Lecture (2)
Heavy-Ion Collisions:
Experiments, Models and Phenomenology

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Phenomenology

As a philosophical movement:
From Wikipedia:
There are several assumptions behind phenomenology that help explain its foundations:
1. Phenomenologists reject the concept of objective research. They prefer grouping assumptions through a process called phenomenological epoché.
2. They believe that analyzing daily human behavior can provide one with a greater understanding of nature.
3. They assert that persons should be explored. This is because persons can be understood through the unique ways they reflect the society they live in.
4. Phenomenologists prefer to gather "capta", or conscious experience, rather than traditional data.
5. They consider phenomenology to be oriented toward discovery, and therefore they research using methods that are far less restrictive than in other sciences.

To a physicist:
• Experiment (momenta and IDs of tracks) → Evolution of $\varepsilon, P, v, \rho$...
• Can be heuristic or semi-quantitative
Facilities

Pb + Pb
E/A=160 GeV

-9 fm/c

resonances
mesons
baryons anti-baryons
super-hadronic matter

AGS(11A GeV), SPS(160A GeV), RHIC(100A+100A GeV), LHC(1.4A+1.4A TeV)
<table>
<thead>
<tr>
<th>Facility</th>
<th>Operating</th>
<th>Equiv. pp c.o.m. Energy</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGS at BNL</td>
<td>1990s</td>
<td>≲ 6 GeV</td>
<td>≲ 160 MeV</td>
</tr>
<tr>
<td>SPS at CERN</td>
<td>1990s —</td>
<td>≲ 20 GeV</td>
<td>≲ 200 MeV</td>
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<tr>
<td>RHIC at BNL</td>
<td>2000 —</td>
<td>≲ 200 GeV</td>
<td>≲ 300 MeV</td>
</tr>
<tr>
<td>LHC at CERN</td>
<td>2010 —</td>
<td>≲ 15 TeV</td>
<td>≲ 400 MeV</td>
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</table>

SPS: Briefly visits QGP
RHIC and LHC: Well into QGP
ASIDE: 3 kinds of rapidity

1. “y”, the rapidity, is a measure of velocity along beam axis — rapidities add, just like Newtonian velocities
2. “η”, the pseudo-rapidity is approximation to “y”
   — depends on θ, angle relative to beam axis
   — for massless particles $y = \eta$
3. “$\eta_s$” Is the spatial rapidity
   — measure of position along z axis (Bjorken coordinates)
ASIDE: 3 kinds of rapidity

Consider $v$ along $z$ axis

\[ \gamma^2 - \gamma^2 v^2 = \frac{1}{1 - v^2} - \frac{v^2}{1 - v^2} = 1 \]

\[ u_0 = \gamma, \quad u_z = \gamma v, \quad u_0^2 - u_z^2 = 1 \]

\[ u^\alpha = (\gamma, \gamma v) = (\cosh y, \sinh y) \]

\[ u_B^\alpha = (\gamma_B, \gamma_B v_B) = (\cosh y_B, \sinh y_B) \]

\[ u^{\alpha B} = (u_0 \gamma_B + u_z \gamma_B v_B, u_z \gamma_B + u_0 \gamma_B v_B) \]

\[ = (\cosh y \cosh y_B + \sinh y \sinh y_B, \sinh y \cosh y_B, \sinh y_B \cosh y) = (\cosh(y + y_B), \sinh(y + y_B)) \]

Rapidity defined as:

\[ y = \sinh^{-1}(\gamma v) = \tanh^{-1}(v) = \frac{1}{2} \ln \left[ \frac{1 + v}{1 - v} \right] \]

RHIC beams: ± 5.4 units of $y$
LHC beams: ± 9.5 units of $y$
Experiments measure mid-rapidity

\[ y = \frac{1}{2} \ln \left( \frac{1 + v_z}{1 - v_z} \right) \]

\[ \eta = \frac{1}{2} \ln \left( \frac{1 + \cos \theta}{1 - \cos \theta} \right) \]

STAR/ALICE measure best for \(-1 < \eta < 1\)
1. Pre-equilibrium, $\tau \lesssim 0.5$ fm/c
   - no good quasi-particles, off-shell
   - flux tubes or classical Yang-Mills field
   - parametric

2. Hydrodynamics ($T \gtrsim 160$ MeV, $1 \approx \tau \approx 5$ fm/c)
   - QGP

3. Hadron simulation ($T \lesssim 160$ MeV)
   - hadrons struggle to maintain chemical/kinetic equilibrium

4. Superimposed on 1 - 3:
   - Femtoscopic correlations, jets, heavy-flavor dynamics
   - correlations…
Pre-equilibrium

Two kinds of energy deposition:
1. Partonic Scattering
2. Fields

Energy increases over time (negative pressure)

\[ T_{00} = \frac{E^2}{2}, \quad T_{xx} = T_{yy} = \frac{E^2}{2}, \quad T_{zz} = -\frac{E^2}{2} \]

QCD similar, but with 8 interacting fields (color-glass condensate)
Hydrodynamics and the QGP

1. Justified because of strong interaction and nearly all light quasi-particles
2. Eq. of state can come from lattice
3. Must account for viscosity

Israel-Stewart equations (several variants)

\[ \partial_t \pi_{ij} = - \frac{1}{\tau_{IS}} \left( \pi_{ij} - \pi_{ij}^{(NS)} \right) + \cdots \]

Arbitrary initial anisotropy of SE tensor
Parameters are viscosity and relaxation times
Why is hydro valid?

Hydro is based on:
  a) energy-momentum conservation
  b) profiles are smooth on scale of system
  c) $T_{ij}$ is not too far from equilibrium
  d) different species don’t flow differently

Should work for QGP
  a) Israel-Stewart is flexible
  b) dominated by light degrees of freedom — not true for hadron gas
Two-dimensional reduction of hydro

“Translational” invariance along beam axis
Small boosts along beam axis don’t change physics

Bjorken coordinates

\[ v_z = \frac{z}{t} \]
\[ \tau = t \sqrt{1 - v_z^2} = \sqrt{t^2 - z^2} \]
\[ \eta_s = \tanh^{-1} \left( \frac{z}{t} \right) \]

\[ t, z \rightarrow \tau, \eta_s \]

\( \varepsilon(\tau) \) — Nothing depends on \( \eta_s \), no longitudinal acceleration
Hydro becomes effectively 2-D
Doesn’t apply for lower RHIC energies
Collective Flow

1. Hallmark of hydrodynamic behavior
2. Reduced by viscosity
3. Radial, elliptic, etc
Radial flow

Non-relativistically,

\[ \left\langle \frac{p_x^2}{2m} + \frac{p_y^2}{2m} \right\rangle = T + \frac{m}{2} v_{\text{coll}}^2 \]

Spectra hotter for protons than pions
More pressure — more flow
Flow velocities \( \sim 0.7c \)
Elliptic Flow

\( v_2 \equiv \langle \cos 2\phi \rangle \)

Suggests low viscosity (close to uncertainty limit)
P.Danielewicz and M.Gyulessy, PRD(1985)
Higher Moments of Flow

\[ \nu_n \equiv \langle \cos(n\phi) \rangle \]

Reflects on lumpiness of initial conditions

V3

MUSIC vs PHENIX
Femtoscopic Correlations

\[ f(\vec{p}, \vec{r}, t) \]

\[
P_2(p_a, p_b) = P_1(p_a)P_1(p_b) + \frac{1}{(2\pi\hbar)^6} \int d^3r_ad^3r_b f(\vec{p}, \vec{r}_a, t)f(\vec{p}, \vec{r}_b, t) \left\{|\phi(q, r_a - r_b)|^2 - 1\right\}
\]

\[
C(p_a, p_b) = \frac{P_2(p_a, p_b)}{P_1(p_a)P_1(p_b)}
\]

Low pressure: \( R_{out} >> R_{side} \) and \( R_{long} \) is large

High pressure: \( R_{out} \sim R_{side} \) and \( R_{long} \) is small
Femtoscopic Correlations

- Stiffer EoS (blue to green)

\[ C(Q) \text{ (fit)} \]

\[ C(Q_{out}) \]

\[ C(Q_{side}) \]

\[ C(Q_{long}) \]

\[ Q_{o,s,l} \] [GeV/c]

\[ R_{out}/R_{side} \]

\[ R_{out} \text{ (fm)} \]

\[ R_{long} \text{ (fm)} \]

\[ k_t \text{ (MeV/c)} \]

S.P. PRL 2009

\[ \epsilon \]

\[ L \]

\[ \Delta a \text{ from the STAR collaboration} \]

\[ \frac{P}{\epsilon} \text{ never falls below 0.1 and the } c \frac{dP}{d\epsilon} \text{ stays fixed throughout mixed phase, and these conditions would have exceeding 20 fm/} \]

\[ P/\epsilon \text{ increases continuously with energy density between } 3 \text{ and } 6 \text{ GeV/fm}^3 \text{. Such solutions of equation of state with a first order phase transition the pressure associated with a hadronic gas with a temperature of 170 MeV, and an entropy density of sound, including parton and field effects of including Coulomb and strong interactions tends to overstate the amount of time required for the Lorentz contracted race energy tensor, including parton and field effects. Since the source functions are not truly Gaussian, this can lead to different Gaussian radii. These were then fit to correlation functions. Since the source functions are not truly Gaussian, this can lead to different Gaussian radii. These were then fit to correlation functions. Since the source functions are not truly Gaussian, this can lead to different Gaussian radii. These were then fit to correlation functions. 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Six dimensions $C(p_a, p_b)$ analyzed

- $R_{\text{out/long/side}}$ as functions of $p_t, y, \varphi$
- Directions of ellipse
- non-Gaussian details of source
- Source sizes for pions, kaons, protons, Lambdas
- Relative offset for different species, e.g. $\pi p$, $K p$, $K \pi$
  — At low energy, correlations with $d, t, \alpha, Li, C$...

All consistent with large collective flow!
Correlations from Charge Conservation — Balance Functions

\[ B(p_b|p_a) = \frac{P_{+-}(p_a, p_b) - P_{++}(p_a, p_b)}{2P_+(p_a)} + \frac{P_{-+}(p_a, p_b) - P_{--}(p_a, p_b)}{2P_-(p_a)} \]

- Integrates to unity
- Early production broader BFs
- Larger diffusion broader BFs
- Can be indexed on species
  — strangeness/baryons made early, electric charge made late
  — Narrow \( \pi \pi \) BFs, broad pp and KK BFs
Charge balance functions verify chemistry

- $\pi\pi$ narrower than KK
- KK narrower than pp

Matches expectations if susceptibility/chemistry equilibrated
\[ C_{ab}(t, \mathbf{r}_1, \mathbf{r}_2) = \langle \delta \rho_a(t, \mathbf{r}_1) \delta \rho_b(t, \mathbf{r}_2) \rangle \]
\[ \partial_t C_{ab} + \nabla_1 \cdot (\mathbf{v}_1 C_{ab}) + \nabla_2 \cdot (\mathbf{v}_2 C_{ab}) - D \nabla_1^2 C_{ab} - D \nabla_2^2 C_{ab} = S_{ab}(t, \mathbf{r}_1) \delta(\mathbf{r}_1 - \mathbf{r}_2) \]
\[ S_{ab}(t, \mathbf{r}) = -s \frac{D}{Dt} \frac{\chi_{ab}(t, \mathbf{r})}{s} \]

**Charge-balance correlations**

Early production of charge → broader correlation

**IV. Phenomenology**

**Susceptibility**

\[ \chi_{ab}/S \]

\[ T (\text{MeV}) \]

\[ 150 \quad 200 \quad 250 \quad 300 \quad 350 \quad 400 \]

S.P. and C.Plumberg, PRC(2019)
Phenomenology — Diffusivity

Strangeness made early
∴ kaon separation determined by diffusivity

$\Delta \phi$

$K^+$

$K^-$

$\Delta \phi$

K+K- Balance Function

Increasing D

Similar work already done for charm, Bernhard & Bass
Electromagnetic Signals
Penetrating Probes

• Photon has ~90% chance of traversing fireball
• Direct photons
  — Must subtract contribution from meson decays ($\pi_0$)
• Dileptons
  — Function of invariant mass
Direct Photons (not from hadron decays)

Puzzling:
- Yield seems high
- High flow
- From hadronization?

Large yield and large anisotropy is observed at PHENIX—>

challenge to theoretical models:
- Large yield -> Early emission
- Large v₂ -> Late emission

In order to understand this, PHENIX has measure data in:
- Large systems:  Au+Au 200, 62, 39 GeV and Cu+Cu at 200 GeV
- Small systems: p+p, p+Au, d+Au (MB) at 200 GeV
Dileptons

Excess at 500 MeV
Meson masses
Hadronization? Shifted $\rho$?

$\rho$ peak not 5x pp!!
Doesn't exist in QGP
Beam Energy Scan at RHIC: 2019-2021

- Energies from 7.7 GeV up (to 200)
- Less T, significantly more $\rho_B$
- Search for phase transition
  — correlations and fluctuations
- Difficult to model:
  — 3D
  — Larger corona
  — EoS depends on baryon density
  — Hadron simulation needs mean fields
  — Stopping 4-dimensional
  — Phase separation/critical phenomena dynamics difficult
Modeling Phase Dynamics

• Need gradient terms and thermal noise to
  a) generate critical correlations
  b) generate surface energies
  c) finite-size droplets

Ideas from:
Stephanov (hydro+),
Steinheimer, Young,
Kapusta, S.P.,...

\[ \epsilon_\kappa = \epsilon + \frac{1}{2} \rho \nabla^2 \rho, \]
\[ s = \bar{s}(\epsilon_\kappa, \rho), \]
\[ \beta = \bar{\beta}(\epsilon_\kappa, \rho), \quad \text{S.P. PRC 2018} \]
\[ \alpha = \bar{\alpha}(\epsilon_\kappa, \rho) + \frac{\bar{\beta} \kappa}{2} \nabla^2 \rho + \frac{\kappa}{2} \nabla^2 (\bar{\beta} \rho), \]
\[ M_i = -\frac{\kappa}{2} \rho (\partial_j \rho) (\partial_i v_j + \partial_j v_i) - \frac{\kappa}{2} \rho^2 \partial_i \nabla \cdot \mathbf{v} + \frac{\kappa}{2} \rho (\partial_i \rho) \nabla \cdot \mathbf{v}, \]
\[ T_{ij} = \bar{P} \delta_{ij} - \kappa \left[ \rho \nabla^2 \rho + \frac{1}{2} (\nabla \rho)^2 \right] \delta_{ij} + \kappa (\partial_i \rho)(\partial_j \rho), \]
Open questions & puzzles
(soft physics)

How is flow generated in small systems?
Why so many direct photons? (hadronization?)
What is the source of soft dileptons?
Can we infer EoS for $B \neq 0$? (beam energy scan)
Can we model signals of critical point or phase separation?
— if so, could signals be observable?

Jets and heavy flavor: Coming soon to a theatre near you