Fundamental Neutron Physics I

Neutron Properties

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Why focus on the neutron?

The neutron exhibits much of the richness of nuclear physics, but is vastly simpler, and thus more interpretable, than complex nuclei.

The neutron can be used to probe Strong, Weak, EM and Gravitational phenomena.

Neutron decay is the archetype for all nuclear beta decay and is a key process in astrophysics.

The neutron is well suited as a laboratory for tests of physics beyond the Standard Model.

Overview of the Lectures

I Introduction to the basic physics of the neutron as a particle

II Overview of neutron facilities and techniques

III The Neutron Electric Dipole Moment

IV Neutron beta decay

V Some Selected Additional Projects (time permitting)

Some General References

Fermi, *Lecture Notes on Nuclear Physics*

Byrne, *Neutrons, Nuclei and Matter*

Golub, Lamoreaux, Richardson, *Ultracold Neutrons*

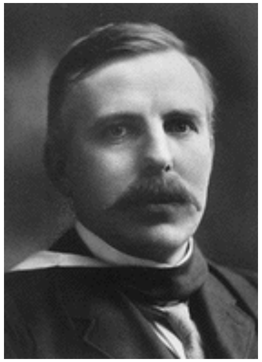
Commins and Bucksbaum, *Weak Interactions of Quarks and Leptons*

Particle Data Group, **pdg.lbl.gov**

Acknowledgements for images

Mike Snow, Fred Weitfeldt, Brad Fillipone, Jeff Nico, Paul Huffman, Sam Werner, Pieter Mumm, Scott Dewey, Chris Crawford, Harmut Abele, Yuri Kamyskov, Tim Chupp, Brad Filippone, Nadia, Fomin,…

Historical Introduction



Ernest Rutherford

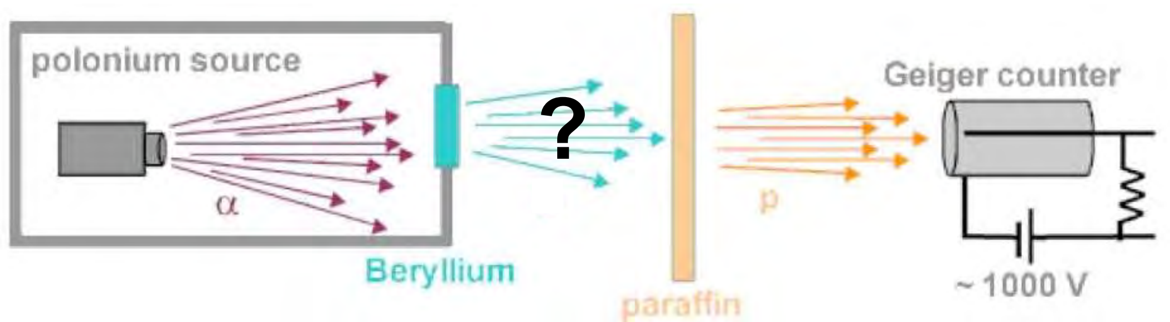
Walter Bothe

Irene Curie

1920 ***Noting that atomic number (Z) does not correspond to atomic weight, Rutherford suggests that, in addition to "bare" protons, the nucleus contains some tightly bound "proton-electron pairs" or neutrons.***

1930 ***Bothe and Becker discovered a penetrating, neutral radiation when alpha particles hit a Be target.***

1931 ***Mme Curie shows that they are not gamma rays and they have sufficient momentum to eject p's from paraffin.***

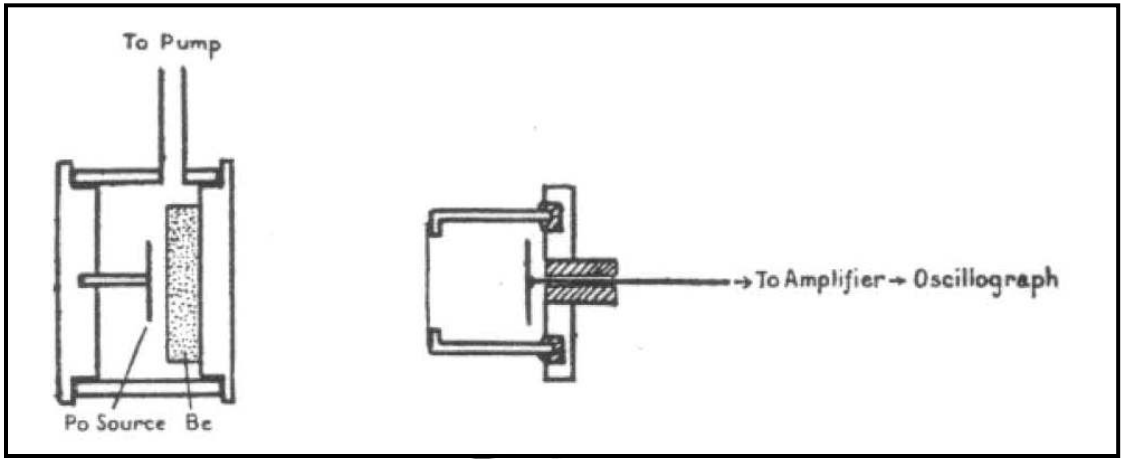


$$\alpha + {}^{9}Be \rightarrow {}^{12}C + n$$

1932 ***Chadwick replaced the paraffin with a variety of other targets and, by measuring the recoil energies of the ejected particles was able to determine the mass of the neutral particle***

James Chadwick

M = 1.15 ± ~10%



J. Chadwick, Proc. Roy. Soc., A ***136*** *692 (1932)*

Chadwick claimed this was Rutherford's "Neutron"

1933 ***Bainbridge makes precision measurements of the atomic masses of the proton and the deuteron using the mass spectrograph***

1934 ***Chadwick and Goldhaber make the first "precision" measurement of the neutron mass by looking at the photo-disassociation of the deuteron***

$$h\nu + {}_1D^2 \rightarrow {}_1H^2 + {}_1n^0$$

Using 2.62MeV gammas from Thorium and determining the recoil energy of the protons they we re able to determine*:

$$M_n = 1.0080 \pm 0.0005$$

KEY OBSERVATION: $M_n > M_p + M_e$

1. The neutron cannot be a bound "proton-electron pair"

2. It is energetically possible for a neutron to decay to e + p

**Chadwick and Goldhaber, Nature, 134 237 (1934)*

Some Neutron Properties

__Mechanical Properties__

Mass

Gravitational Mass (equivalence principle test)

Spin

__Electromagnetic Properties__

Charge (limit on neutrality)

Magnetic Dipole Moment

Electric Dipole Moment

Internal Charge Distribution

~~***Higher Moments (Quadrupole,Octupole,..)***~~

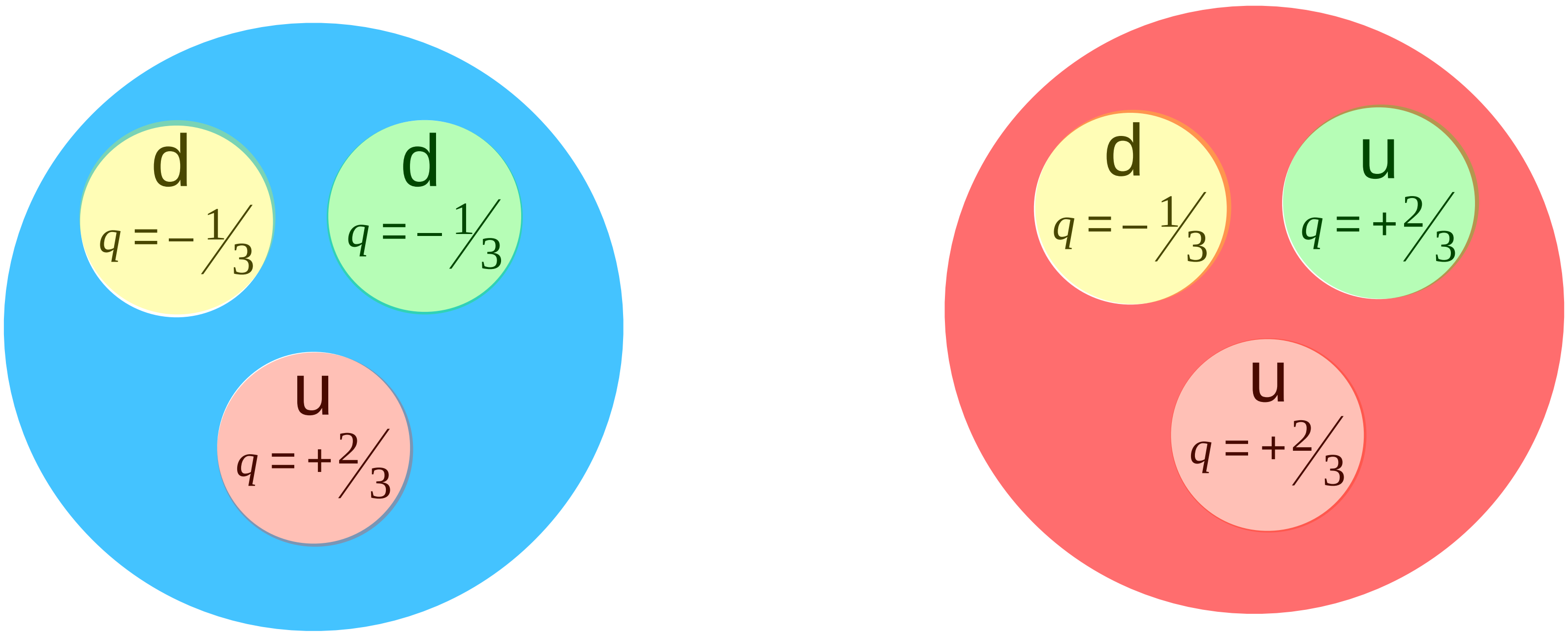
__Miscellaneous Quantum Numbers:__

Intrinsic Parity (P), Isospin (I), Baryon Number (B), Strangeness (S), …

The Neutron Mass

The "Naive" Quark Model

Nucleons are color triplets of spin ½ Quarks



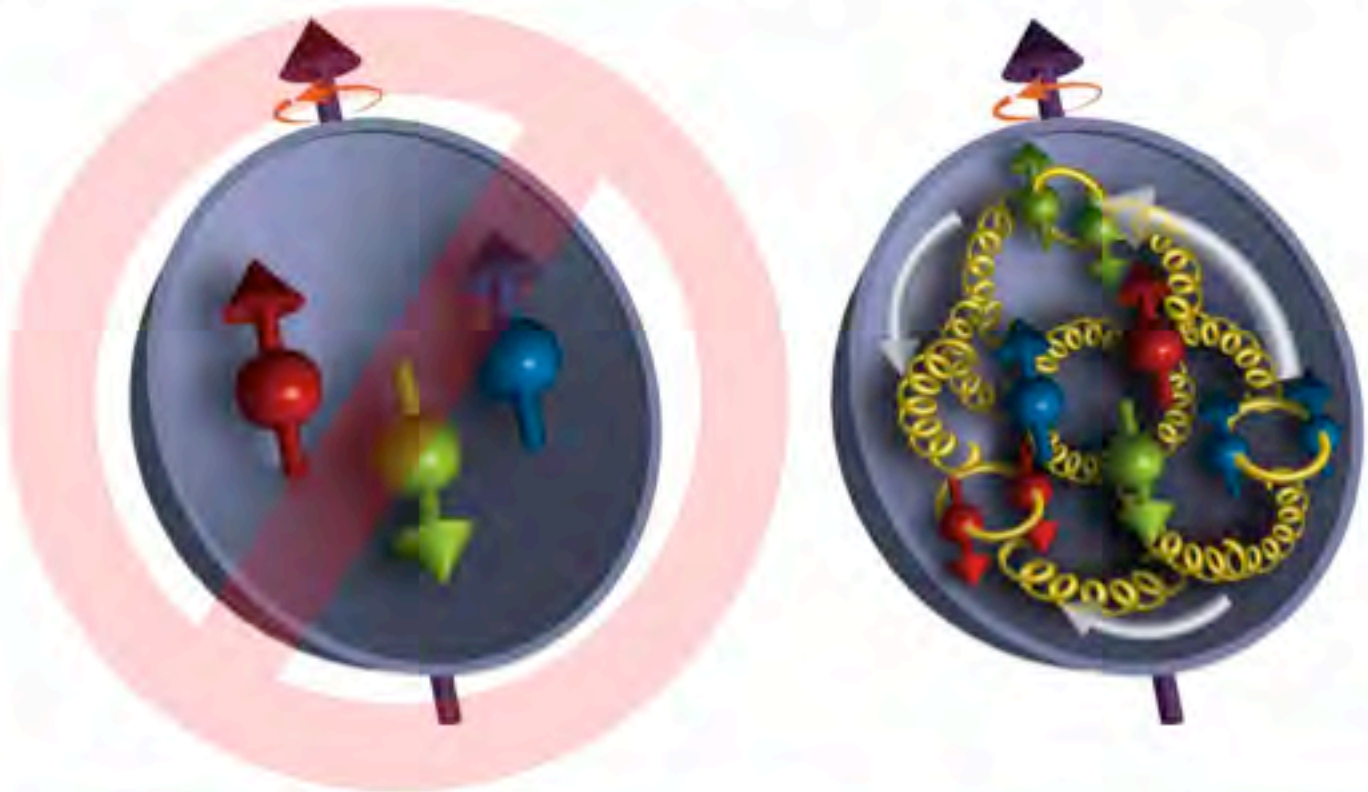
Neutron
q=0

Proton
q=+1

Electrostatic self-energy suggests: $m_p > m_n$?

THE NUCLEON IS COMPLICATED!

Nucleons are NOT simply composed of 3 valence quarks. There are gluons, virtual strange quarks,….lots of stuff…



~99% of the nucleon mass arises from the self-energy of the gluon field

Image: BNL

Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi,[1] S. Durr,[1,2] Z. Fodor,[1,2,3*] C. Hoelbling,[1] S. D. Katz,[3,4] S. Krieg,[1,2]
L. Lellouch,[5] T. Lippert,[1,2] A. Portelli,[5,6] K. K. Szabo,[1,2] B. C. Toth[1]

Science **347**, 1452 (2015)

	ΔM_{TOTAL}	ΔM_{QCD}	ΔM_{QED}	(MeV)
Theory	1.51(28)	2.51	-1.00	
Experiment	1.30			

Determination of the Neutron Mass

The best method for the determination of the neutron mass considers the reaction:

$$n + p \rightarrow d + \gamma$$

and measures two quantities with high accuracy:

1. A gamma ray energy

The actual experiment is an absolute determination of the 2.2MeV gamma ray wavelength in terms of the SI meter.

2. A mass difference

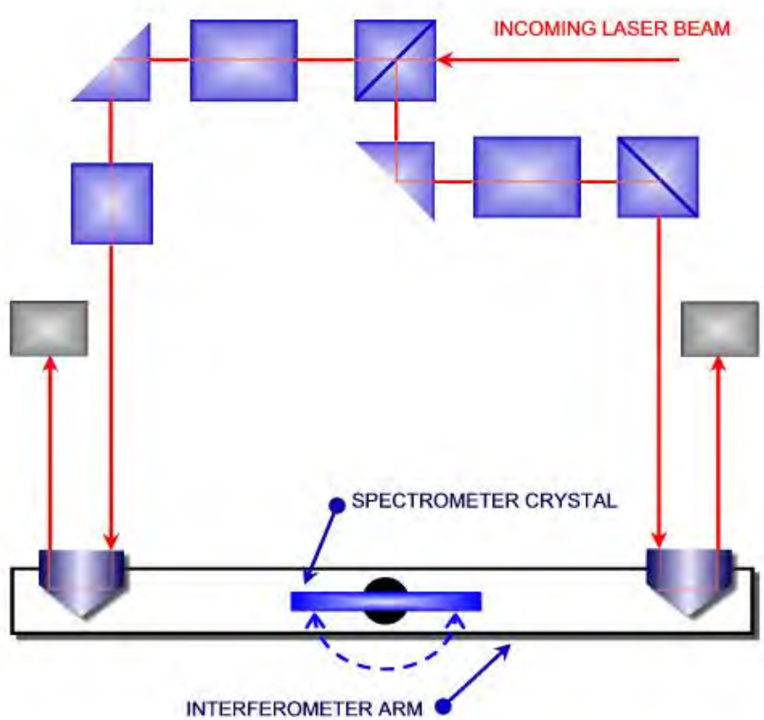
The actual experiment is the determination of the D - H mass difference in atomic mass units.

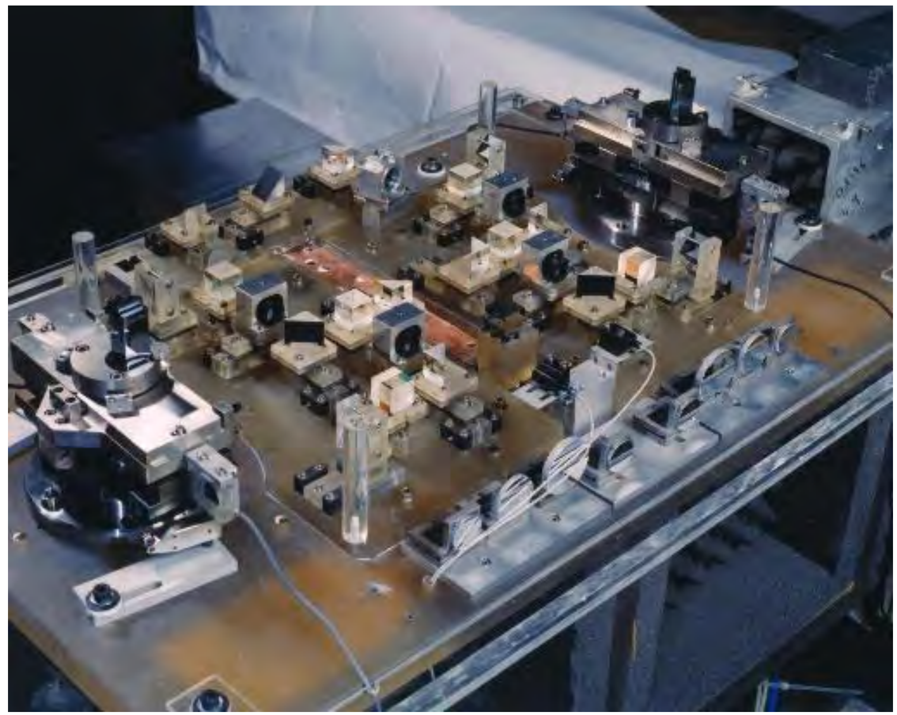
Absolute Measurement of 2.2Mev n-p Capture Gamma Energy

Measure Bragg angle for diffraction of 2.2MeV gamma from a perfect single crystal of Silicon with an accurately measured lattice spacing d.

$$n\lambda^* = 2d\sin\theta \qquad E_\gamma = h\nu = \frac{hc}{\lambda^*}$$

Bragg Angle is a few milli-radian
Need nano-radian precision!

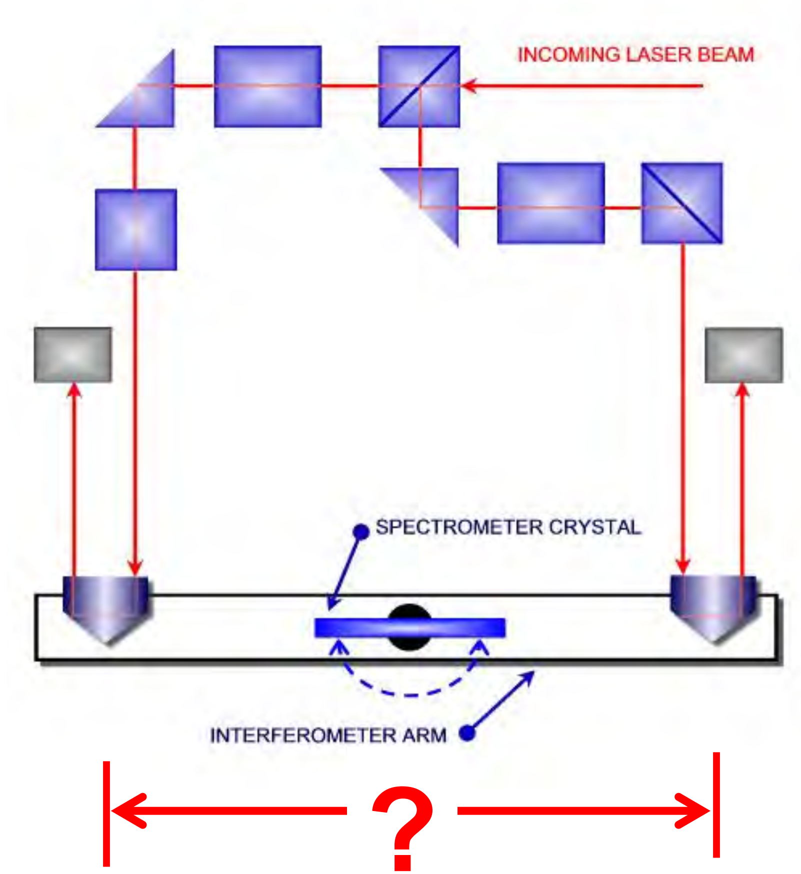




See: http://www.ill.fr/YellowBook/PN3/

Precision vs. Accuracy

Angle Interferometer gives high precision but what about its "calibration"

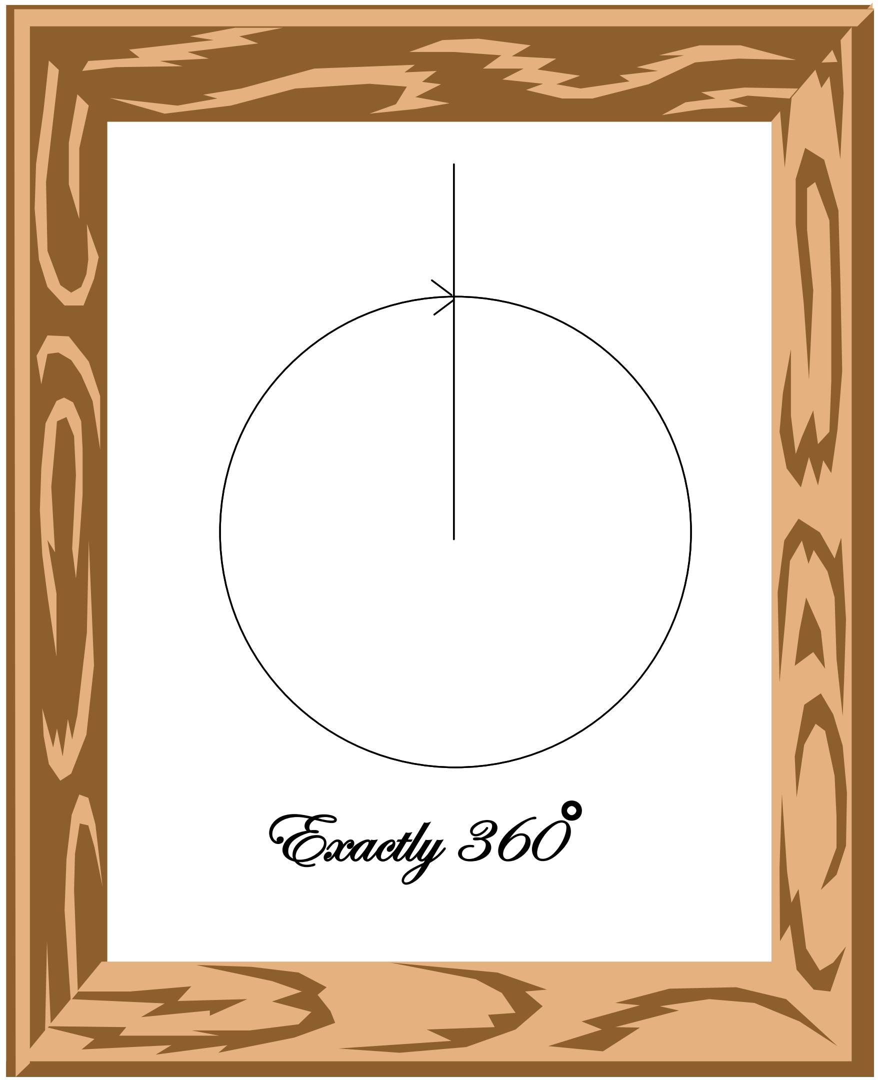


What can we use to calibrate a precision angle device?
Is there a "Standard" for angle measurement?

See: http://www.ill.fr/YellowBook/PN3/

The NIST Standard for the Kilogram (At least until May 2019)

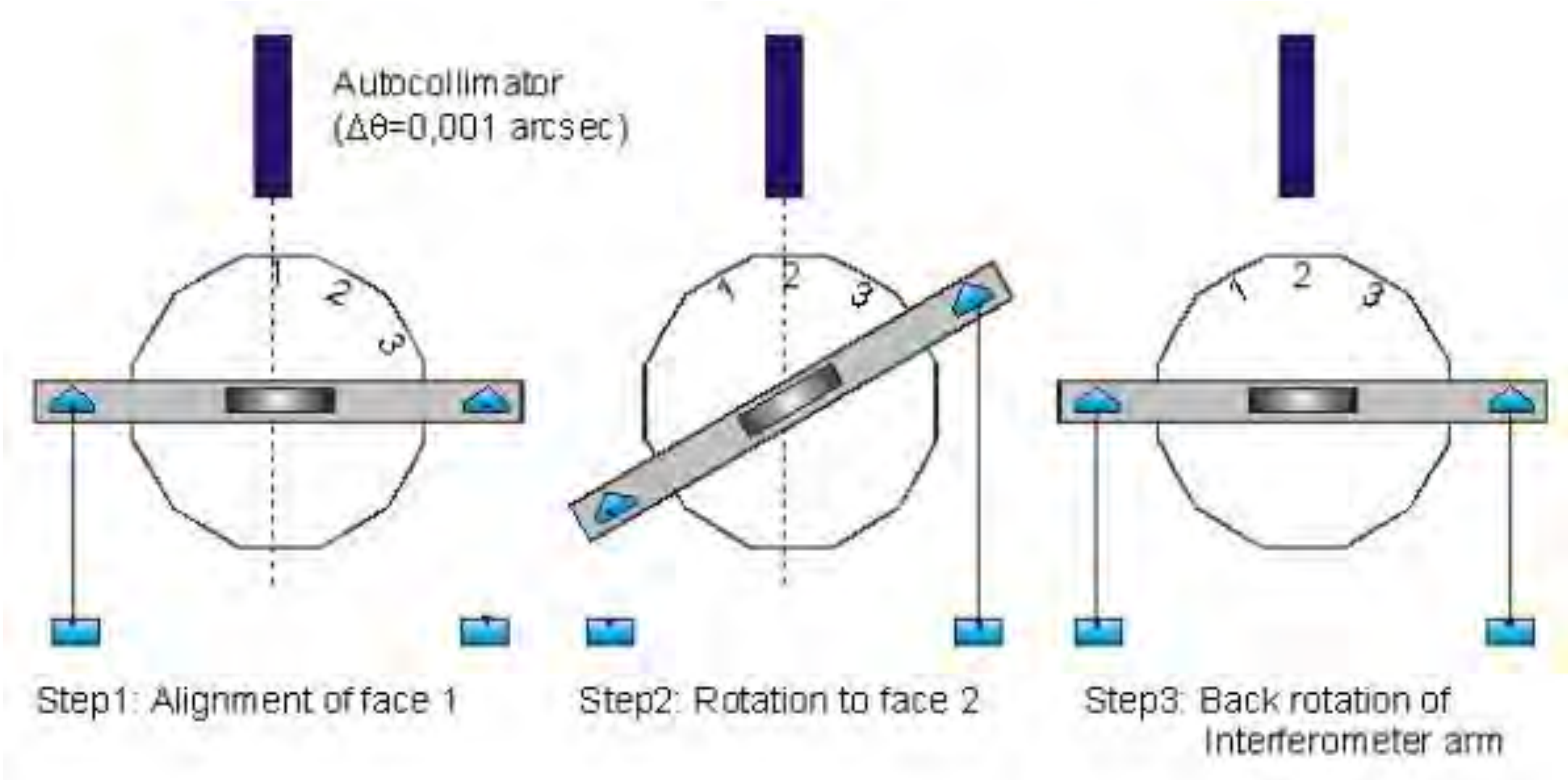
What is the NIST Standard for angles?



The NIST Standard Angle

Absolute Measurement of 2.2Mev n-p Capture Gamma Energy

Step 3: Calibrate Angle interferometer



Measure 24 interfacial angles of a precision quartz optical polygon Since they must sum to 360°, there are only 23 independent quantities. A 24 parameter fit can give the calibration constant.

See: http://www.ill.fr/YellowBook/PN3/

Determination of the Neutron Mass

$$M(n) = 939.5654133(58)\ MeV$$

E. G. Kessler, et. al., Phys Lett A, 255 (1999)
G.L Greene, et. al., Phys. Rev. Lett. 24, 819 (1986)

The Neutron’s Gravitational Mass

Equivalence Principle Test with Neutrons

The measurement of the neutron mass represents a determination of the neutron's INERTIAL mass. To determine the neutron's GRAVITATIONAL mass, one must compare the free fall acceleration of the neutron with the acceleration g of macroscopic test masses:

$$F_n = m_i\, a_n$$

$$m_g\, g = m_i\, a_n$$

$$m_g / m_i = a_n / g \equiv \gamma$$

Falling Neutrons

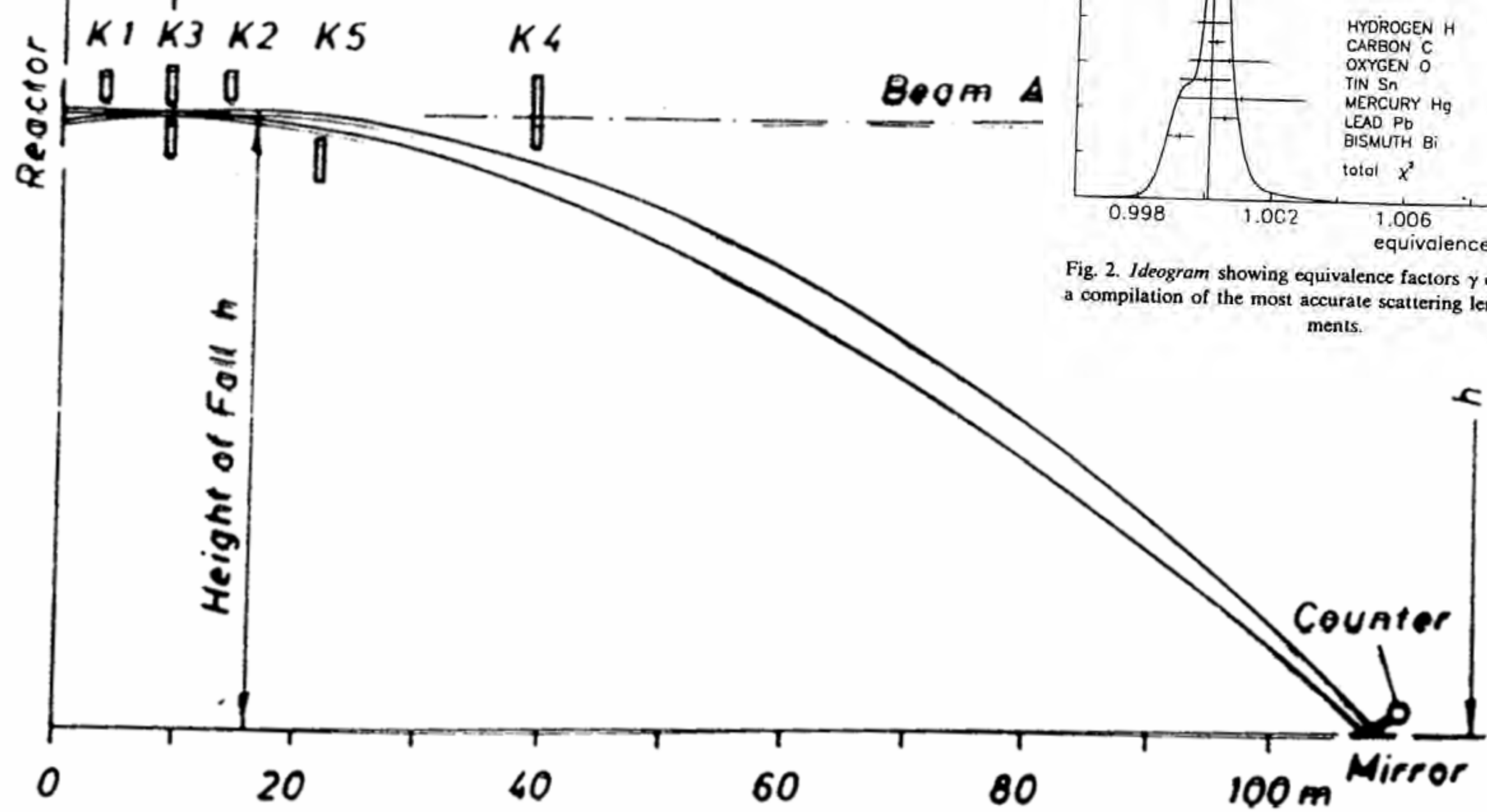


Fig. 1. Principle of the neutron-gravity refractometer

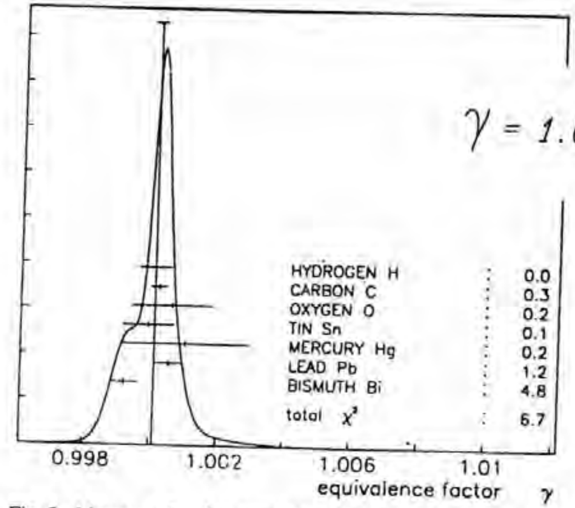
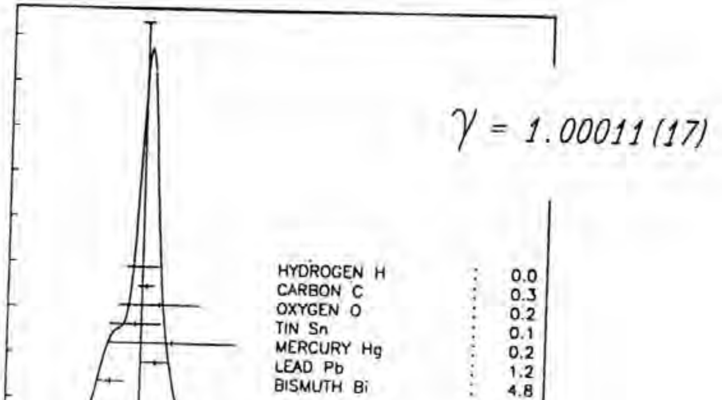


Fig. 2. *Ideogram* showing equivalence factors γ obtained from a compilation of the most accurate scattering length measurements.

$\gamma = 1.00011(17)$

For review see: Schmiedmayer, NIM ***A284****, 59 (1989)*

<u>*Falling Neutrons*</u>



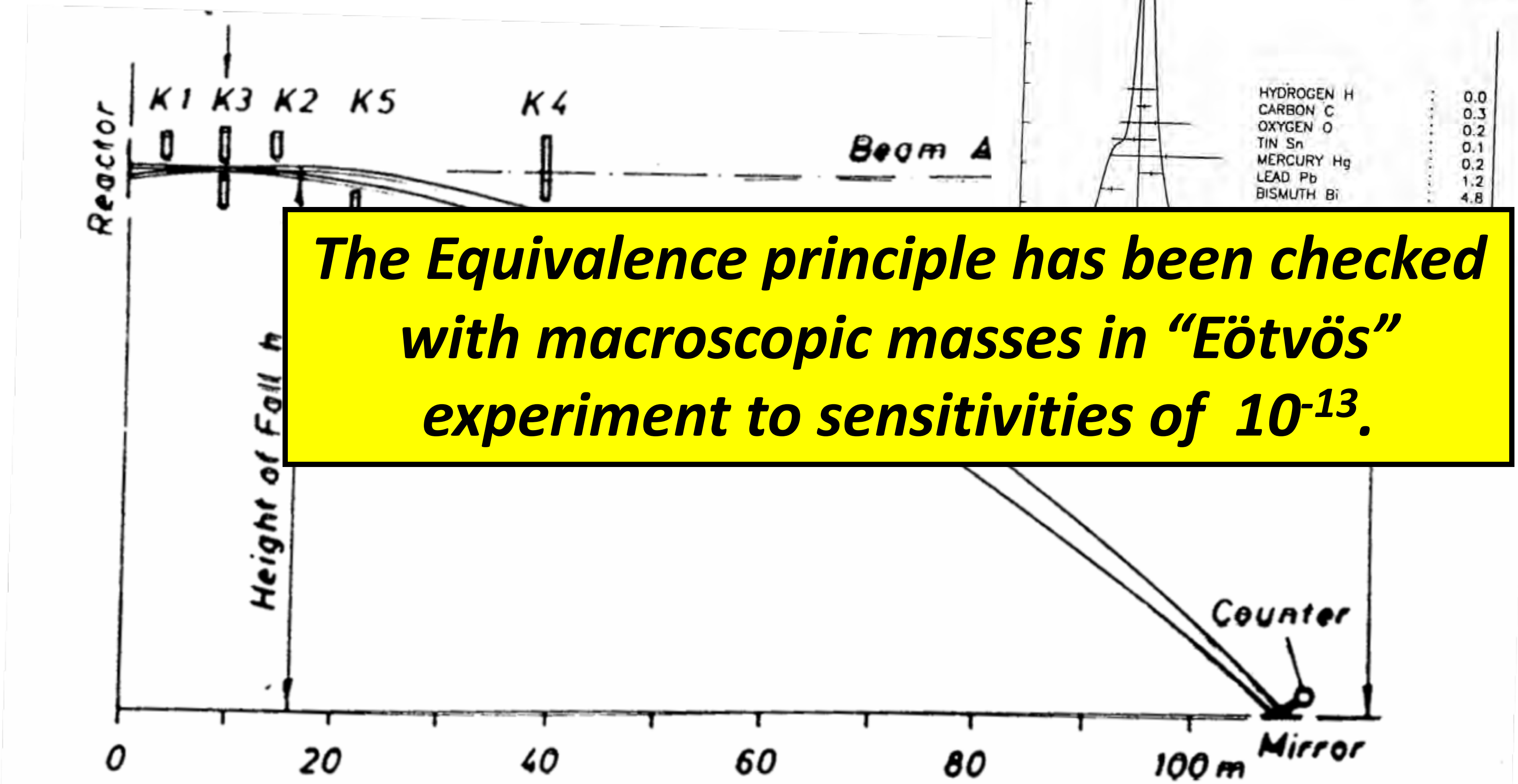


Fig. 1. Principle of the neutron-gravity refractometer

The Equivalence principle has been checked with macroscopic masses in "Eötvös" experiment to sensitivities of 10^{-13}.

*For review see: Schmiedmayer, NIM **A284**, 59 (1989)*

“Quantum Bouncing Ball” *

$$V(x) = \begin{cases} \infty & \text{for} \quad 0 > x \\ mgh & \text{for} \quad x \geq 0 \end{cases}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + mgx\psi = E\psi$$

This differential equation is not soluble in terms of elementary functions. However the equation:

$$\frac{d^2 y}{dx^2} - yx = 0$$

has been extensively studied. As expected there are two linearly independent solutions. These are known as “Airy” functions:

$$Ai(x) \ \text{ and } \ Bi(x)$$

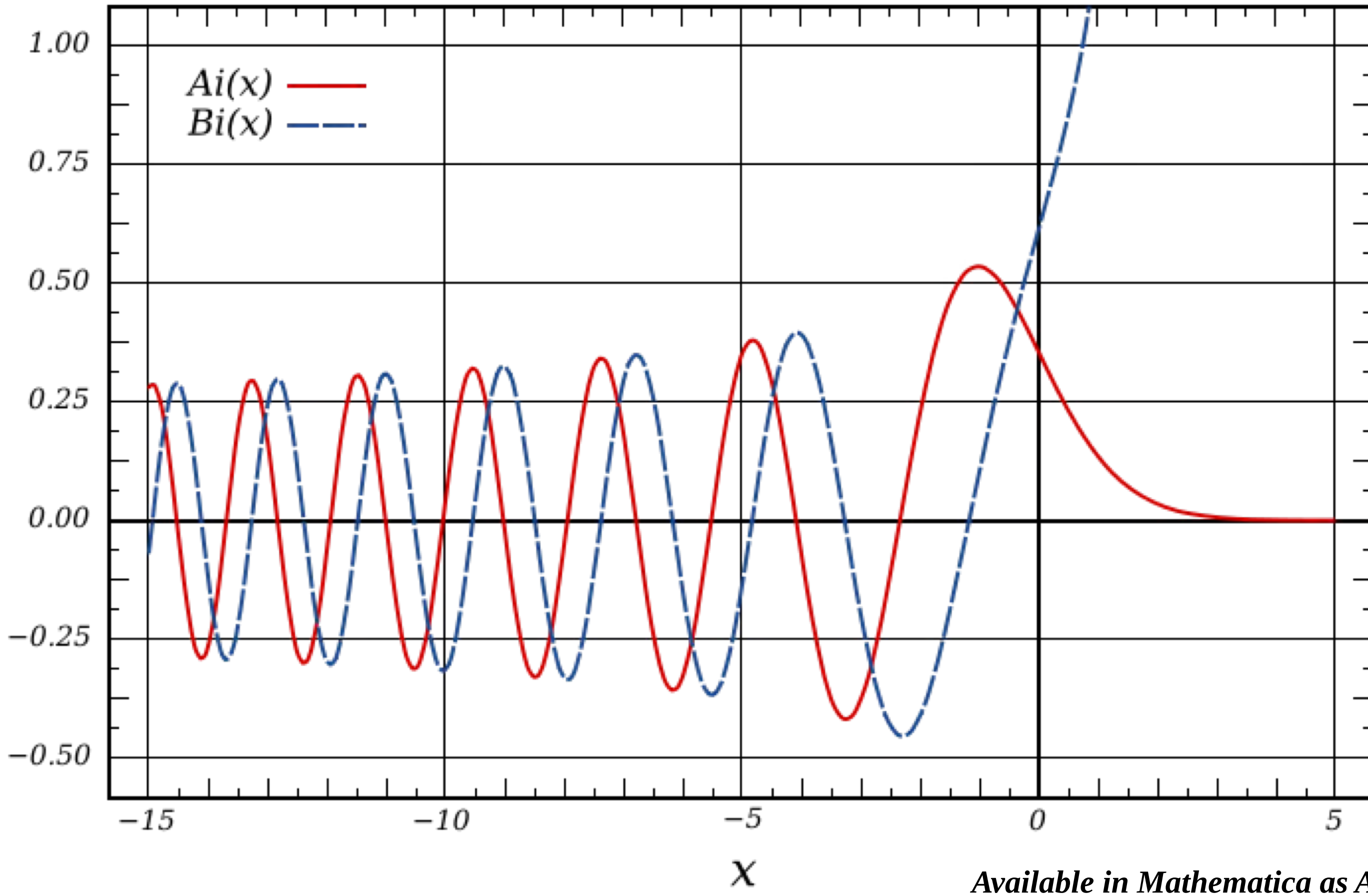
****Nezvizhevsky, et al, Nucl Instrum and Meth, A440, 754 (2000)]***
Nezvizhevsky, et al , Nature 415, 297 (2002)

Airy Functions

$$Ai(x) = \frac{1}{\pi}\int_0^\infty \cos\left(\frac{t^3}{3} + xt\right)dt$$

$$Bi(x) = \frac{1}{\pi}\int_0^\infty \left[\exp\left(\frac{t^3}{3} + xt\right) + \sin\left(\frac{t^3}{3} + xt\right)\right]dt$$

George Airy
1801-1892



Available in Mathematica as AiryAi[z] and AiryBi[z]

Solution to Schrödinger's Equation $$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+mgx\psi=E\psi$$

Stationary States:

$$\psi_n=A_n Ai\left(\frac{x}{x_0}+\alpha_n\right)\text{ for } x>0$$

is the nth stationary state

$$x_0=\left(\hbar^2/2m^2g\right)^{1/3}$$

is the gravitational scale height

$$\alpha_n$$

is the nth zero of the Airy function $Ai(x)$
(note α_n are all negative)

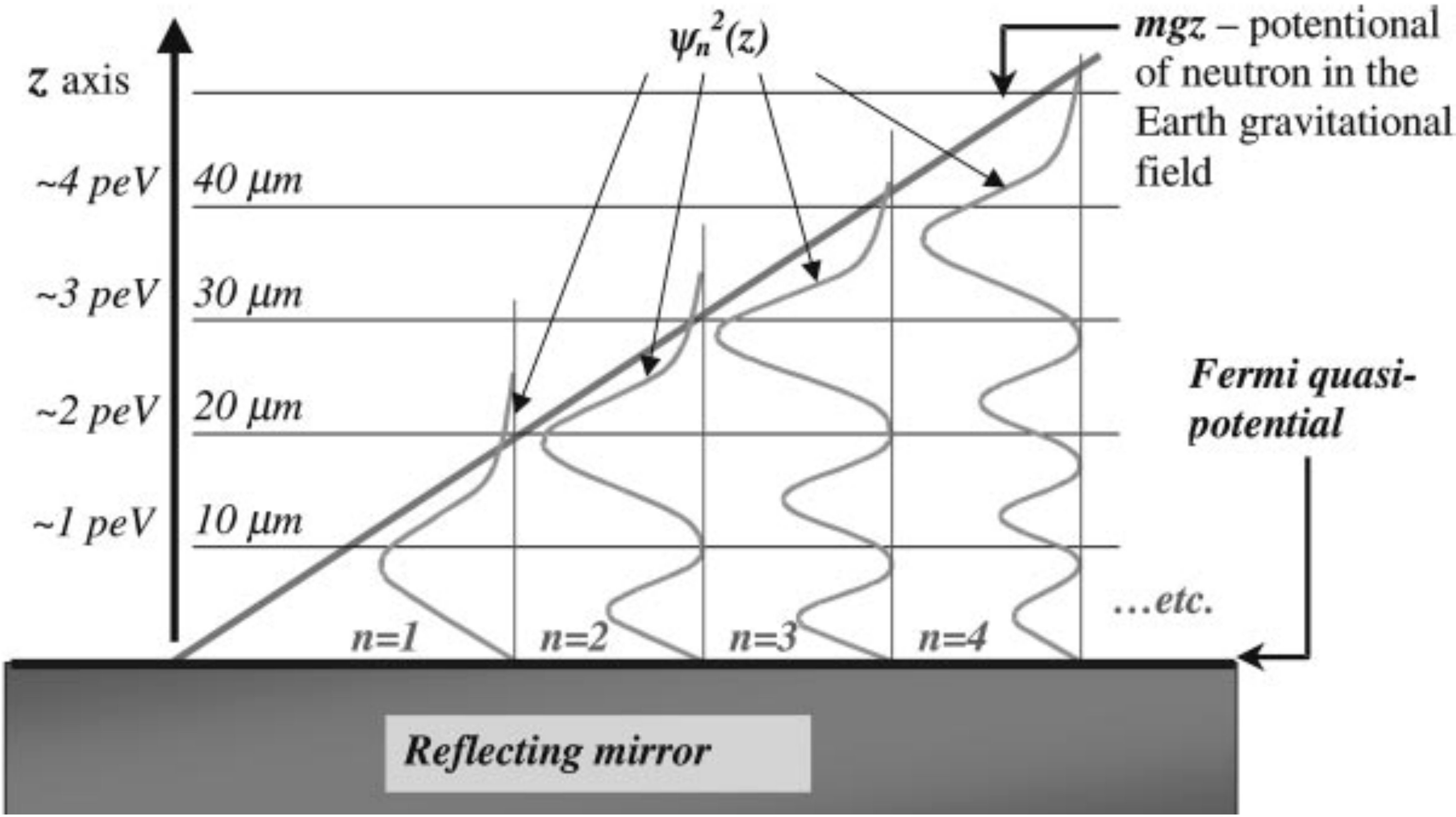
$$A_n=\frac{1}{\sqrt{x_0\int_{\alpha_n}^{\infty}Ai(\xi)^2\,d\xi}}$$

is the normalization constant

Energy Levels are given by the roots of $Ai(x)$

$$Ai\left(-\frac{\sqrt[3]{2}}{\sqrt[3]{mg^2\hbar^2}}E_n\right)=0$$

The Quantum "Bouncing Neutron"



$$E_1 = 1.44 \times 10^{-12} eV$$

$$E_2 = 2.53 \times 10^{-12} eV$$

$$E_3 = 3.42 \times 10^{-12} eV$$

$$E_4 = 4.21 \times 10^{-12} eV$$

The Experiment:

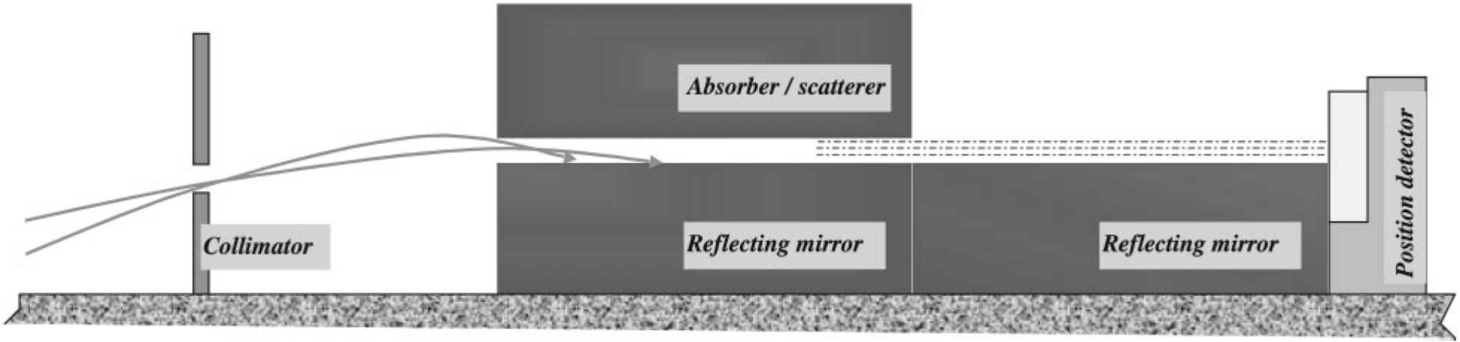


Fig. 3. General scheme of the experiment.

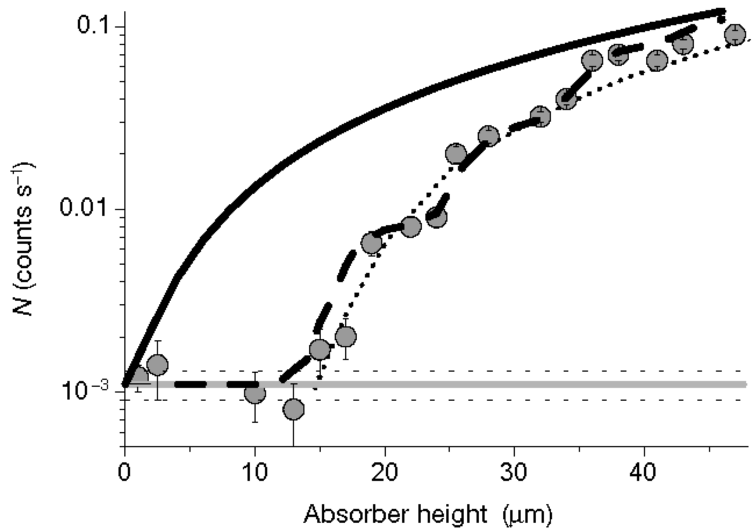
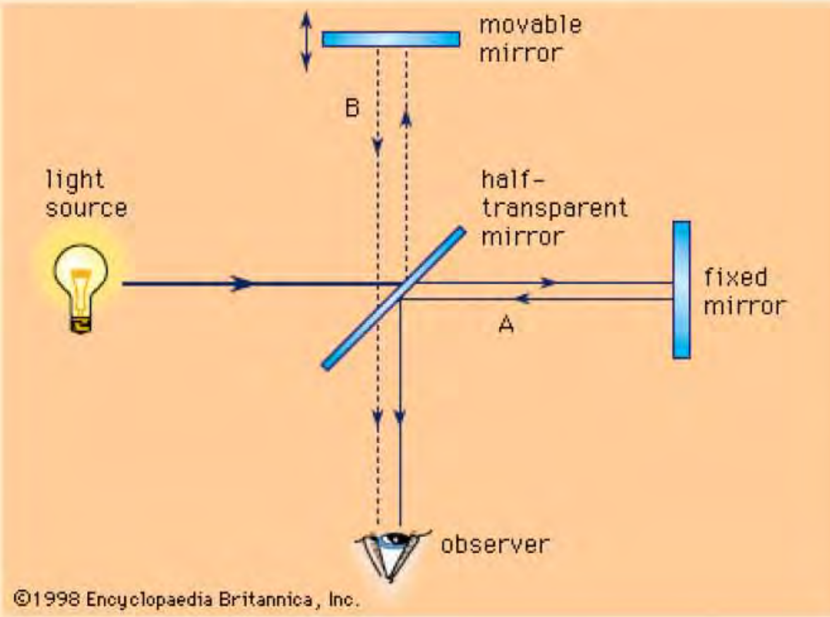


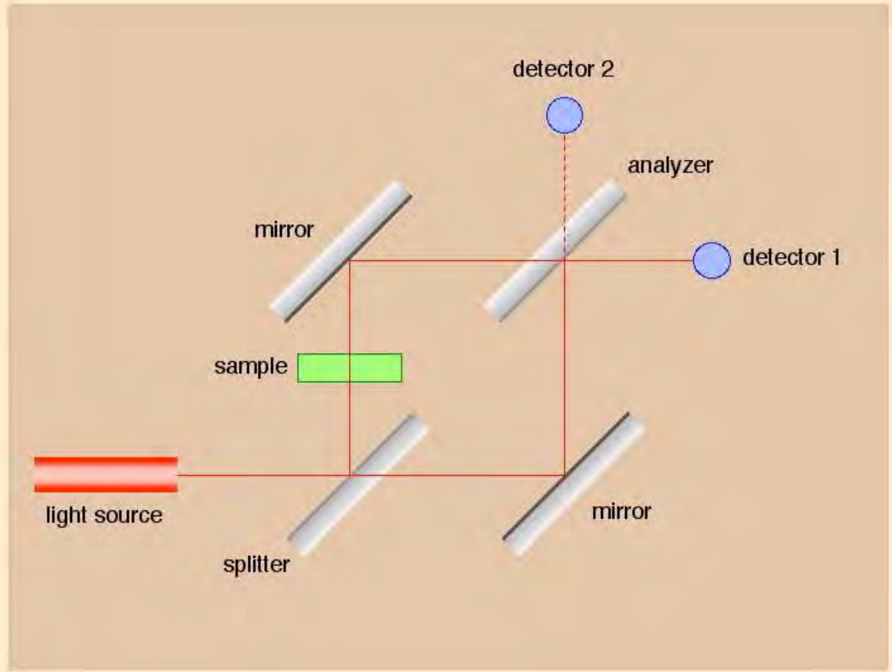
Figure 4 The neutron throughput versus the absorber height at low height values. The data points are summed up in intervals of 2 μm. The dashed curve corresponds to a fit using the quantum-mechanical calculation, in which all level populations and the height resolution are fitted from the experimental data. The solid curve is again the full classical treatment. The dotted line is a truncated fit in which it is assumed that only the lowest quantum state—which leads to the first step—exists.

Neutron Interferometry

Slides Courtesy of Fred Weitfeldt

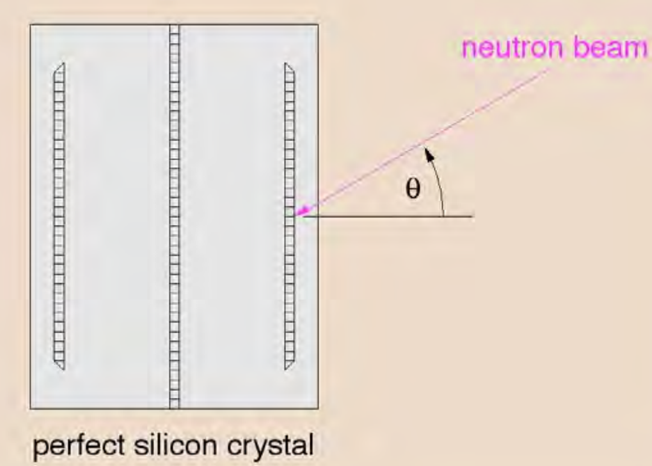
Michelson Interferometer





Mach-Zender Interferometer

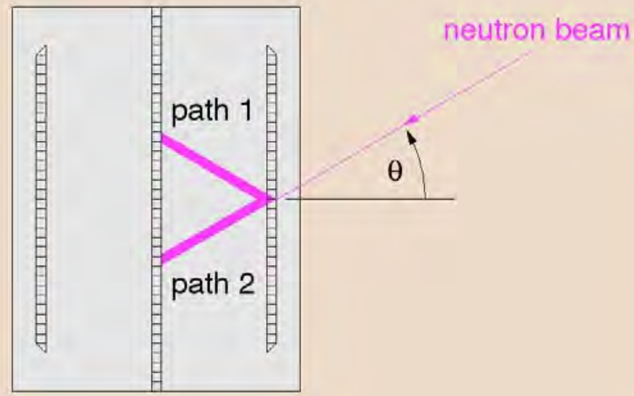
Perfect Crystal LLL Neutron Interferometer



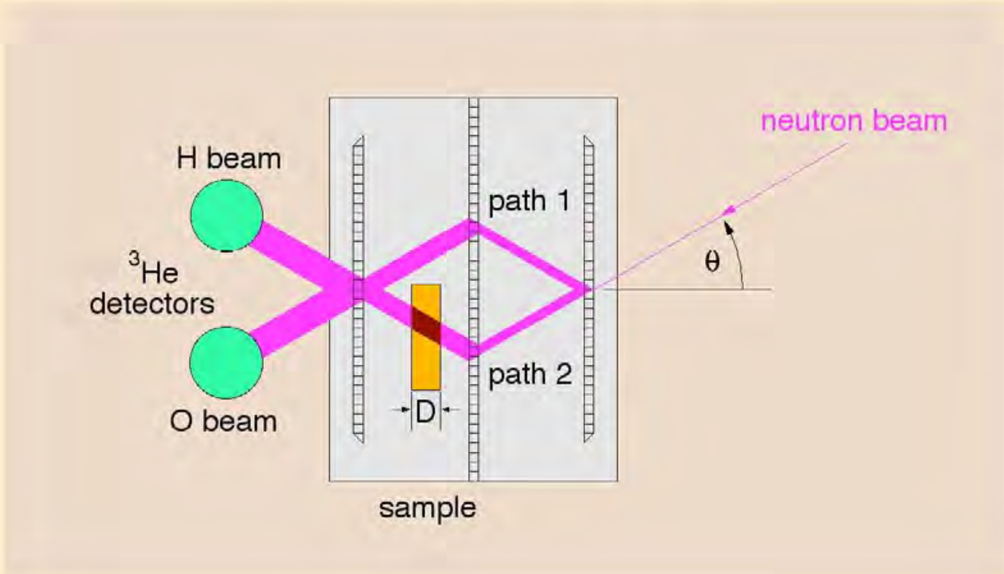
Bragg condition: $n\lambda = 2d\sin\theta$

d = lattice spacing

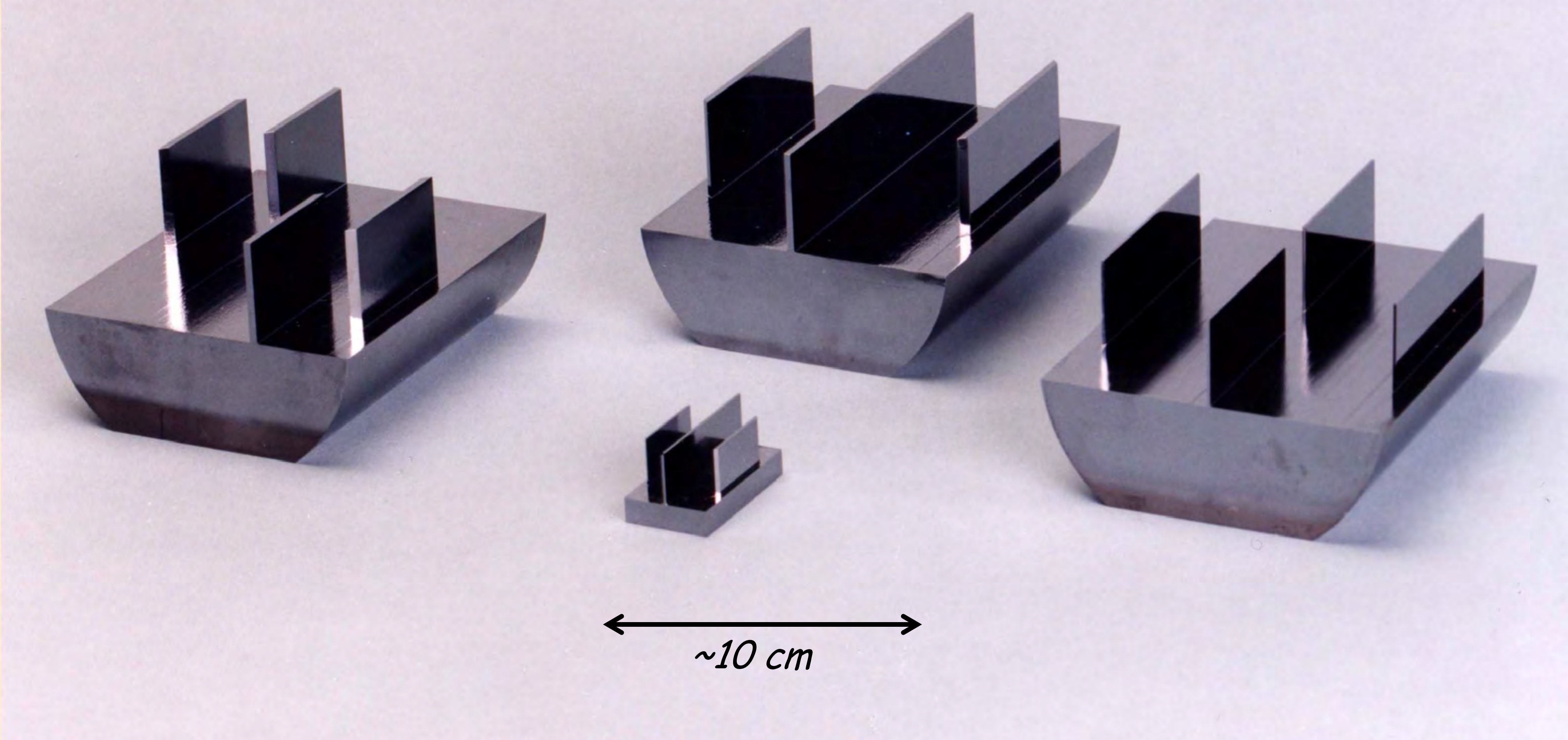
Perfect Crystal LLL Neutron Interferometer



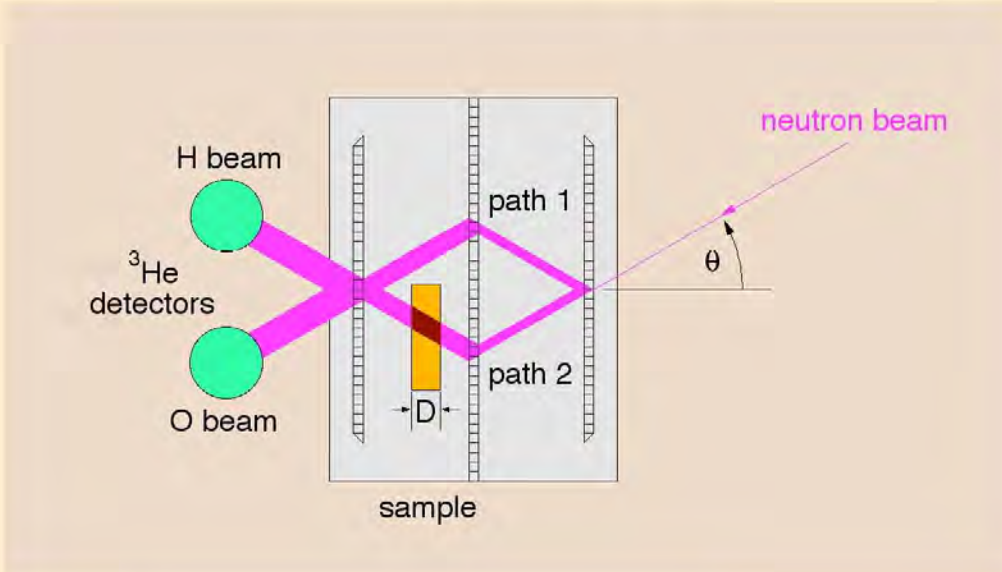
Nuclear Phase Shift

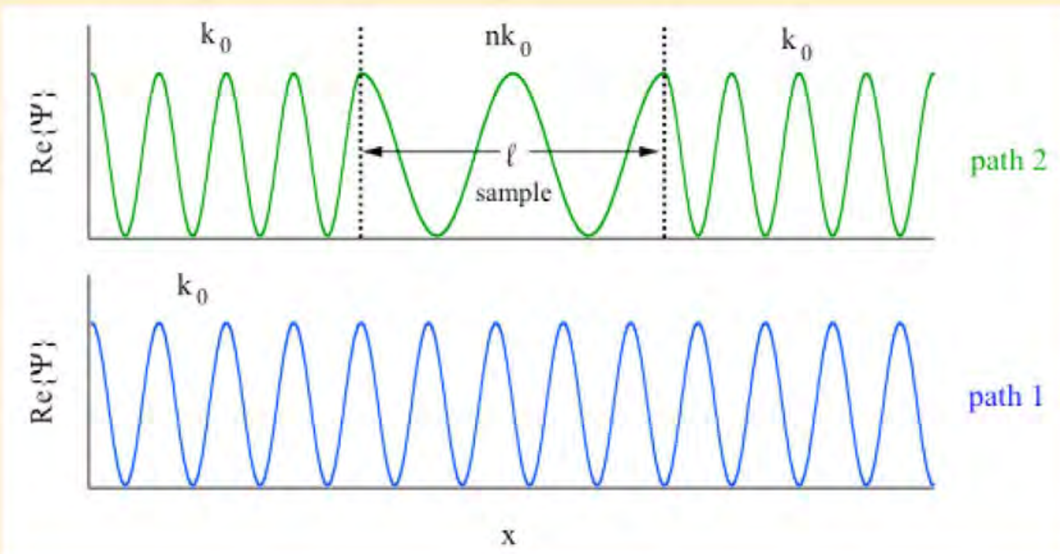


Perfect Crystal Silicon Interferometers



Nuclear Phase Shift

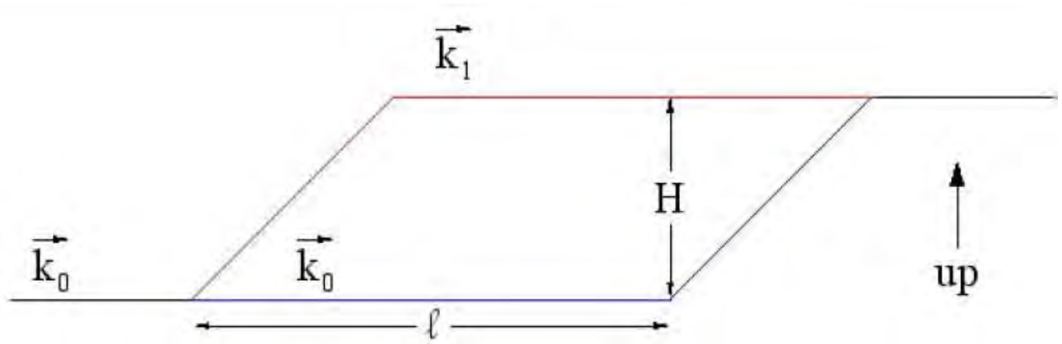




Introduce a Phase Shift into one leg. Could be:

1. **Electromagnetic due to magnetic moment**
2. **Nuclear Force due to coherent scattering amplitude**
3. ***GRAVITY***

Quantum Mechanical Gravitational Phase Shift



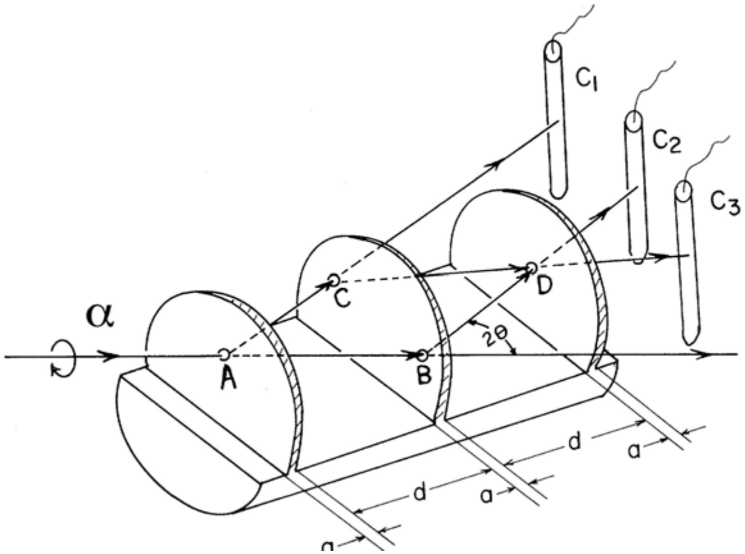
$$\Delta\phi = \frac{2\pi\lambda g A}{h^2} m_i m_g$$

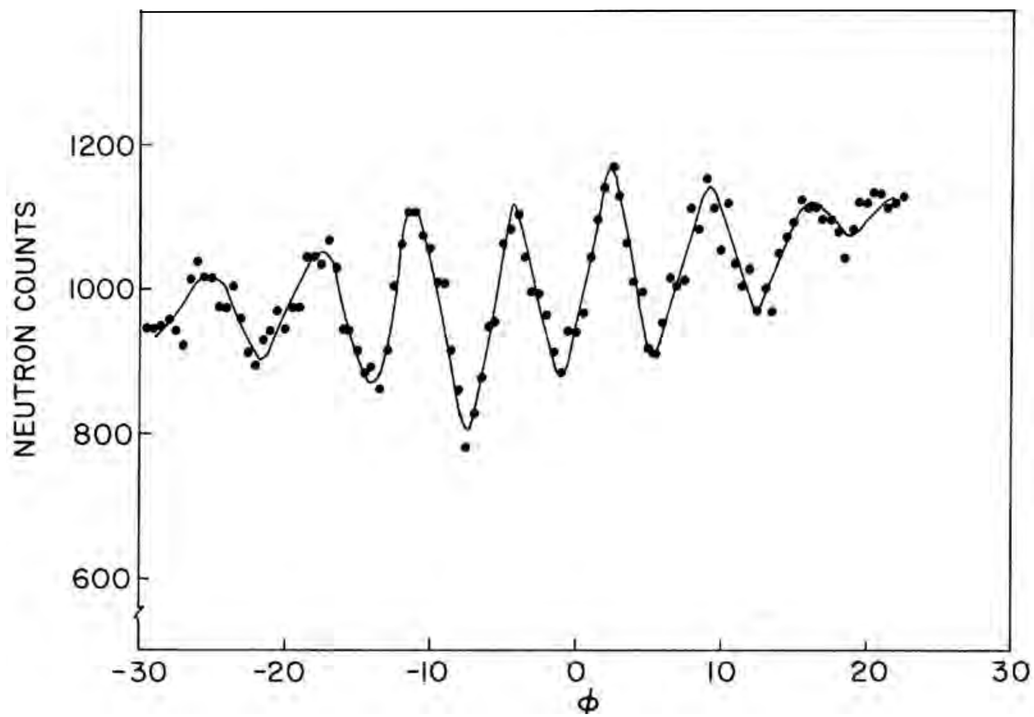
A=Hl = Area of parallelogram

m_i = *neutron inertial mass*
m_g = *neutron gravitational mass*

Test of the Weak Equivalence Principle in the Quantum Limit

The Collela-Overhauser-Werner Experiment COW





*Collela, Overhauser, Werner, PRL **34**, 1472 (1975)*

…

…

*Littrell, Allman, Werner, Phys Rev **A56**, 1767 (1997)*

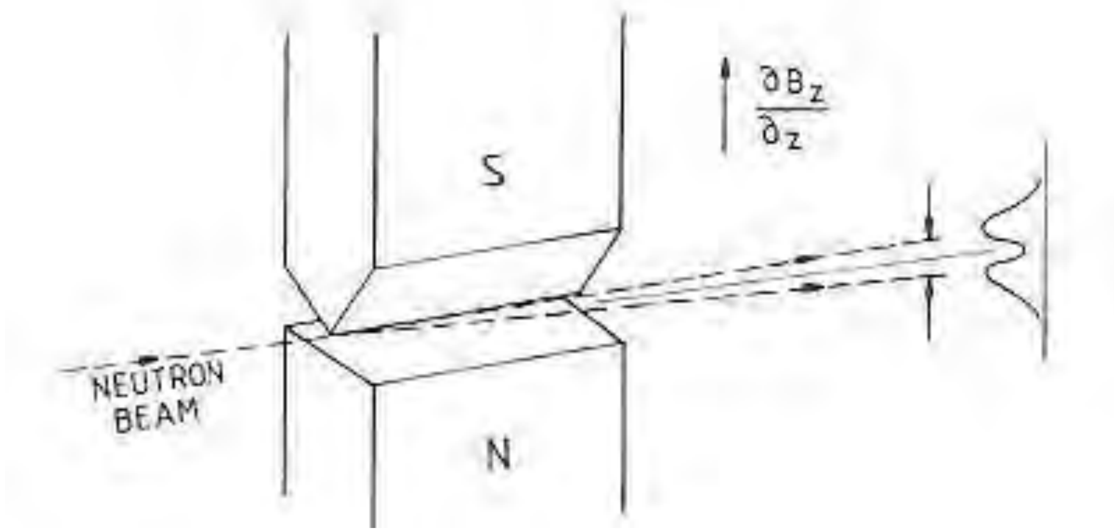
The Neutron Spin

The Neutron has an Intrinsic Spin of s=½

1934 ***Schwinger concludes that s=½ based on the band spectrum of molecular D_2 and the scattering of neutrons from ortho and para H_2.***

1949 ***Hughes and Burgey observe the mirror reflection of neutrons from magnetized iron. They observe 2 critical angles definitively showing the neutron has two magnetic sub-levels.***

1954 ***Neutron Stern-Gerlach experiment explicitly demonstrates s=½ .***

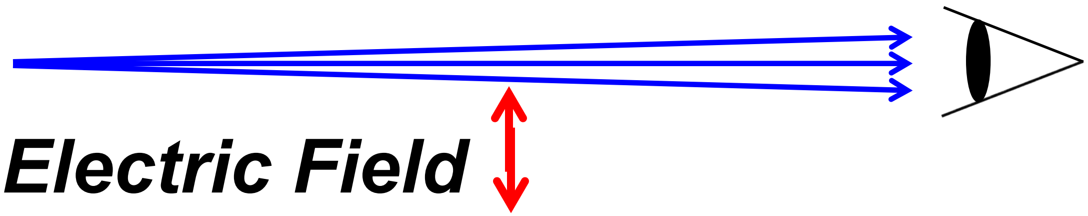


*See also Fischbach, Greene, Hughes, PRL **66**, 256 (1991) showing* $\vec{L} = \hbar\vec{s}$

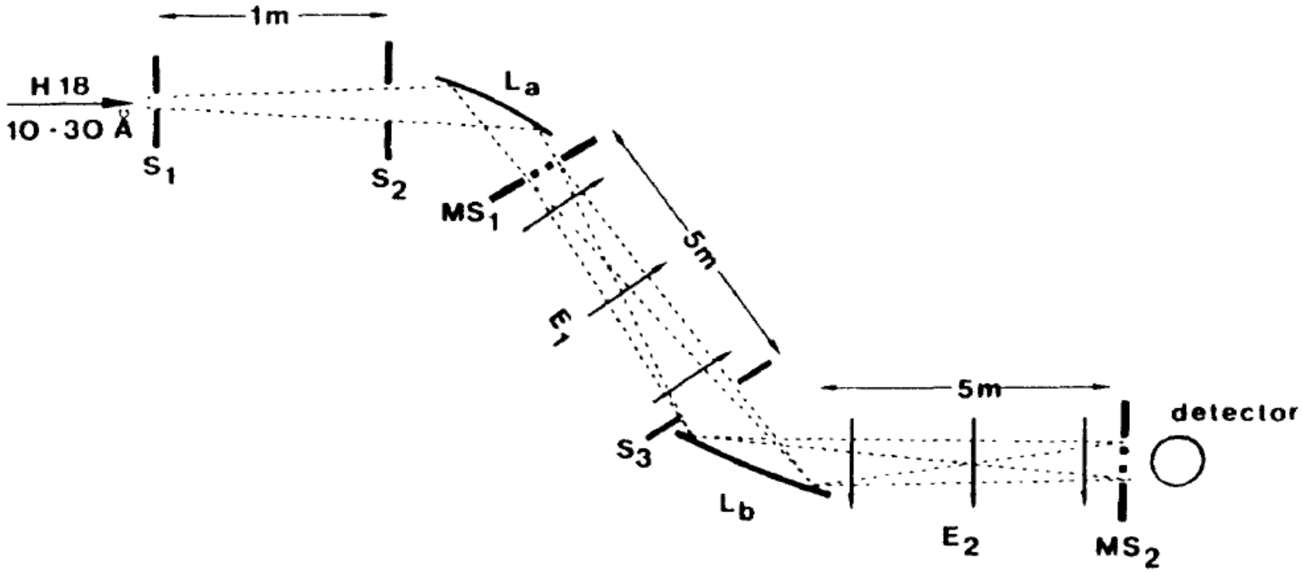
The Neutron Charge ?

The Neutron Appears to be Neutral

Neutron beam



Electric Field



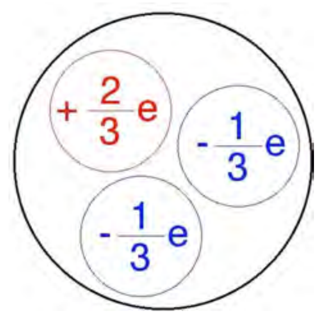
$$\boxed{Q_n = (-0.4 \pm 1.1) \times 10^{-21}\, e}$$

J. Baumann et al., Phys. Rev. D 37, 3107 (1988)

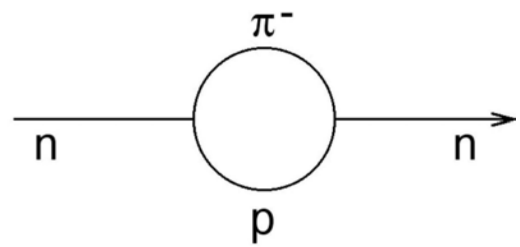
The Neutron Charge Distribution

Neutrality does not Imply Uniformity

The neutron is a composite structure of charged quarks which may be distributed non uniformly within the neutron.



Fermi & Marshall suggested that the neutron should have a positive "core" and a negative "skin" due to virtual pion emission



Neutron Mean Charge Radius:

$$\left\langle r_n^2 \right\rangle = \int \rho\left(r\right) r^2 dr^3$$

Neutron Electric Scattering Form Factor

The Fourier transform of the neutron charge density $G_E^n\left(Q^2\right)$ is accessible from electron scattering.

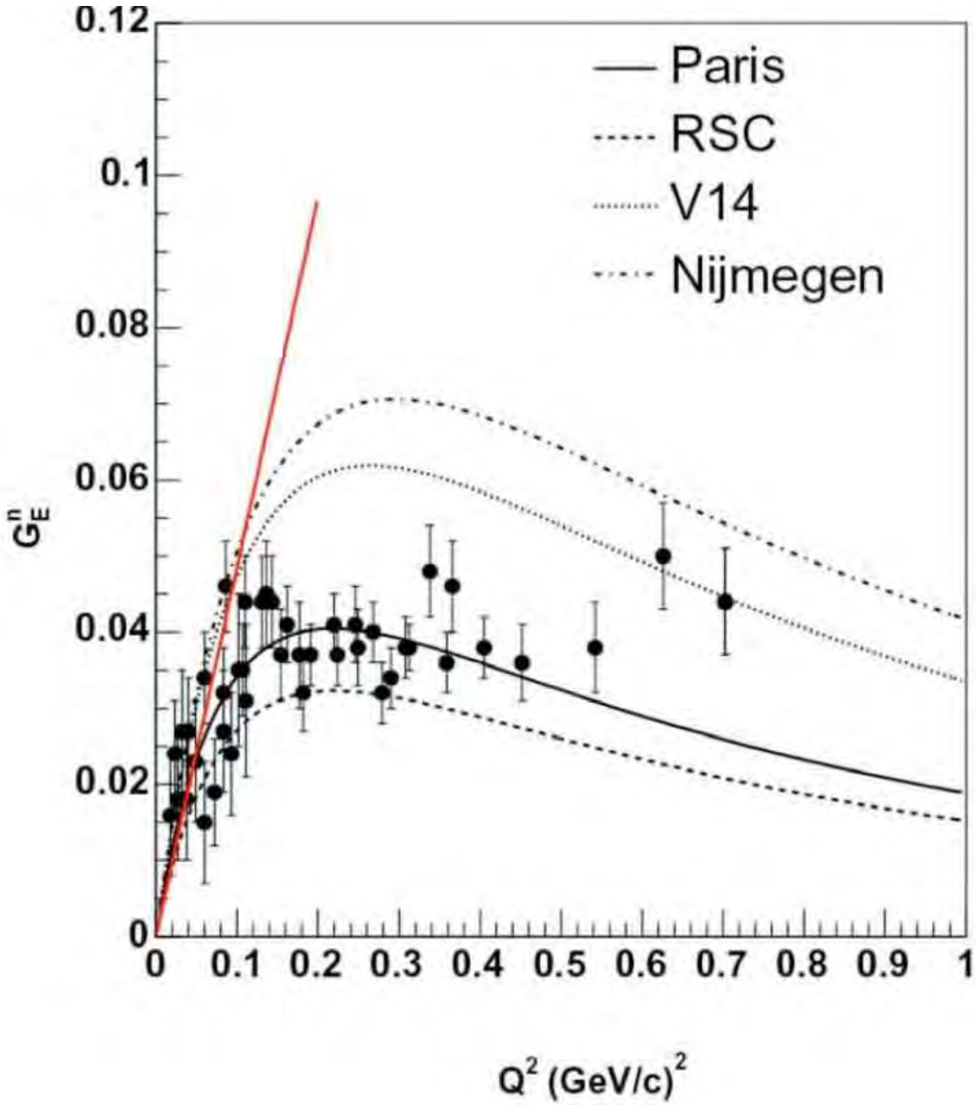
Expanding in the momentum transfer Q^2 :

$$G_E^n\left(Q^2\right) = q_n - \frac{1}{6}\left\langle r_n^2 \right\rangle Q^2$$

In the limit of low Q^2 :

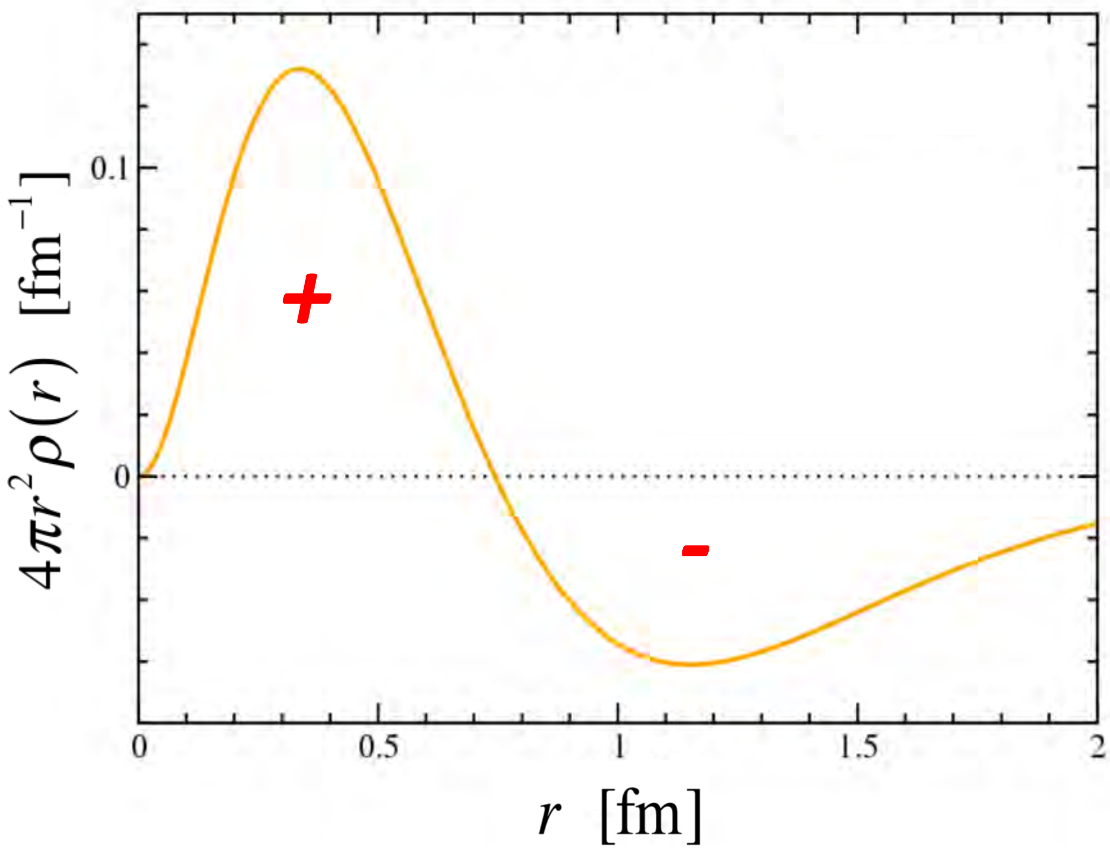
$$\left\langle r_n^2 \right\rangle = -6\frac{d}{dQ^2} G_E^n\left(Q^2\right)\Big|_{Q^2=0}$$

The Mean Square Neutron Charge Radius $G_E^n(Q^2)$.
Constrains the Slope In Electron Scattering Experiments.
(e.g. Bates, JLab,…)



V. Ziskin, Ph.D. thesis, 2005

The neutron does have a static charge distribution with a positive "core" and a negative "halo"



G.A. Miller and J. Arrington Phys. Rev. C 78, 032201 (2008)

Experimental Situation is in Disarray

2230 S. KOPECKY *et al.* 56

TABLE I. Experimental results of b_{ne} in units of 10^{-3} fm.

Experiment	Target	Result	Reference
Angular scattering	Ar	-0.1 ± 1.8	1947 [7] Fermi
Transmission	Bi	-1.9 ± 0.4	1951 [8] Havens
Angular scattering	Kr, Xe	-1.5 ± 0.4	1952 [9] Hamermesh
Mirror reflection	Bi/O	-1.39 ± 0.13	1953 [10] Hughes
Angular scattering	Kr, Xe	-1.4 ± 0.3	1956 [11] Crouch
Crystal spectrometer transmission	Bi	-1.56 ± 0.05	1959 [2] Melkonian
		-1.49 ± 0.05	1976 in Ref. [15]
		$-1.44 \pm 0.033 \pm 0.06$	1997 this work
Angular scattering	Ne, Ar, Kr, Xe	-1.34 ± 0.03	1966 [12] Krohn
Angular scattering	Ne, Ar, Kr, Xe	-1.30 ± 0.03	1973 [13] Krohn
Single crystal scattering	^{186}W	-1.60 ± 0.05	1975 [14] Alexandrov
Filter-transmission, mirror reflection	Pb	-1.364 ± 0.025	1976 [15] Koester
Filter-transmission, mirror reflection	Bi	-1.393 ± 0.025	1976 [15] Koester
n-TOF transmission, mirror reflection Ref. [17]	Bi	-1.55 ± 0.11	1986 [16] Alexandrov
Filter-transmission, mirror reflection	Pb, Bi	-1.32 ± 0.04	1986 [17] Koester
n-TOF transmission	thorogenic ^{208}Pb	$-1.31 \pm 0.03 \pm 0.04$	1995 [1] Kopecky
		$-1.33 \pm 0.027 \pm 0.03$	1997 this work
Filter-transmission, mirror reflection	Pb-isotopes, Bi	-1.32 ± 0.03	1995 [5] Koester
Garching-Argonne compilation	[12,13,15,17]	-1.31 ± 0.03	1986 [3] Sears
Dubna compilation	[14,16]	-1.59 ± 0.04	1989 [19] Alexandrov
Foldy approximation, b_F		-1.468	1952 [18] Foldy

The Neutron Magnetic Moment

"Naive" Quark Model

Static SU(6) Model:

1. *Baryons wavefunctions are quark color singlets with correct symmetry*

2. *Baryon magnetic moments arise solely from the static sum of the quark moments*

3. *Individual quark moments are proportional to quark charges (i.e. $\mu_u = -2\mu_d$)*

1.

$$n_\uparrow = \sqrt{\tfrac{2}{3}} d_\uparrow d_\uparrow u_\downarrow - \sqrt{\tfrac{1}{3}} \left(\frac{d_\uparrow d_\downarrow + d_\downarrow d_\uparrow}{\sqrt{2}} \right) u_\uparrow$$

$$p_\uparrow = \sqrt{\tfrac{2}{3}} u_\uparrow u_\uparrow d_\downarrow - \sqrt{\tfrac{1}{3}} \left(\frac{u_\uparrow u_\downarrow + u_\downarrow u_\uparrow}{\sqrt{2}} \right) d_\uparrow$$

2.

$$\mu_n = -\tfrac{1}{3}\mu_u + \tfrac{4}{3}\mu_d$$

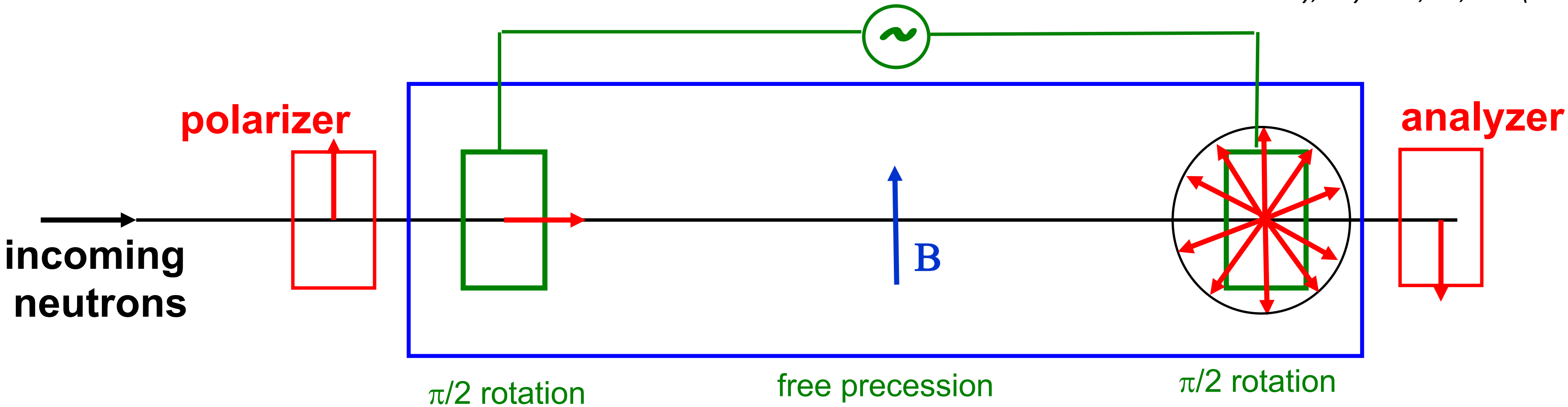
$$\mu_p = -\tfrac{1}{3}\mu_d + \tfrac{4}{3}\mu_u$$

3.

$$\boxed{\frac{\mu_n}{\mu_p} = -\frac{2}{3}}$$

Method of Separated Oscillatory Fields

*N.F.Ramsey, Phys Rev, **76**, 996 (1949)*



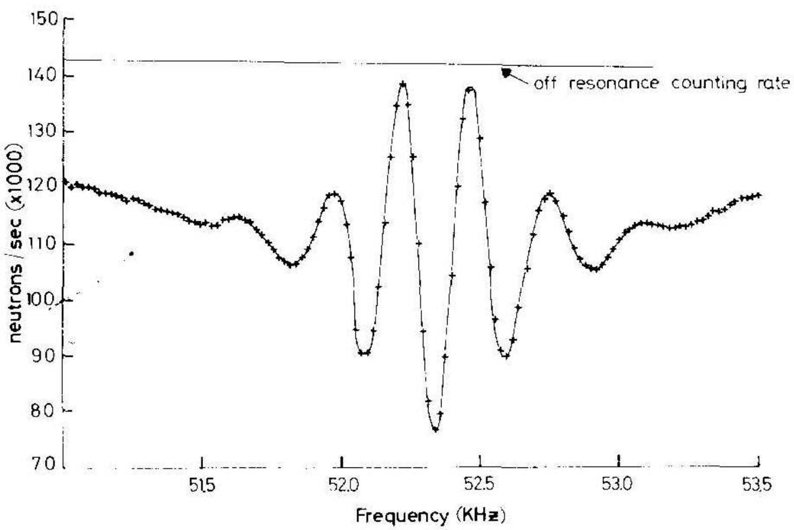


Fig. 1. Neutron resonance.

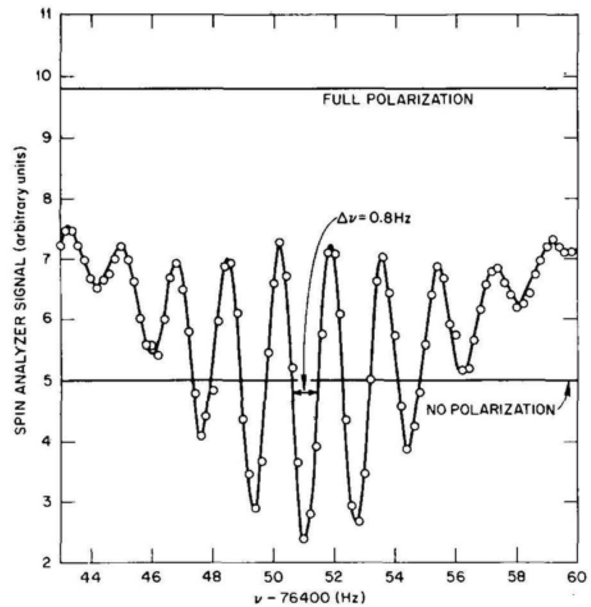


Fig. 2. Proton resonance.

$$\boxed{\frac{\mu_n}{\mu_p} = -0.68497935(17)}$$

*G. L. Greene, et. al. Physics Letters, **71B**, 297 (1977)*

WHY IS THE AGREEMENT SO GOOD?

$$\boxed{\frac{\mu_n}{\mu_p} = -0.68497935(17)}$$ **experiment**

vs.

$$\boxed{\frac{\mu_n}{\mu_p} = -0.67}$$ **theory**

Polarized electron, proton, and muon scattering experiments on H, D and ^{3}He indicate that only 20-30% of the nucleon spin comes from the intrinsic spin of the quarks.

The spin structure of the nucleon is one of the outstanding problems at the interface between nuclear and particle physics.

Over the past 20 years more than 1000 theoretical papers have been published and major experiments have been carried out at practically all major accelerator laboratories.

The work is ongoing…

The Neutron Electric Dipole Moment

Wait for Lecture 4

End of Lecture 1

Fundamental Neutron Physics I

Neutron Properties

Geoffrey Greene
University of Tennessee / Oak Ridge National Laboratory

Why focus on the neutron?

The neutron exhibits much of the richness of nuclear physics, but is vastly simpler, and thus more interpretable, than complex nuclei.

The neutron can be used to probe Strong, Weak, EM and Gravitational phenomena.

Neutron decay is the archetype for all nuclear beta decay and is a key process in astrophysics.

The neutron is well suited as a laboratory for tests of physics beyond the Standard Model.

Overview of the Lectures

I Introduction to the basic physics of the neutron as a particle

II Overview of neutron facilities and techniques

III The Neutron Electric Dipole Moment

IV Neutron beta decay

V Some Selected Additional Projects (time permitting)

Some General References

Fermi, *Lecture Notes on Nuclear Physics*

Byrne, *Neutrons, Nuclei and Matter*

Golub, Lamoreaux, Richardson, *Ultracold Neutrons*

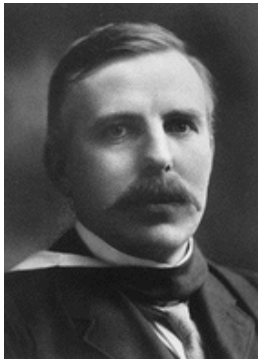
Commins and Bucksbaum, *Weak Interactions of Quarks and Leptons*

Particle Data Group, **pdg.lbl.gov**

Acknowledgements for images

Mike Snow, Fred Weitfeldt, Brad Fillipone, Jeff Nico, Paul Huffman, Sam Werner, Pieter Mumm, Scott Dewey, Chris Crawford, Harmut Abele, Yuri Kamyskov, Tim Chupp, Brad Filippone, Nadia, Fomin,…

Historical Introduction



Ernest Rutherford

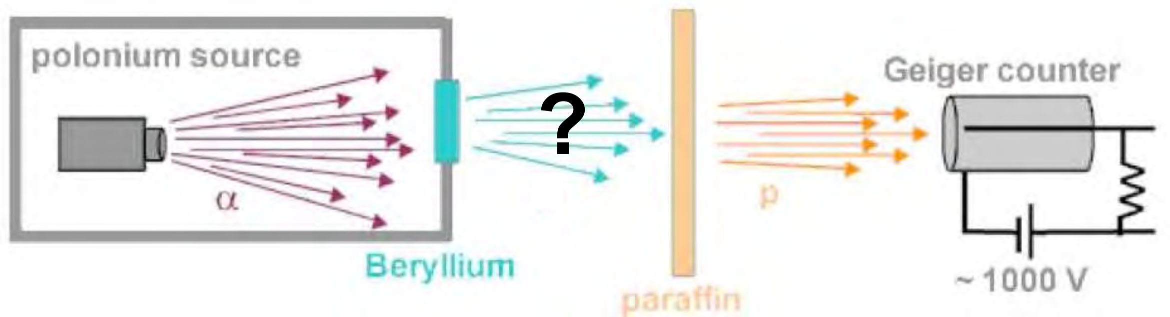
1920 ***Noting that atomic number (Z) does not correspond to atomic weight, Rutherford suggests that, in addition to "bare" protons, the nucleus contains some tightly bound "proton-electron pairs" or neutrons.***

Walter Bothe

1930 ***Bothe and Becker discovered a penetrating, neutral radiation when alpha particles hit a Be target.***

1931 ***Mme Curie shows that they are not gamma rays and they have sufficient momentum to eject p's from paraffin.***

Irene Curie

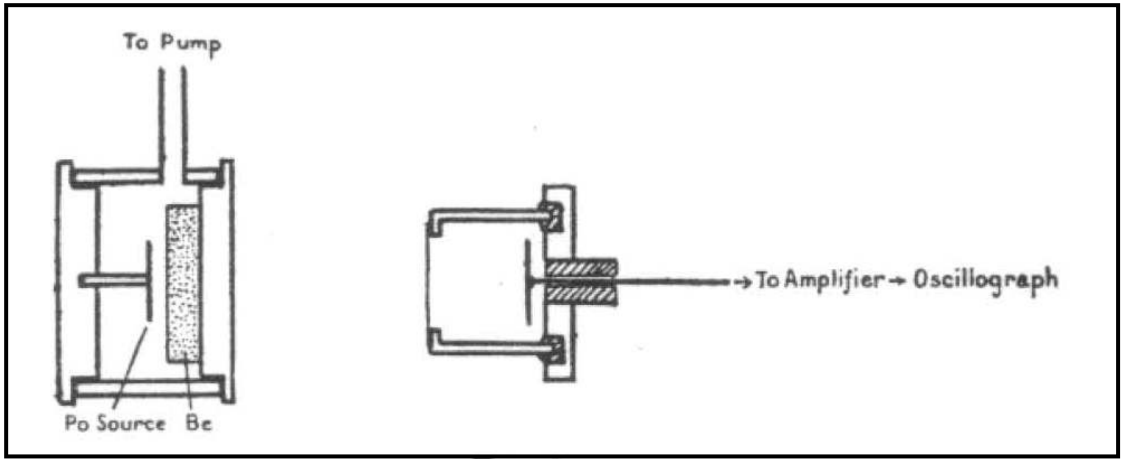


$$\alpha + {}^{9}Be \rightarrow {}^{12}C + n$$

1932 ***Chadwick replaced the paraffin with a variety of other targets and, by measuring the recoil energies of the ejected particles was able to determine the mass of the neutral particle***

James Chadwick

$$M = 1.15 \pm \sim 10\%$$



J. Chadwick, Proc. Roy. Soc., A ***136*** *692 (1932)*

Chadwick claimed this was Rutherford's "Neutron"

1933 ***Bainbridge makes precision measurements of the atomic masses of the proton and the deuteron using the mass spectrograph***

1934 ***Chadwick and Goldhaber make the first "precision" measurement of the neutron mass by looking at the photo-disassociation of the deuteron***

$$h\nu + {}_1D^2 \rightarrow {}_1H^2 + {}_1n^0$$

Using 2.62MeV gammas from Thorium and determining the recoil energy of the protons they we re able to determine*:

$$M_n = 1.0080 \pm 0.0005$$

KEY OBSERVATION: $M_n > M_p + M_e$

1. The neutron cannot be a bound "proton-electron pair"

2. It is energetically possible for a neutron to decay to e + p

**Chadwick and Goldhaber, Nature, 134 237 (1934)*

Some Neutron Properties

Mechanical Properties

Mass

Gravitational Mass (equivalence principle test)

Spin

Electromagnetic Properties

Charge (limit on neutrality)

Magnetic Dipole Moment

Electric Dipole Moment

Internal Charge Distribution

~~*Higher Moments (Quadrupole,Octupole,..)*~~

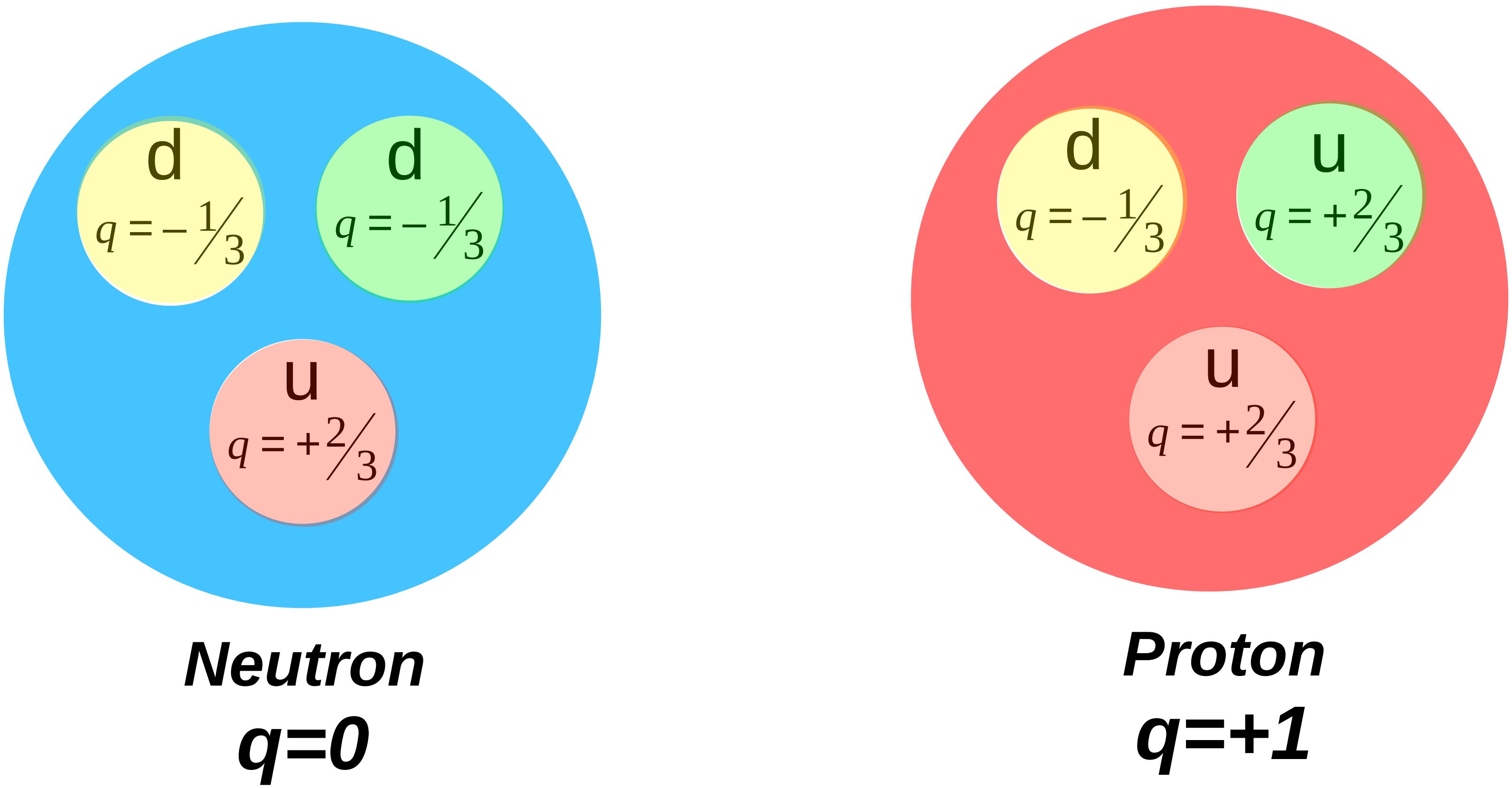
Miscellaneous Quantum Numbers:

Intrinsic Parity (P), Isospin (I), Baryon Number (B), Strangeness (S), …

The Neutron Mass

The "Naive" Quark Model

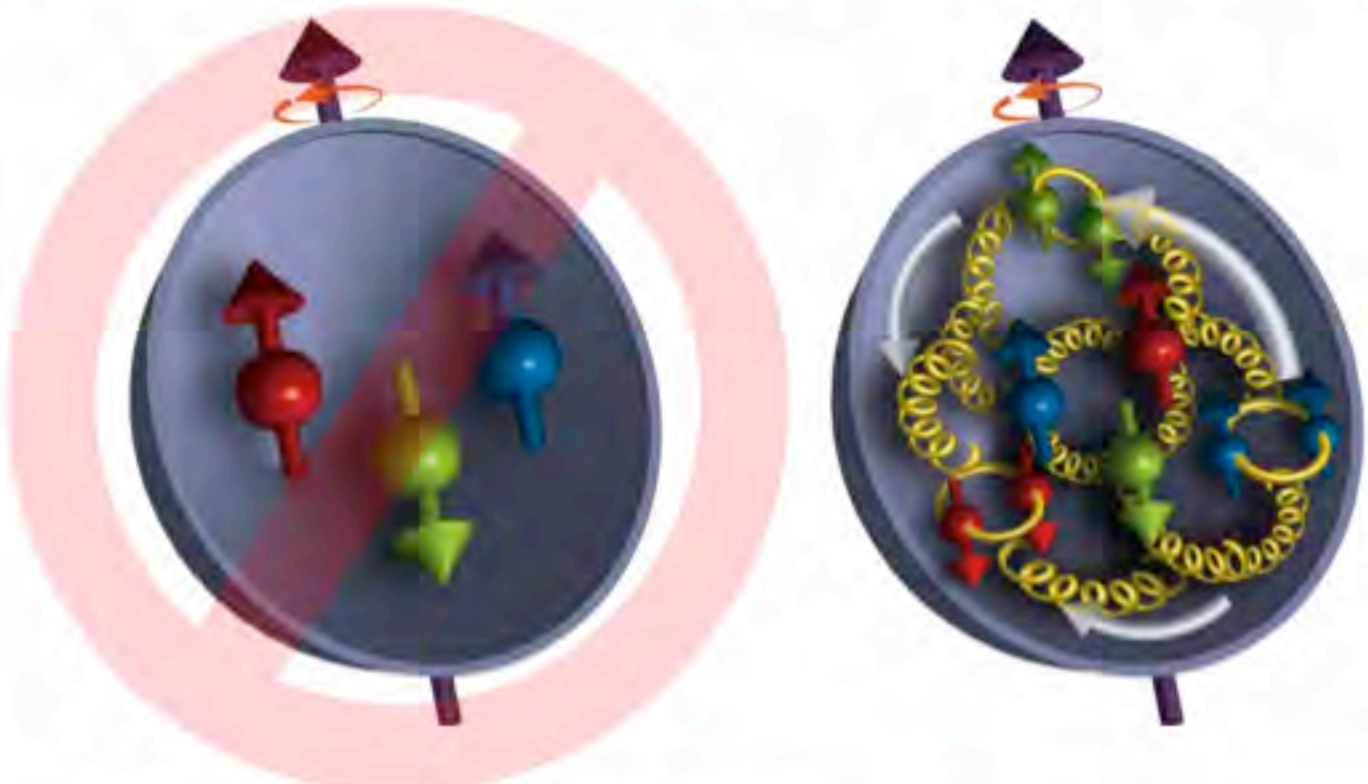
Nucleons are color triplets of spin ½ Quarks



Electrostatic self-energy suggests: $m_p > m_n$?

THE NUCLEON IS COMPLICATED!

Nucleons are NOT simply composed of 3 valence quarks.
There are gluons, virtual strange quarks,…lots of stuff…



~99% of the nucleon mass arises from the self-energy of the gluon field

Image: BNL

Ab initio calculation of the neutron-proton mass difference

Sz. Borsanyi,[1] S. Durr,[1,2] Z. Fodor,[1,2,3*] C. Hoelbling,[1] S. D. Katz,[3,4] S. Krieg,[1,2] L. Lellouch,[5] T. Lippert,[1,2] A. Portelli,[5,6] K. K. Szabo,[1,2] B. C. Toth[1]

Science **347**, 1452 (2015)

	ΔM_{TOTAL}	ΔM_{QCD}	ΔM_{QED}	(MeV)
Theory	1.51(28)	2.51	-1.00	
Experiment	1.30			

Determination of the Neutron Mass

The best method for the determination of the neutron mass considers the reaction:

$$n + p \rightarrow d + \gamma$$

and measures two quantities with high accuracy:

1. A gamma ray energy

The actual experiment is an absolute determination of the 2.2MeV gamma ray wavelength in terms of the SI meter.

2. A mass difference

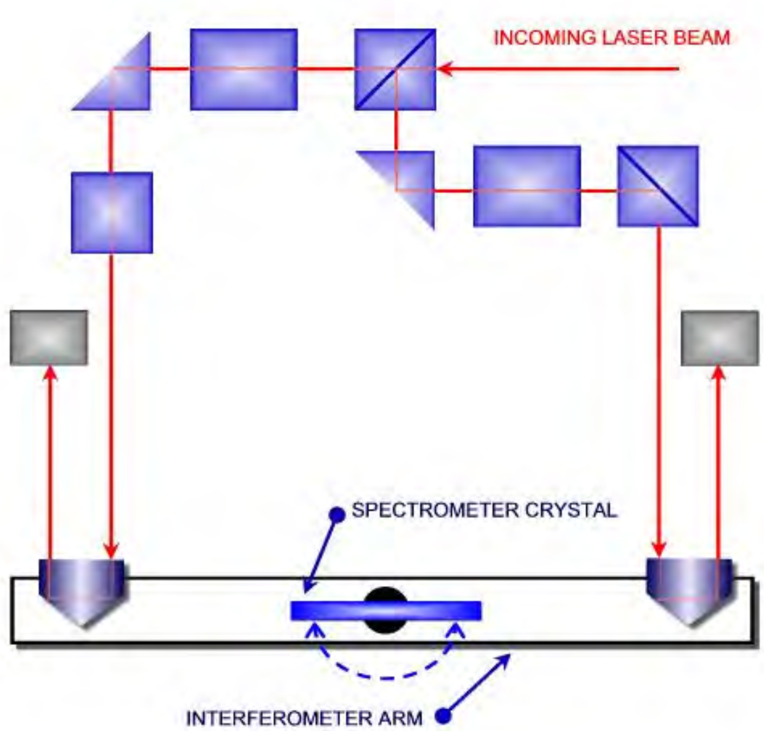
The actual experiment is the determination of the D - H mass difference in atomic mass units.

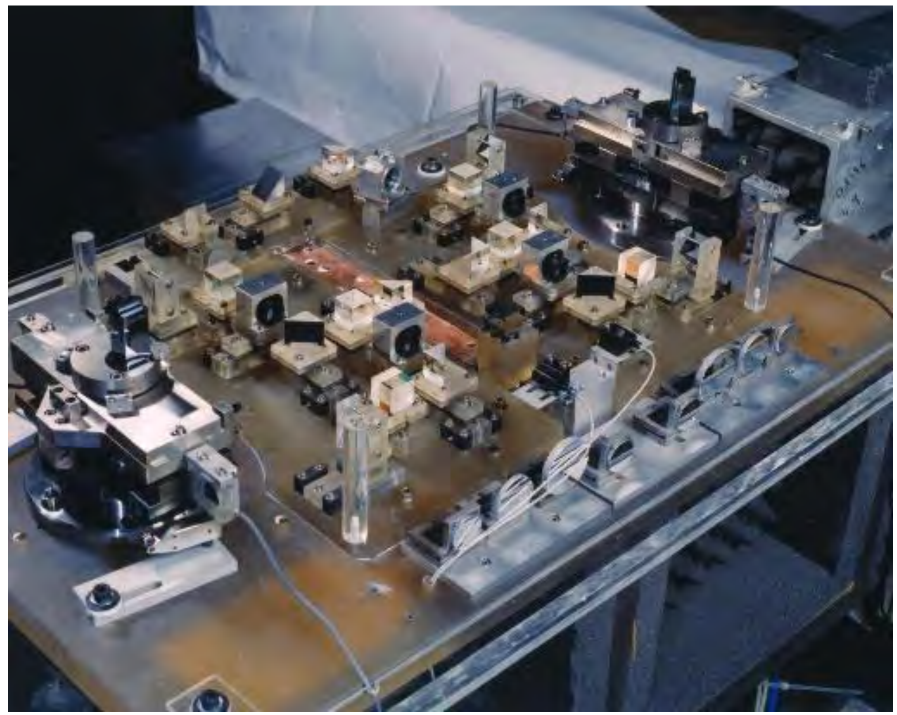
Absolute Measurement of 2.2Mev n-p Capture Gamma Energy

Measure Bragg angle for diffraction of 2.2MeV gamma from a perfect single crystal of Silicon with an accurately measured lattice spacing d.

$$n\lambda^* = 2d\sin\theta \qquad E_\gamma = h\nu = \frac{hc}{\lambda^*}$$

Bragg Angle is a few milli-radian
Need nano-radian precision!

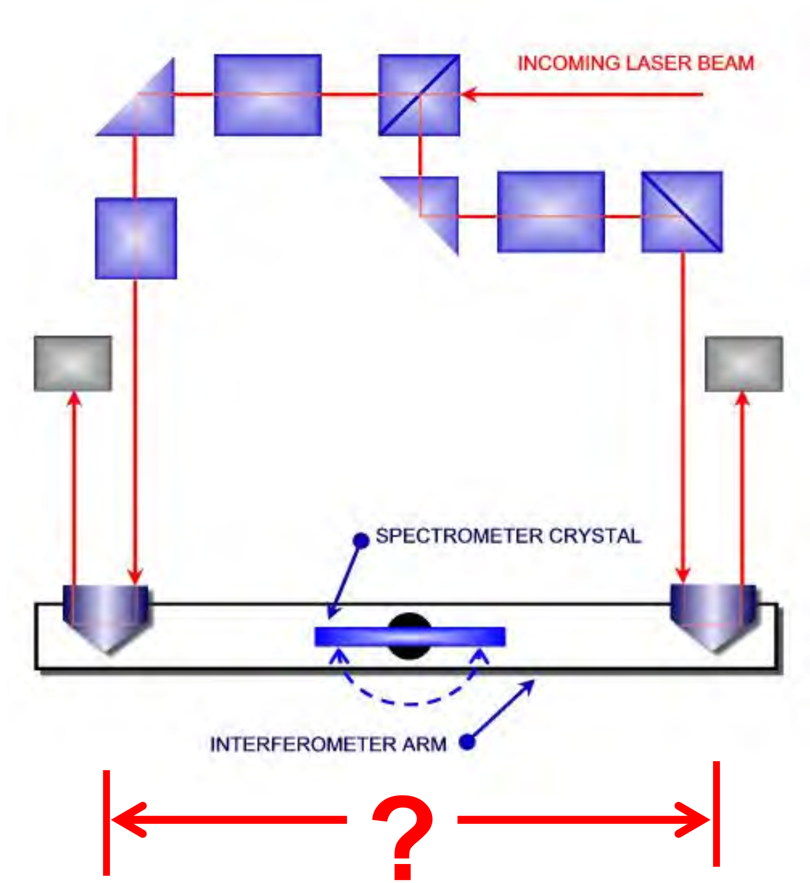




See: http://www.ill.fr/YellowBook/PN3/

Precision vs. Accuracy

Angle Interferometer gives high precision but what about its "calibration"

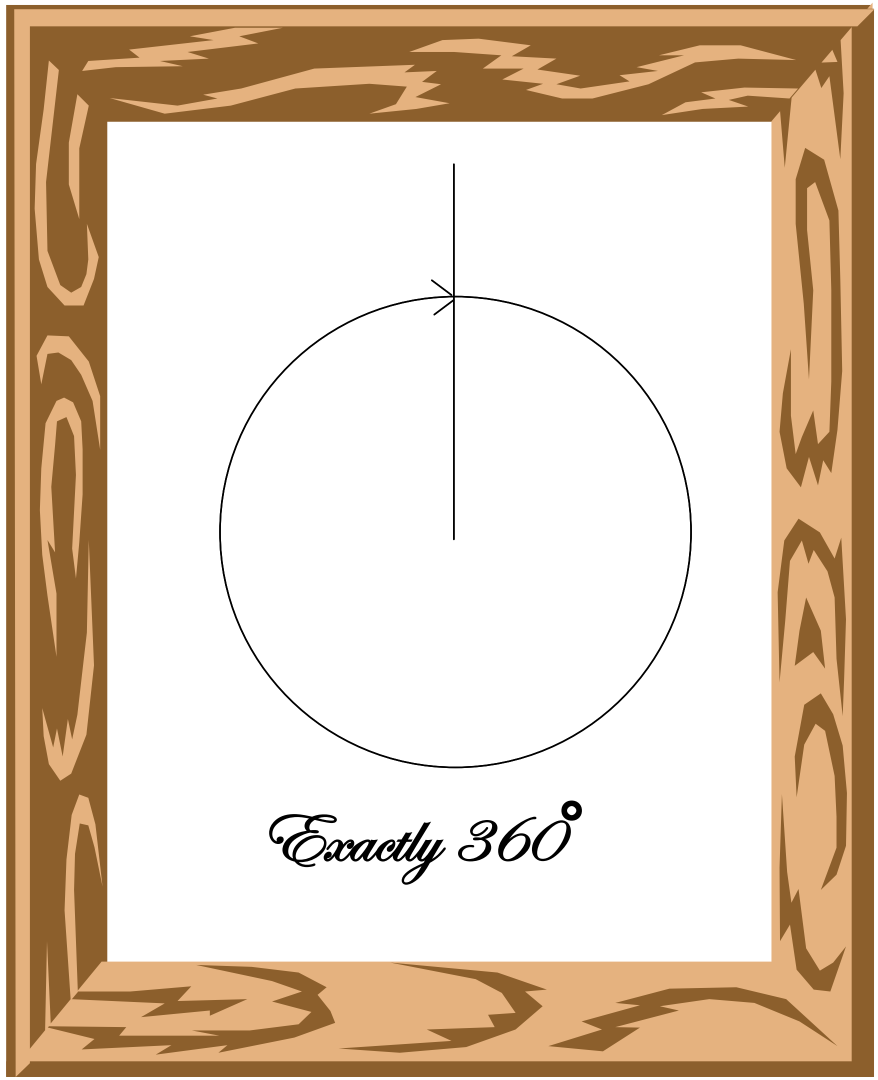


What can we use to calibrate a precision angle device?
Is there a "Standard" for angle measurement?

See: http://www.ill.fr/YellowBook/PN3/

The NIST Standard for the Kilogram
(At least until May 2019)

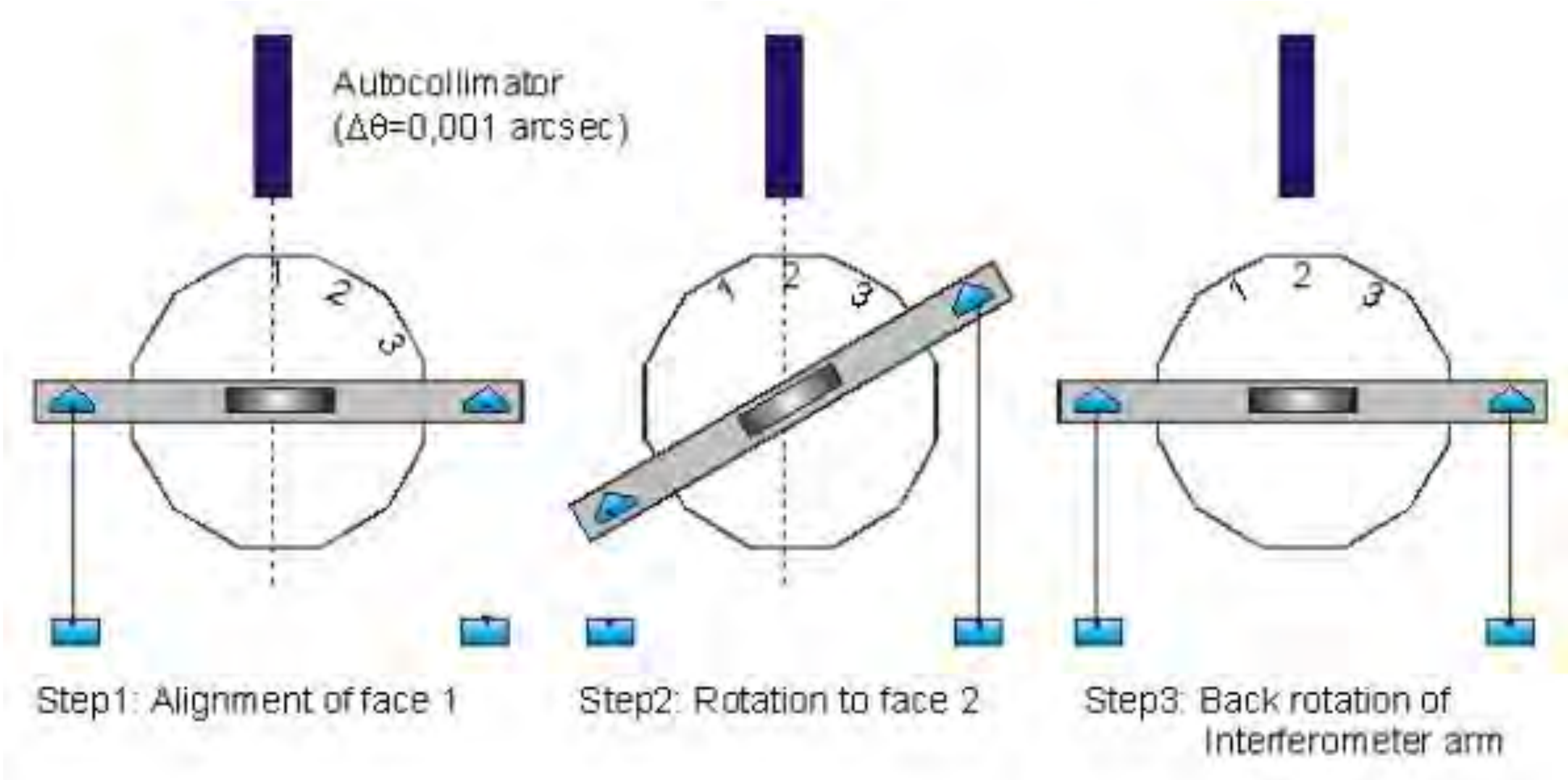
What is the NIST Standard for angles?



The NIST Standard Angle

Absolute Measurement of 2.2Mev n-p Capture Gamma Energy

Step 3: Calibrate Angle interferometer



Measure 24 interfacial angles of a precision quartz optical polygon
Since they must sum to 360°, there are only 23 independent quantities. A 24 parameter fit can give the calibration constant.

See: http://www.ill.fr/YellowBook/PN3/

Determination of the Neutron Mass

$$M(n) = 939.5654133(58)\ MeV$$

E. G. Kessler, et. al., Phys Lett A, 255 (1999)
G.L Greene, et. al., Phys. Rev. Lett. 24, 819 (1986)

The Neutron's Gravitational Mass

Equivalence Principle Test with Neutrons

The measurement of the neutron mass represents a determination of the neutron's INERTIAL mass. To determine the neutron's GRAVITATIONAL mass, one must compare the free fall acceleration of the neutron with the acceleration g of macroscopic test masses:

$$F_n = m_i \, a_n$$

$$m_g \, g = m_i \, a_n$$

$$m_g \, / \, m_i = a_n \, / g \equiv \gamma$$

Falling Neutrons

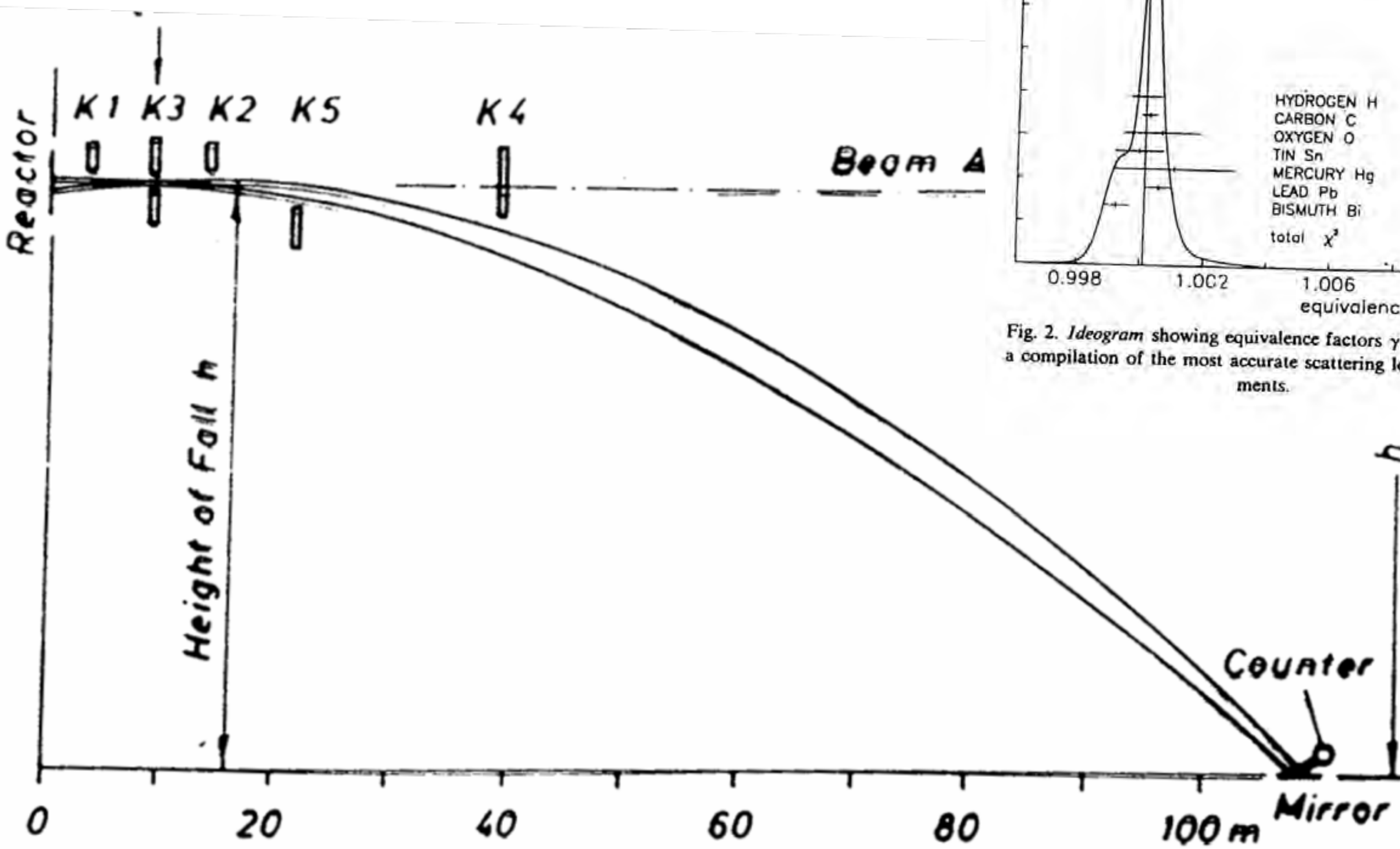


Fig. 1. Principle of the neutron-gravity refractometer

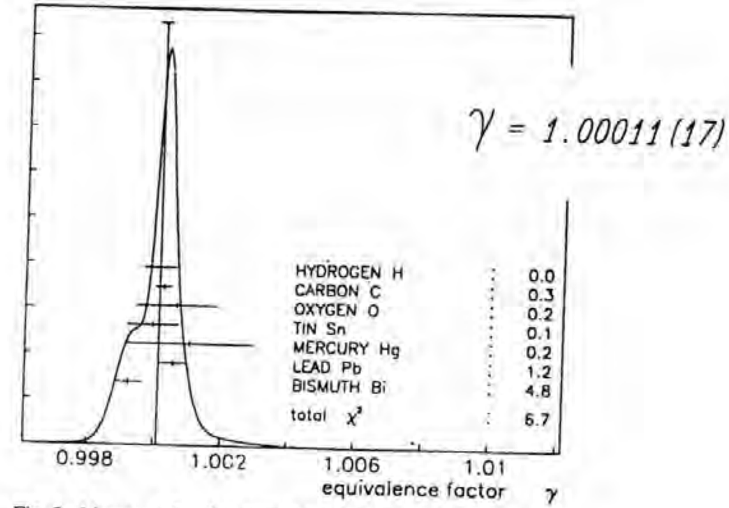


Fig. 2. *Ideogram* showing equivalence factors γ obtained from a compilation of the most accurate scattering length measurements.

For review see: Schmiedmayer, NIM ***A284****, 59 (1989)*

Falling Neutrons

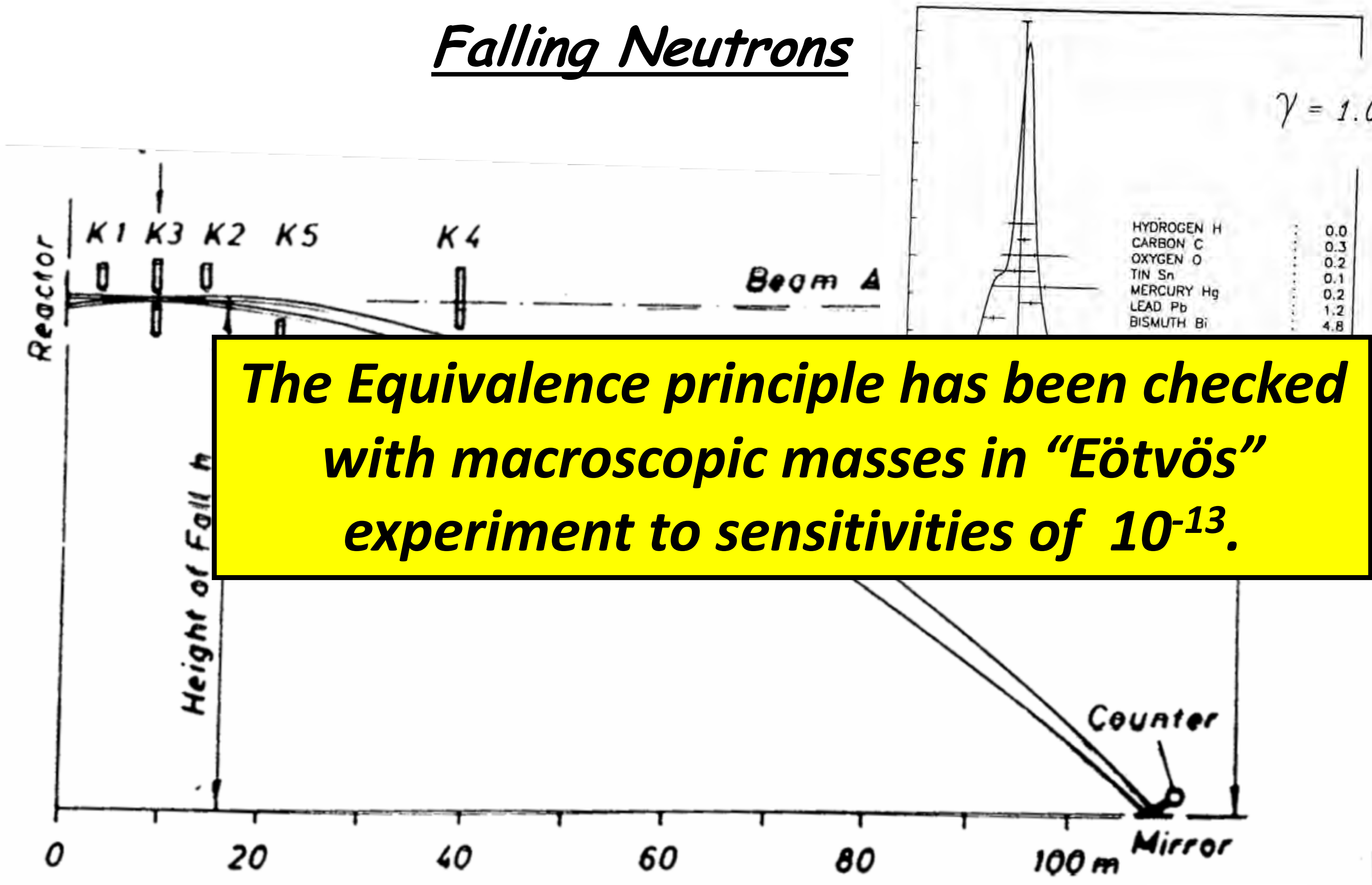


Fig. 1. Principle of the neutron-gravity refractometer

The Equivalence principle has been checked with macroscopic masses in "Eötvös" experiment to sensitivities of 10^{-13}.

*For review see: Schmiedmayer, NIM **A284**, 59 (1989)*

*"Quantum Bouncing Ball" **

$$V(x) = \begin{cases} \infty & \text{for} \quad 0 > x \\ mgh & \text{for} \quad x \geq 0 \end{cases}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + mgx\psi = E\psi$$

This differential equation is not soluble in terms of elementary functions. However the equation:

$$\frac{d^2 y}{dx^2} - yx = 0$$

has been extensively studied. As expected there are two linearly independent solutions. These are known as "Airy" functions:

$$Ai(x) \text{ and } Bi(x)$$

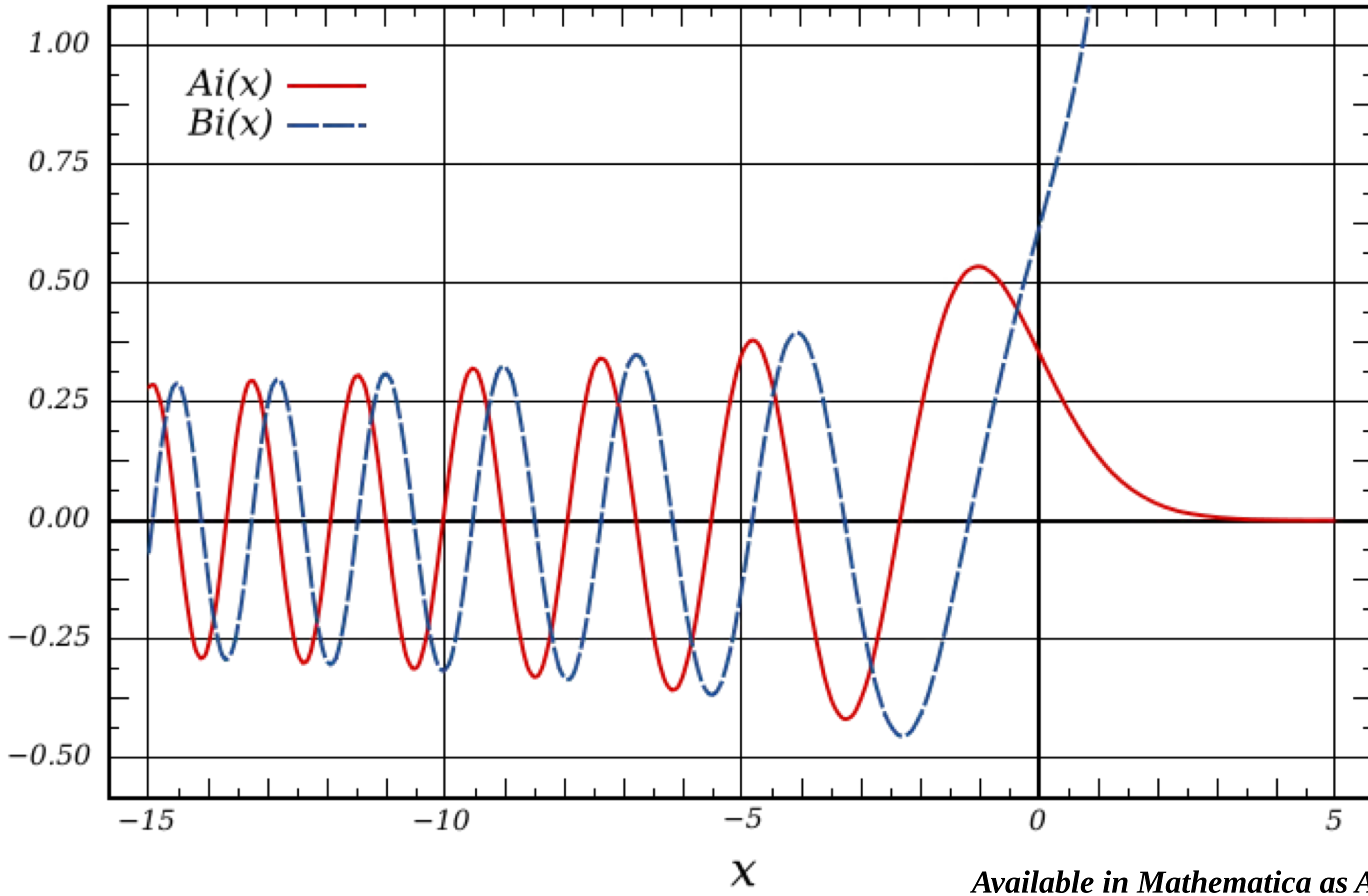
***Nezvizhevsky, et al, Nucl Instrum and Meth, A440, 754 (2000)]**
Nezvizhevsky, et al , Nature 415, 297 (2002)*

Airy Functions

$$Ai(x) = \frac{1}{\pi}\int_0^\infty \cos\left(\frac{t^3}{3} + xt\right) dt$$

$$Bi(x) = \frac{1}{\pi}\int_0^\infty \left[\exp\left(\frac{t^3}{3} + xt\right) + \sin\left(\frac{t^3}{3} + xt\right)\right] dt$$

George Airy
1801-1892



Available in Mathematica as AiryAi[z] and AiryBi[z]

Solution to Schrödinger's Equation $$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}+mgx\psi=E\psi$$

Stationary States:

$$\psi_n = A_n Ai\left(\frac{x}{x_0}+\alpha_n\right)\ \text{for } x>0$$

is the nth stationary state

$$x_0=\left(\hbar^2 \big/ 2m^2 g\right)^{1/3}$$

is the gravitational scale height

$$\alpha_n$$

is the nth zero of the Airy function $Ai(x)$

(note α_n are all negative)

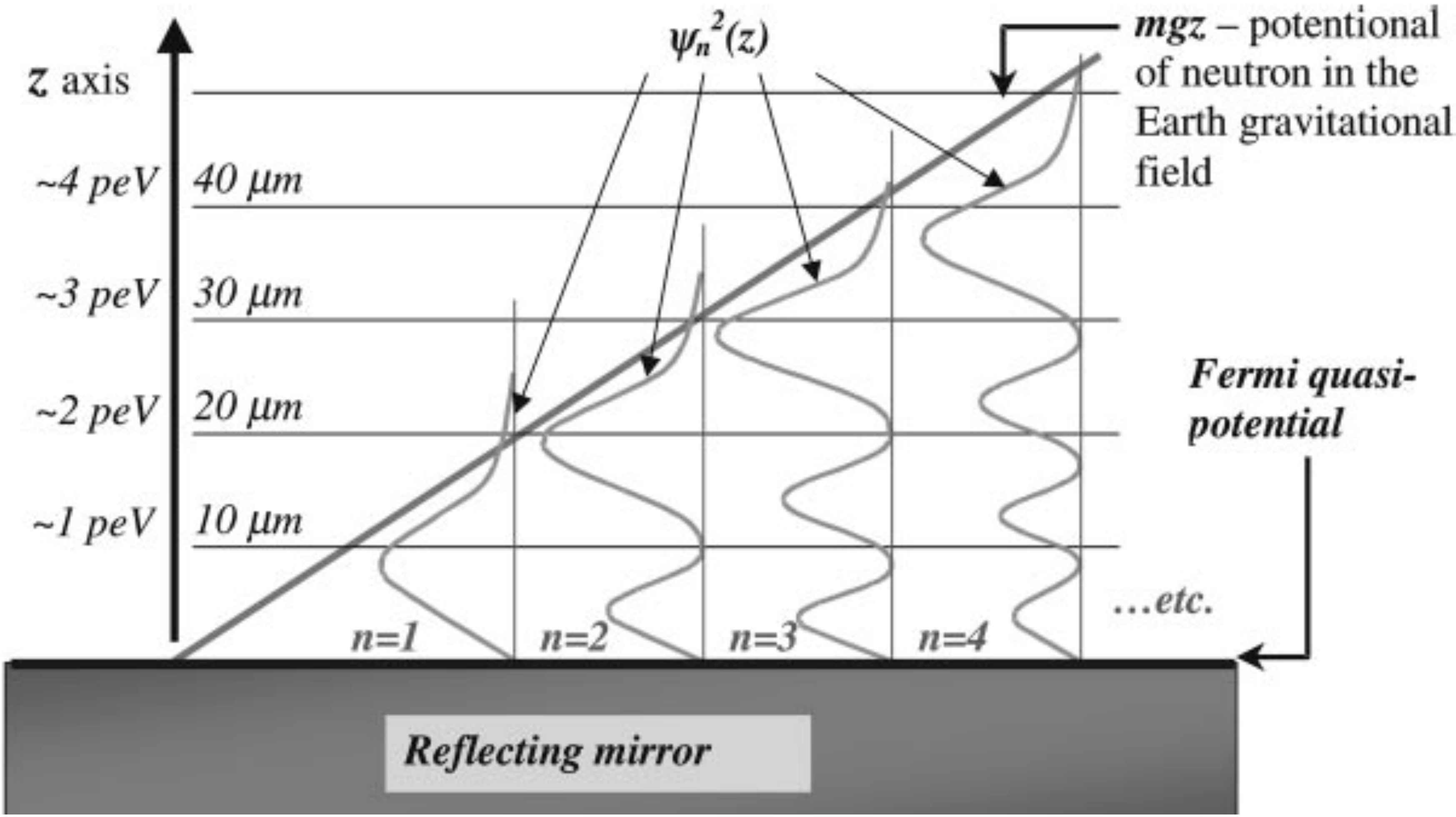
$$A_n=\frac{1}{\sqrt{x_0\int_{\alpha_n}^{\infty}Ai(\xi)^2\,d\xi}}$$

is the normalization constant

Energy Levels are given by the roots of $Ai(x)$

$$Ai\left(-\frac{\sqrt[3]{2}}{\sqrt[3]{mg^2\hbar^2}}E_n\right)=0$$

The Quantum "Bouncing Neutron"



$$E_1 = 1.44 \times 10^{-12}\, eV$$

$$E_2 = 2.53 \times 10^{-12}\, eV$$

$$E_3 = 3.42 \times 10^{-12}\, eV$$

$$E_4 = 4.21 \times 10^{-12}\, eV$$

The Experiment:

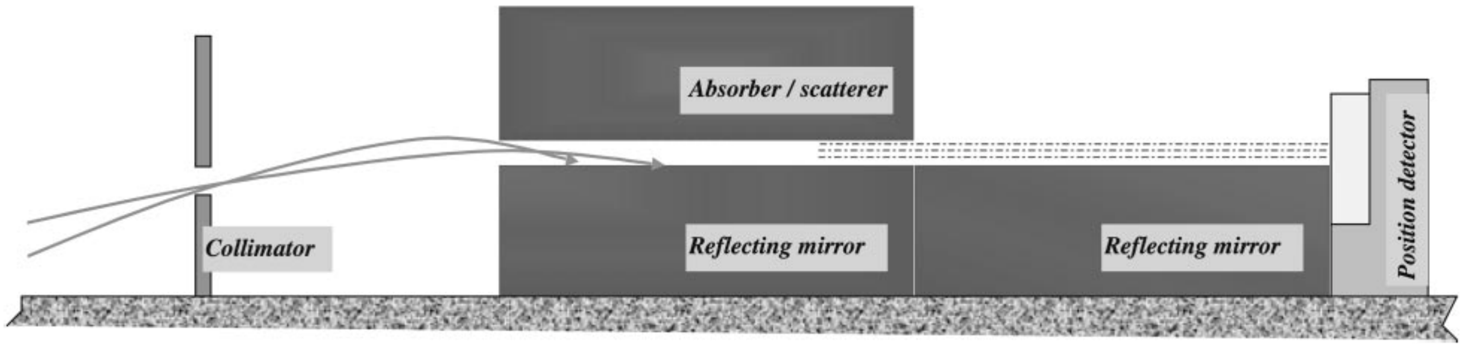


Fig. 3. General scheme of the experiment.

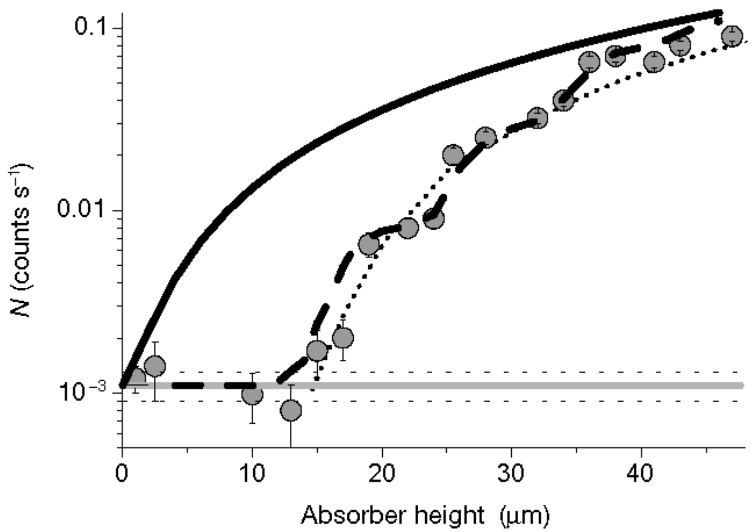
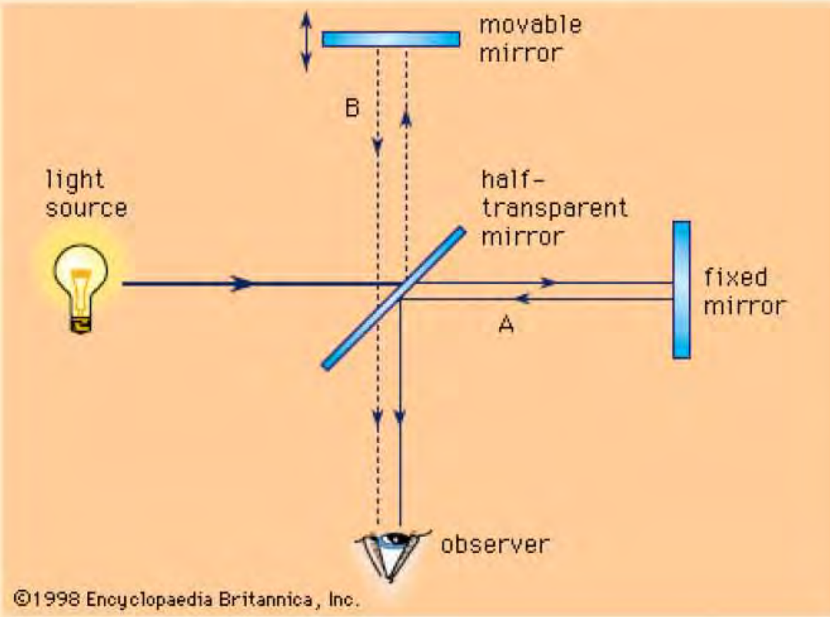


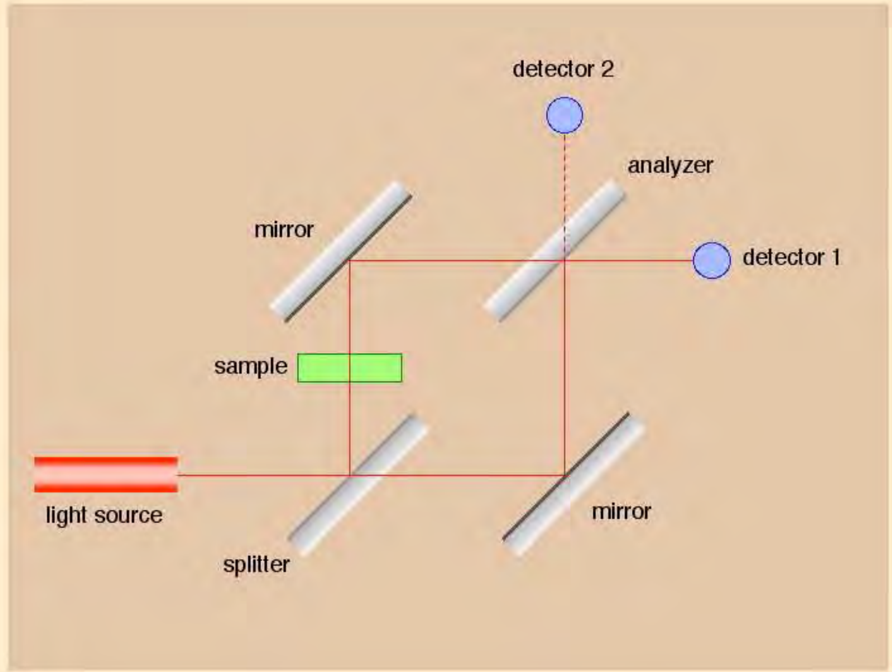
Figure 4 The neutron throughput versus the absorber height at low height values. The data points are summed up in intervals of 2 μm. The dashed curve corresponds to a fit using the quantum-mechanical calculation, in which all level populations and the height resolution are fitted from the experimental data. The solid curve is again the full classical treatment. The dotted line is a truncated fit in which it is assumed that only the lowest quantum state—which leads to the first step—exists.

Neutron Interferometry

Slides Courtesy of Fred Weitfeldt

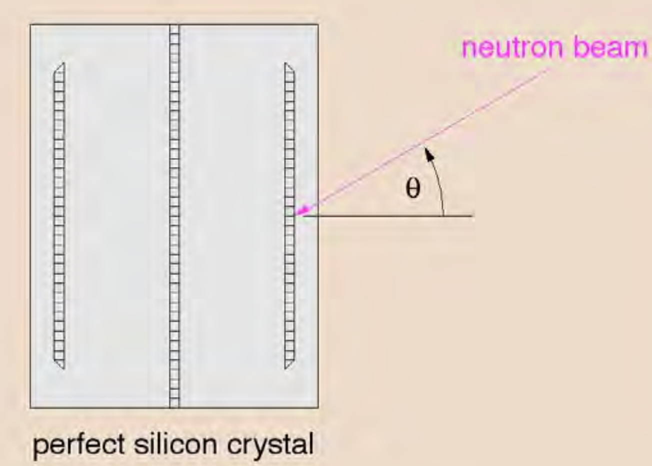
Michelson Interferometer





Mach-Zender Interferometer

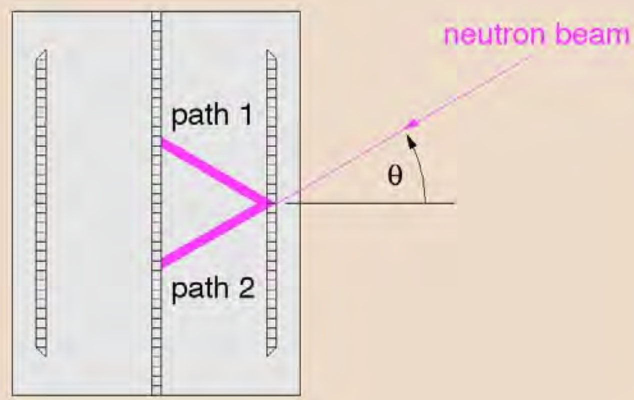
Perfect Crystal LLL Neutron Interferometer



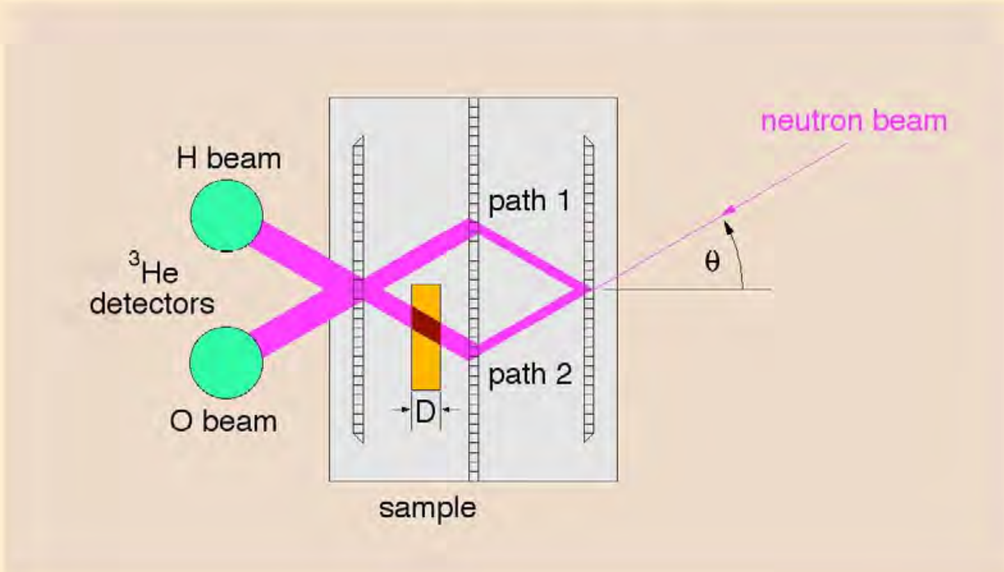
Bragg condition: $n\lambda = 2d\sin\theta$

d = lattice spacing

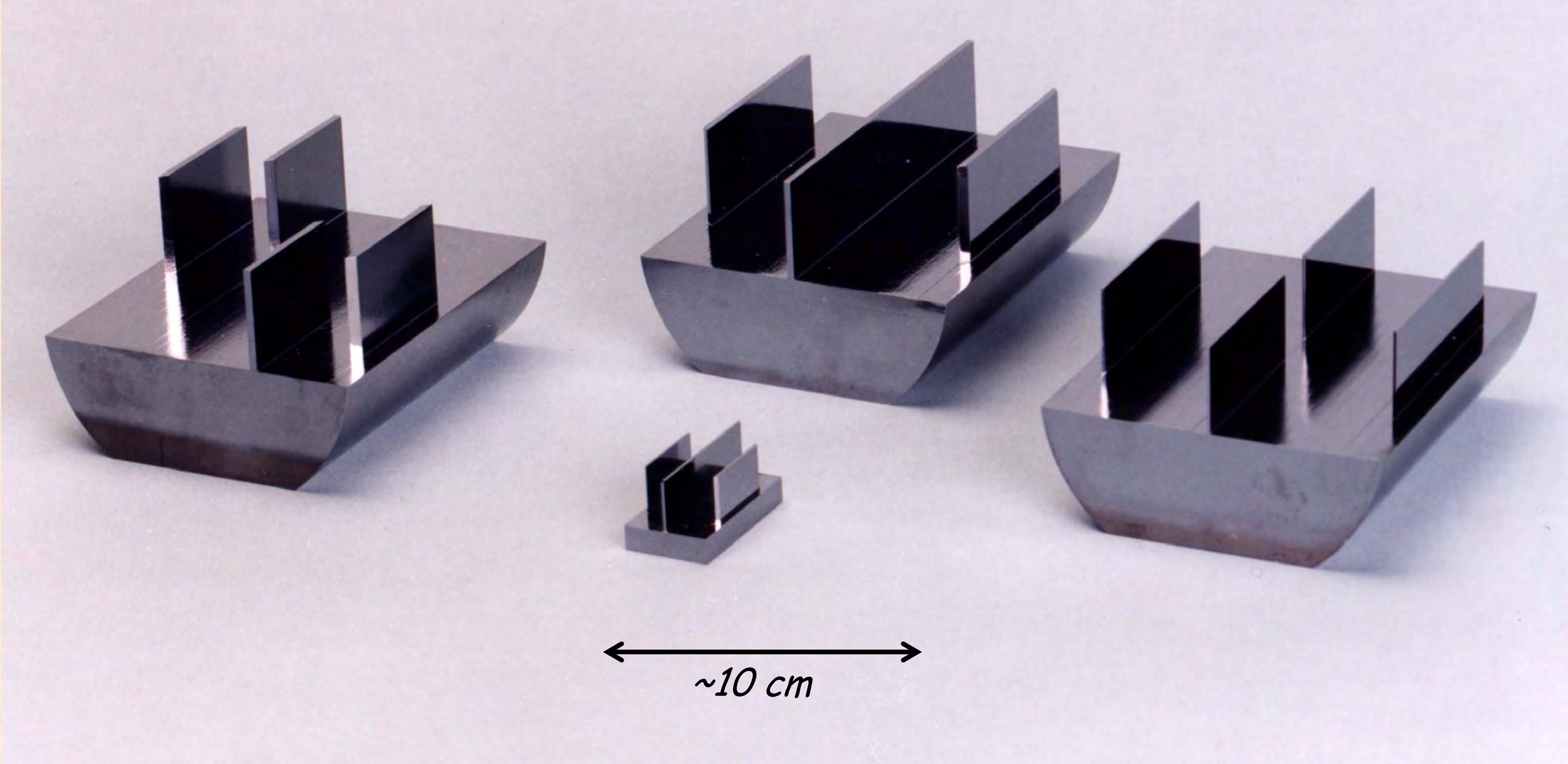
Perfect Crystal LLL Neutron Interferometer



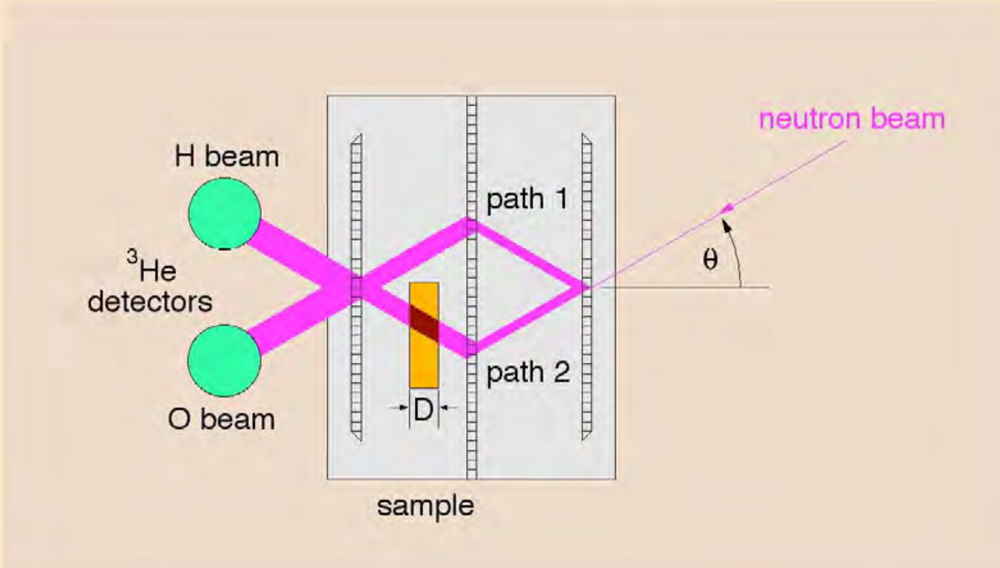
Nuclear Phase Shift

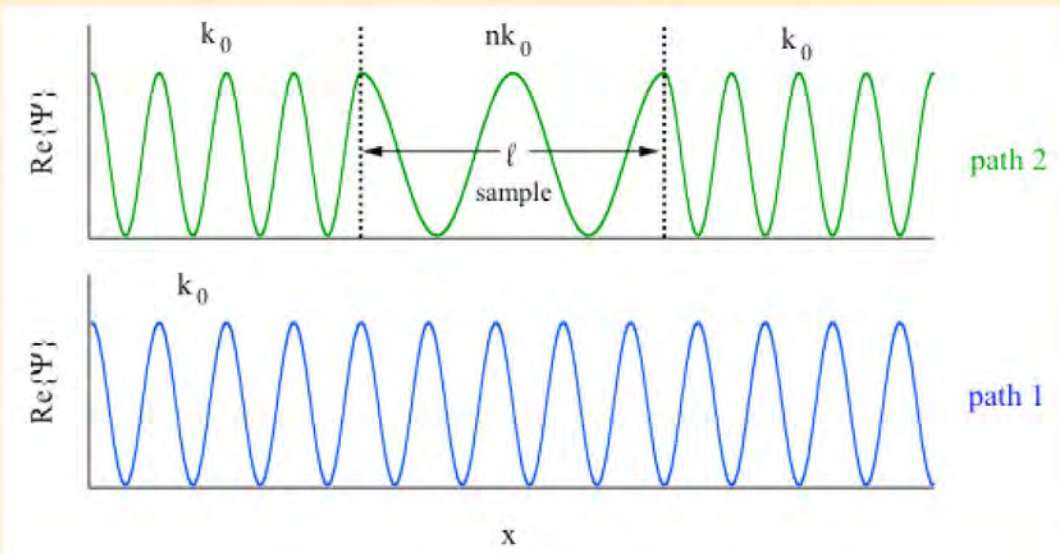


Perfect Crystal Silicon Interferometers



Nuclear Phase Shift

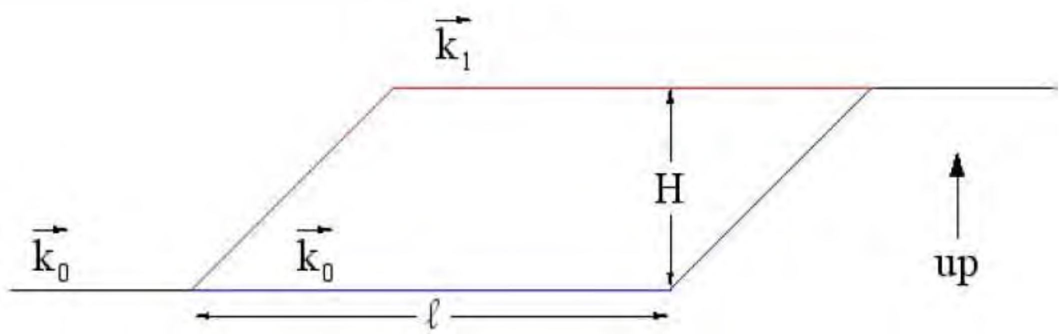




Introduce a Phase Shift into one leg. Could be:

1. **Electromagnetic due to magnetic moment**
2. **Nuclear Force due to coherent scattering amplitude**
3. ***GRAVITY***

Quantum Mechanical Gravitational Phase Shift



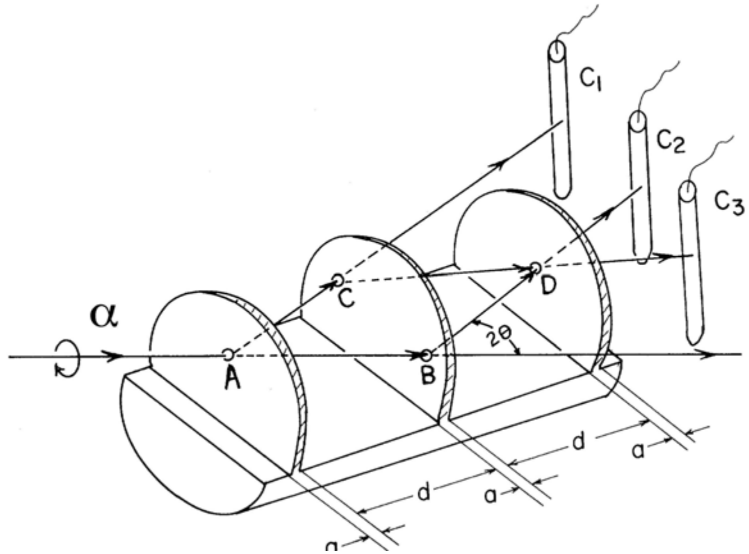
$$\Delta\phi = \frac{2\pi\lambda g A}{h^2} m_i m_g$$

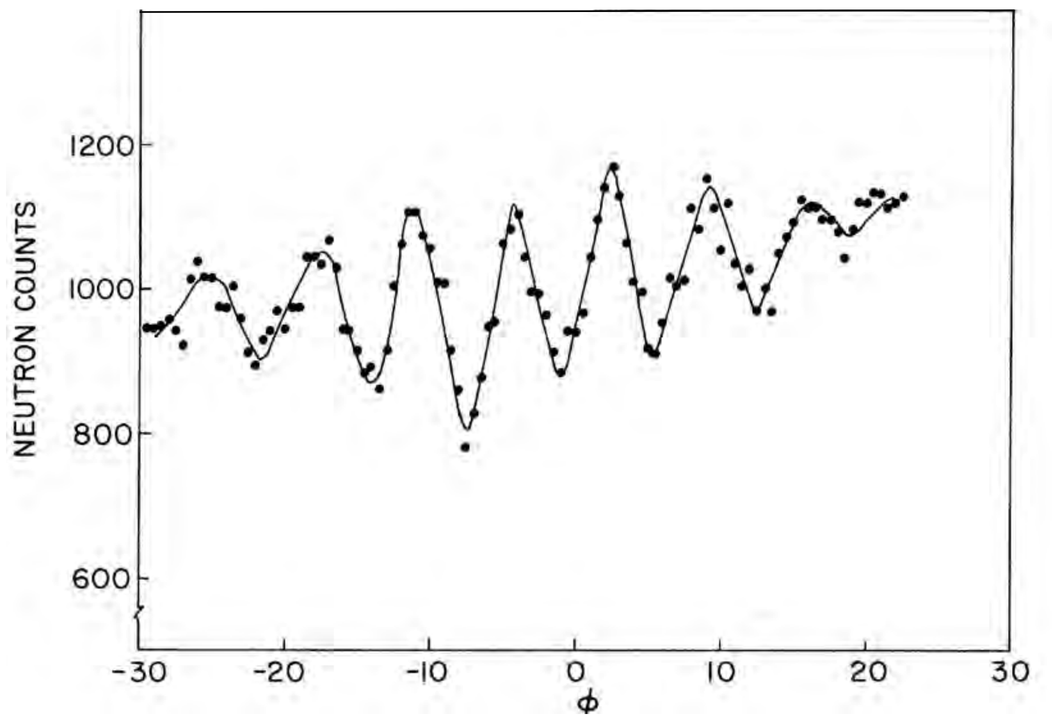
A=Hl = Area of parallelogram

m_i = neutron inertial mass
m_g = neutron gravitational mass

Test of the Weak Equivalence Principle in the Quantum Limit

The Collela-Overhauser-Werner Experiment COW





*Collela, Overhauser, Werner, PRL **34**, 1472 (1975)*

…

…

*Littrell, Allman, Werner, Phys Rev **A56**, 1767 (1997)*

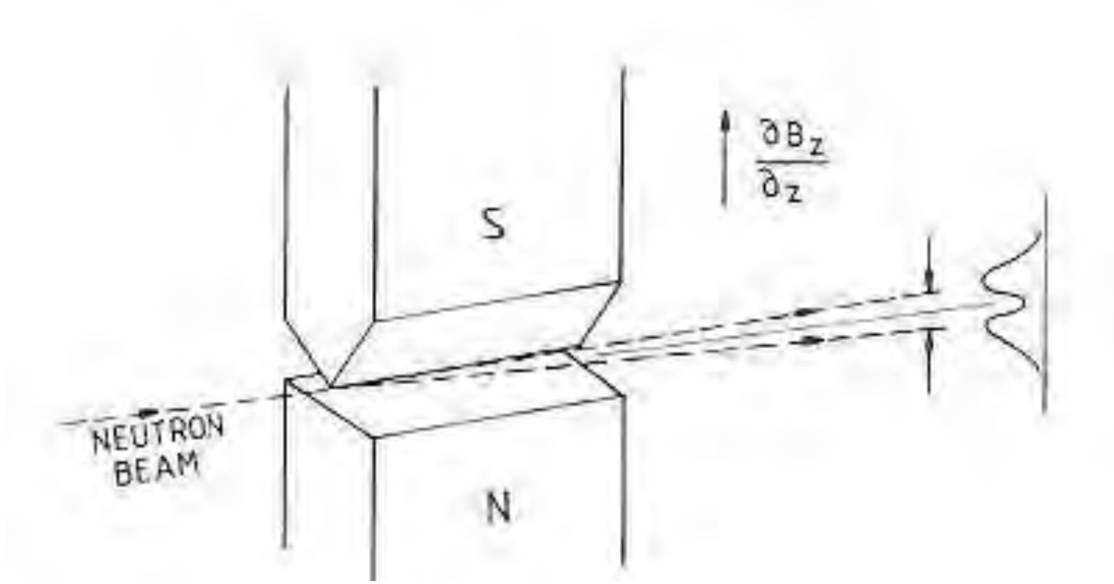
The Neutron Spin

The Neutron has an Intrinsic Spin of $s=\frac{1}{2}$

1934 ***Schwinger concludes that s=½ based on the band spectrum of molecular D_2 and the scattering of neutrons from ortho and para H_2.***

1949 ***Hughes and Burgey observe the mirror reflection of neutrons from magnetized iron. They observe 2 critical angles definitively showing the neutron has two magnetic sub-levels.***

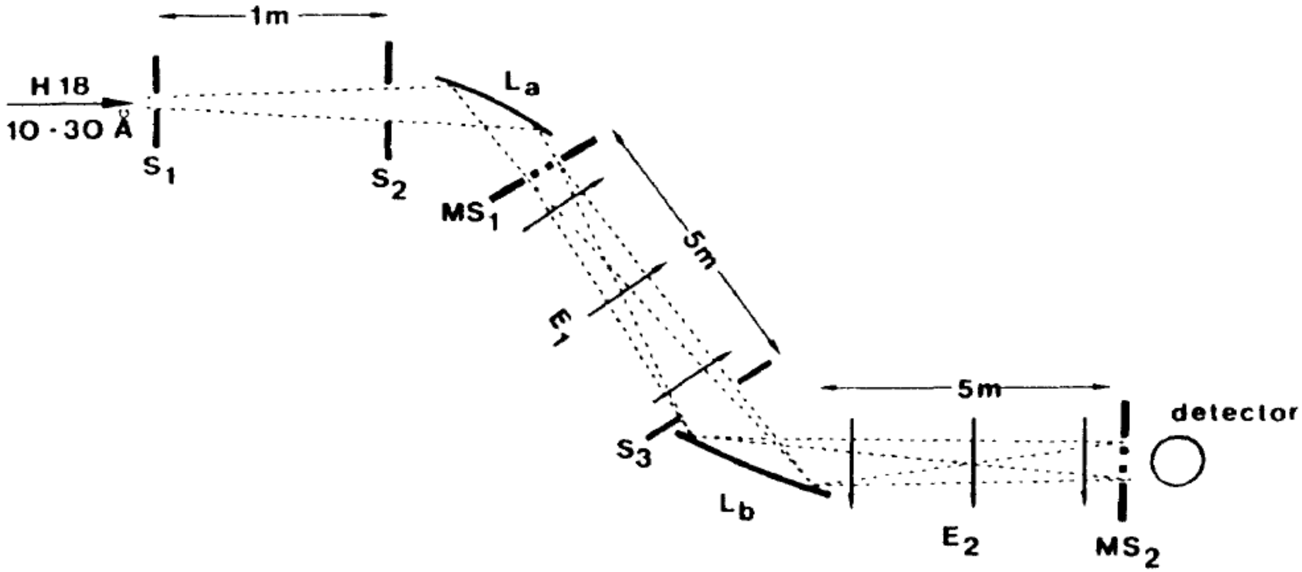
1954 ***Neutron Stern-Gerlach experiment explicitly demonstrates s=½ .***



See also Fischbach, Greene, Hughes, PRL ***66****, 256 (1991) showing* $\vec{L} = \hbar \vec{s}$

The Neutron Charge ?

The Neutron Appears to be Neutral



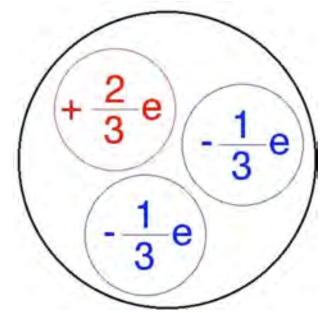
$$\boxed{Q_n = (-0.4 \pm 1.1) \times 10^{-21}\, e}$$

J. Baumann et al., Phys. Rev. D 37, 3107 (1988)

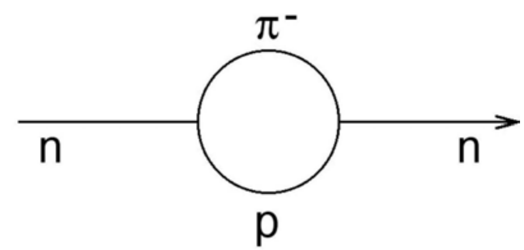
The Neutron Charge Distribution

Neutrality does not Imply Uniformity

The neutron is a composite structure of charged quarks which may be distributed non uniformly within the neutron.



Fermi & Marshall suggested that the neutron should have a positive "core" and a negative "skin" due to virtual pion emission



Neutron Mean Charge Radius:

$$\langle r_n^2 \rangle = \int \rho(r) r^2 dr^3$$

Neutron Electric Scattering Form Factor

The Fourier transform of the neutron charge density $G_E^n\left(Q^2\right)$ is accessible from electron scattering.

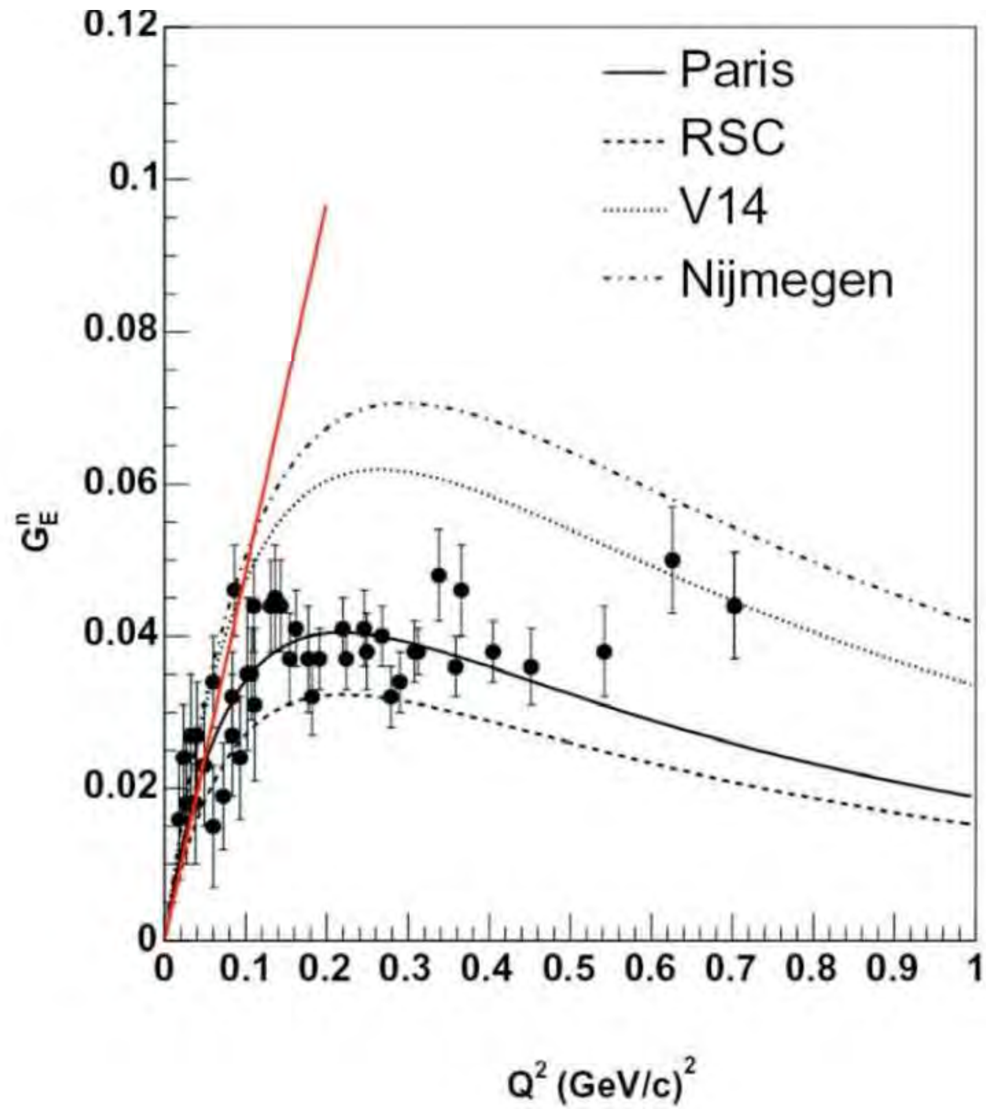
Expanding in the momentum transfer Q^2 :

$$G_E^n\left(Q^2\right) = q_n - \frac{1}{6}\left\langle r_n^2 \right\rangle Q^2$$

In the limit of low Q^2 :

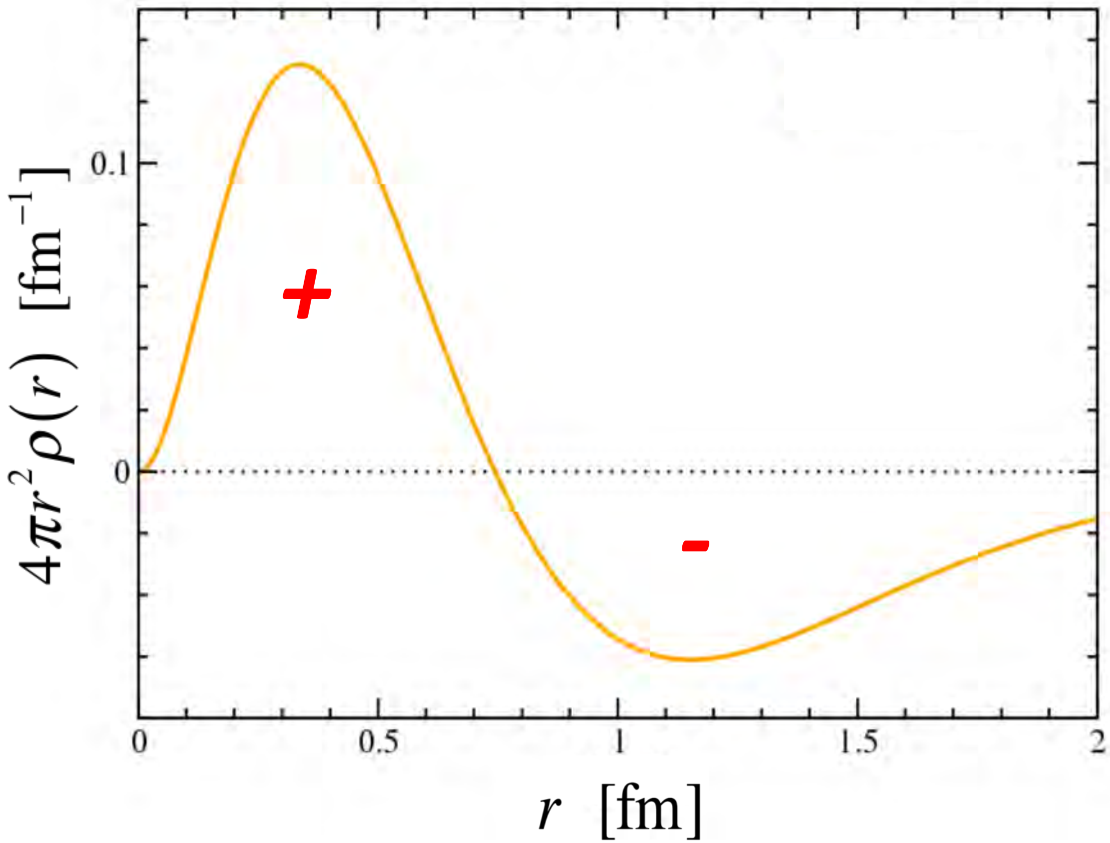
$$\left\langle r_n^2 \right\rangle = -6 \frac{d}{dQ^2} G_E^n\left(Q^2\right)\Big|_{Q^2=0}$$

The Mean Square Neutron Charge Radius $G_E^n\left(Q^2\right)$.
Constrains the Slope In Electron Scattering Experiments.
(e.g. Bates, JLab,…)



V. Ziskin, Ph.D. thesis, 2005

The neutron does have a static charge distribution with a positive "core" and a negative "halo"



G.A. Miller and J. Arrington Phys. Rev. C 78, 032201 (2008)

Experimental Situation is in Disarray

TABLE I. Experimental results of b_{ne} in units of 10^{-3} fm.

Experiment	Target	Result	Reference
Angular scattering	Ar	-0.1 ± 1.8	1947 [7] Fermi
Transmission	Bi	-1.9 ± 0.4	1951 [8] Havens
Angular scattering	Kr, Xe	-1.5 ± 0.4	1952 [9] Hamermesh
Mirror reflection	Bi/O	-1.39 ± 0.13	1953 [10] Hughes
Angular scattering	Kr, Xe	-1.4 ± 0.3	1956 [11] Crouch
Crystal spectrometer transmission	Bi	-1.56 ± 0.05	1959 [2] Melkonian
		-1.49 ± 0.05	1976 in Ref. [15]
		$-1.44\pm0.033\pm0.06$	1997 this work
Angular scattering	Ne, Ar, Kr, Xe	-1.34 ± 0.03	1966 [12] Krohn
Angular scattering	Ne, Ar, Kr, Xe	-1.30 ± 0.03	1973 [13] Krohn
Single crystal scattering	^{186}W	-1.60 ± 0.05	1975 [14] Alexandrov
Filter-transmission, mirror reflection	Pb	-1.364 ± 0.025	1976 [15] Koester
Filter-transmission, mirror reflection	Bi	-1.393 ± 0.025	1976 [15] Koester
n-TOF transmission, mirror reflection Ref. [17]	Bi	-1.55 ± 0.11	1986 [16] Alexandrov
Filter-transmission, mirror reflection	Pb, Bi	-1.32 ± 0.04	1986 [17] Koester
n-TOF transmission	thorogenic ^{208}Pb	$-1.31\pm0.03\pm0.04$	1995 [1] Kopecky
		$-1.33\pm0.027\pm0.03$	1997 this work
Filter-transmission, mirror reflection	Pb-isotopes, Bi	-1.32 ± 0.03	1995 [5] Koester
Garching-Argonne compilation	[12,13,15,17]	-1.31 ± 0.03	1986 [3] Sears
Dubna compilation	[14,16]	-1.59 ± 0.04	1989 [19] Alexandrov
Foldy approximation, b_F		-1.468	1952 [18] Foldy

The Neutron Magnetic Moment

"Naive" Quark Model

Static SU(6) Model:

1. *Baryons wavefunctions are quark color singlets with correct symmetry*

2. *Baryon magnetic moments arise solely from the static sum of the quark moments*

3. *Individual quark moments are proportional to quark charges (i.e.* $\mu_u = -2\mu_d$ *)*

1.

$$n_\uparrow = \sqrt{\tfrac{2}{3}} d_\uparrow d_\uparrow u_\downarrow - \sqrt{\tfrac{1}{3}} \left(\frac{d_\uparrow d_\downarrow + d_\downarrow d_\uparrow}{\sqrt{2}} \right) u_\uparrow$$

$$p_\uparrow = \sqrt{\tfrac{2}{3}} u_\uparrow u_\uparrow d_\downarrow - \sqrt{\tfrac{1}{3}} \left(\frac{u_\uparrow u_\downarrow + u_\downarrow u_\uparrow}{\sqrt{2}} \right) d_\uparrow$$

2.

$$\mu_n = -\tfrac{1}{3}\mu_u + \tfrac{4}{3}\mu_d$$

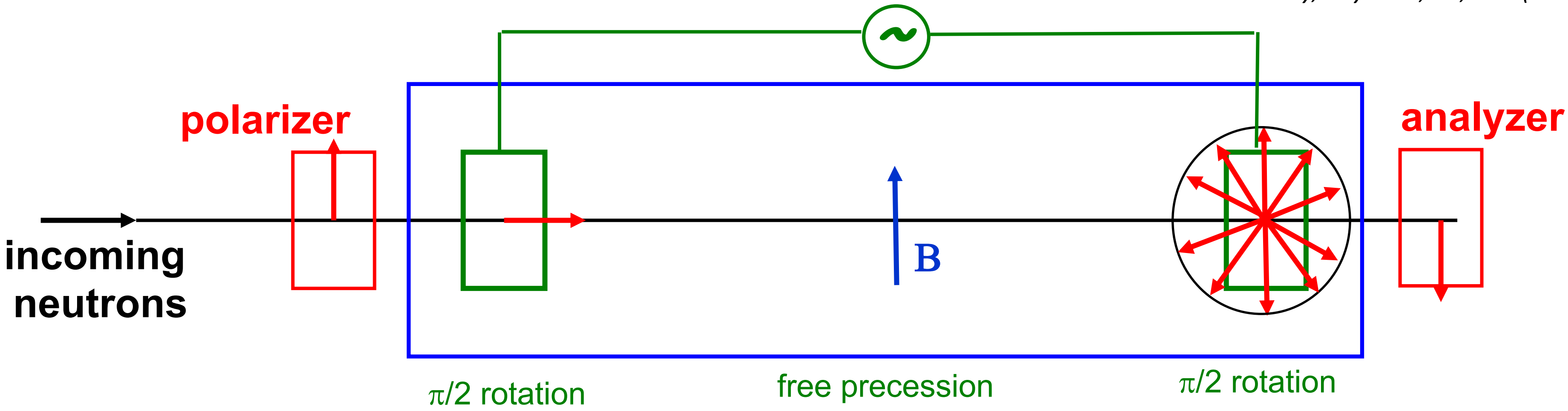
$$\mu_p = -\tfrac{1}{3}\mu_d + \tfrac{4}{3}\mu_u$$

3.

$$\boxed{\frac{\mu_n}{\mu_p} = -\frac{2}{3}}$$

Method of Separated Oscillatory Fields

*N.F.Ramsey, Phys Rev, **76**, 996 (1949)*



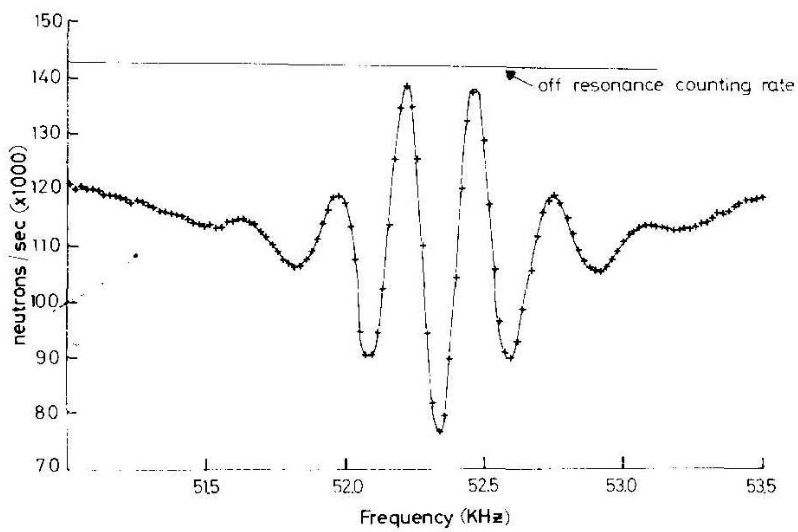


Fig. 1. Neutron resonance.

$$\boxed{\frac{\mu_n}{\mu_p} = -0.68497935(17)}$$

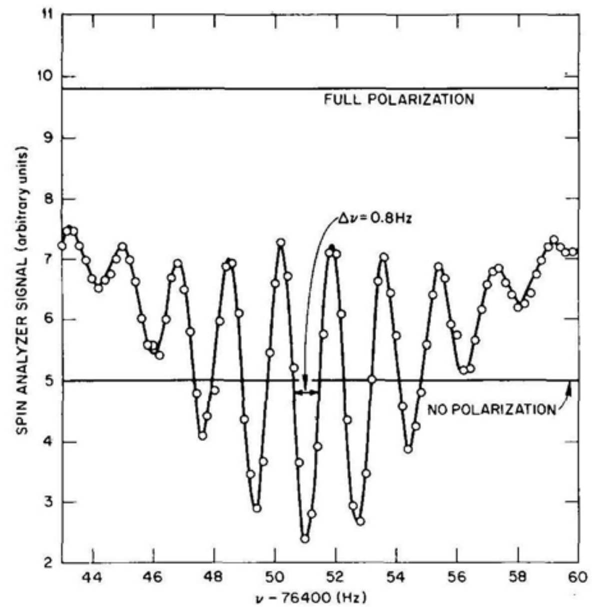


Fig. 2. Proton resonance.

*G. L. Greene, et. al. Physics Letters, **71B**, 297 (1977)*

WHY IS THE AGREEMENT SO GOOD?

$$\boxed{\frac{\mu_n}{\mu_p} = -0.68497935(17)}$$ experiment

vs.

$$\boxed{\frac{\mu_n}{\mu_p} = -0.67}$$ theory

Polarized electron, proton, and muon scattering experiments on H, D and ^{3}He indicate that only 20-30% of the nucleon spin comes from the intrinsic spin of the quarks.

The spin structure of the nucleon is one of the outstanding problems at the interface between nuclear and particle physics.

Over the past 20 years more than 1000 theoretical papers have been published and major experiments have been carried out at practically all major accelerator laboratories.

The work is ongoing…

The Neutron Electric Dipole Moment

Wait for Lecture 4

End of Lecture 1