are possible. [Olive et al., Particle Data Group, Chin. Phys. C, 38, 090001 (2014).]

Symmetries of the Hadronic, Weak Current

The values of the 6 couplings (assuming T invariance) are constrained by symmetry.

- Conserved-Vector Current ("CVC") Hypothesis
- Absence of Second-Class Currents ("SCC")
- Partially Conserved Axial Current ("PCAC") Hypothesis

CVC:
The charged weak current and isovector electromagnetic current form an **isospin triplet.** [Feynman and Gell-Mann, 1958]

$$J_\mu^{em,q} = \frac{2}{3}\bar{\psi}_u \gamma^\mu \psi_u - \frac{1}{3}\bar{\psi}_d \gamma^\mu \psi_d$$

$$J_\mu^{em,q} = e_0 \bar{\psi}_q \gamma^\mu I \psi_q + e_1 \bar{\psi}_q \gamma^\mu \tau_3 \psi_q \quad \text{with} \quad \psi_q = \begin{pmatrix} \psi_u \\ \psi_d \end{pmatrix}$$

$$\tau_3 \begin{pmatrix} \psi_u \\ 0 \end{pmatrix} = \begin{pmatrix} \psi_u \\ 0 \end{pmatrix} \quad ; \quad \tau_3 \begin{pmatrix} 0 \\ \psi_d \end{pmatrix} = - \begin{pmatrix} 0 \\ \psi_d \end{pmatrix} \quad ; \quad e_{0,1} = \frac{1}{2}(e_u \pm e_d)$$

Symmetries of the Hadronic, Weak Current

Thus

$$J_\mu^{em\,N} = \bar{\psi}[F_1^S(q^2)\gamma^\mu - i\frac{F_2^S(q^2)}{M_n}\sigma^{\mu\nu}q_\nu + \frac{F_3^S(q^2)}{M_n}q^\mu]e_0 I\psi$$

$$+\bar{\psi}[F_1^V(q^2)\gamma^\mu - i\frac{F_2^V(q^2)}{M_n}\sigma^{\mu\nu}q_\nu + \frac{F_3^V(q^2)}{M_n}q^\mu]e_1\tau_3\psi$$

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \quad \text{and} \quad \tau_+ \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} = \begin{pmatrix} \psi_p \\ 0 \end{pmatrix}$$

The CVC hypothesis implies

$$f_1(q^2) = F_1^V(q^2) \quad \text{and} \quad f_1(q^2) \to 1 \quad \text{as } q^2 \to 0$$
$$f_2(q^2) = F_2^V(q^2)$$
$$f_3(q^2) = F_3^V(q^2) = 0 \quad \text{(current conservation)}$$

$f_1(0) = 1 + \Delta_R^V$ Δ_R^V starts in $\mathcal{O}(\alpha)$!
[tested to $\mathcal{O}(0.3\%)$ in $0^+ \to 0^+$ decays]
$f_2(0)/f_1(0) = (\kappa_p - \kappa_n)/2 \approx 1.8529$
[tested to $\mathcal{O}(10\%)$ in $A = 8, 12$ systems]
The Ademollo-Gatto theorem makes the second test more interesting.

Symmetries of the Hadronic, Weak Current

SCC: "Wrong" G-parity interactions do not appear if isospin is an exact symmetry. [Weinberg, 1958]
$G \equiv C \exp(i\pi T_2)$ where T_2 is a rotation about the 2-axis in isospin space.

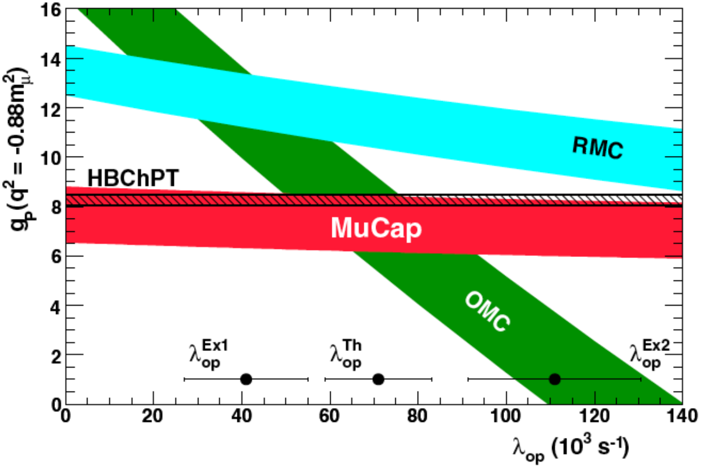
$$\exp(i\pi T_2)\psi = -i\tau_2\psi = \begin{pmatrix} -\psi_n \\ \psi_p \end{pmatrix}$$

$$GV_\mu^{(I)}G^\dagger = +V_\mu^{(I)} \quad ; \quad GA_\mu^{(I)}G^\dagger = -A_\mu^{(I)} \quad \text{“first class”}$$
$$GV_\mu^{(II)}G^\dagger = -V_\mu^{(II)} \quad ; \quad GA_\mu^{(II)}G^\dagger = +A_\mu^{(II)} \quad \text{“second class”}$$

no SCC: $g_2 = 0$ and $f_3 = 0$
(tested to $\mathcal{O}(10\%)$ in $A = 12$ system (combined CVC/SCC test))
PCAC: g_1/f_1 is set by strong-interaction physics:
Goldberger-Treiman relation $\frac{g_1(0)}{f_1(0)} = g_{\pi NN}\frac{f_\pi}{M_N}$
Can test some of these relationships through experiments sensitive to recoil-order effects.

PCAC Tests in Muon Capture

g_3 is also predicted by PCAC (HBChPT) and can be studied in μ capture. MuCap redresses the problem with OMC. [Andreev et al., MuCap, PRL 99, 032002 (2007).]



This can also be viewed as a test of lepton-flavor universality, though π (radiative) β decay yields much more severe constraints! Also K_{l3}, K_{l2}!

on Mass

Mass is key to explaining the relative strength of the weak and electromagnetic interactions.

Unified Electroweak spin = 1		
Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	–1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

In the SM, the gauge boson masses, as well as those of the elementary fermions, arise through the spontaneously breaking of a local gauge symmetry — the “Higgs mechanism”.

The Higgs mechanism is also key to describing the mixing of the quarks under the weak interactions: it gives rise to the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

The Higgs boson has finally been discovered, and measuring its couplings precisely will tell us whether it is “just” a SM Higgs — or not!