1. A 2 kg mass is attached to a horizontal spring and is executing simple harmonic motion with an angular velocity of 10 radians/sec. At a given instant in time it is found to be 10 cm to the left of the equilibrium position and moving to the right with a velocity of 1.732 m/sec.

a. What is the spring constant?

The spring constant, mass and angular velocity are related by

\[ \omega^2 = \frac{k}{m} \]

or solving for the spring constant

\[ k = \omega^2 m = (10)^2 \cdot 2 = 200 \text{ Nts/m} \]

b. What is the total energy of the system?

The total energy of the system is just the sum of the kinetic energy of the mass and the spring potential energy.

\[ E_{\text{Total}} = \frac{1}{2} m v^2 + \frac{1}{2} k x^2 \]

We use the initial conditions for the position and velocity.

\[ E_{\text{Total}} = \frac{1}{2} (2) (1.732)^2 + \frac{1}{2} (200) (0.1)^2 \]

\[ = 3 + 1 \]

\[ = 4 \text{ joules} \]

c. What is the maximum amplitude of oscillation?

At the point of maximum amplitude, all of the energy will be in the spring potential energy. So we can then use that

\[ E_{\text{Total}} = \frac{1}{2} k A^2 \]

\[ A = \sqrt{\frac{2 E_{\text{Total}}}{k}} = \sqrt{\frac{2 \cdot 4}{200}} = \sqrt{0.04} = 0.2 \text{ m} = 20 \text{ cm} \]

d. What is the phase angle?

We need to use the equation for position and velocity as functions of time.

\[ x = A \cos(\omega t + \phi) \]
\[ v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \]

We know that at \( t = 0 \), \( x = -0.1 \text{ m} \) while \( v = 1.732 \text{ m/s} \). So at \( t = 0 \) we have that

\[ -0.1 = 0.2 \cos \phi \]
\[ 1.732 = -10 (0.2) \sin \phi \]

or

\[ \cos \phi = -0.5 \]
\[ \sin \phi = -0.866 \]
Then dividing the cosine equation by the sine equation we then have
\[
\tan \phi = \frac{-0.866}{-0.5} = 1.732 \\
\phi = 60^{\circ}
\]

But since both the cosine and sine terms are negative this puts the angle in the 3rd quadrant, so
\[
\phi = 180 + 60 = 240^{\circ}
\]
2. An infinitely long wire of mass per unit length 1gm/m is under a tension of 10 nts. An observer next to the wire notices 10 peaks pass him in a time of 1 sec moving to the left.

a. **What is the wave velocity on the wire?**

The wave velocity is related to the tension in the wire and the mass per unit length of the wire by

\[ v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10}{0.001}} = 100 \text{ m/s} \]

b. **What is the frequency of the waves?**

Since ten peaks past a given point in 1 second, the frequency is just

\[ f = 10 \text{ Hz} \]

c. **What is the wavelength?**

The wave speed, frequency, and wavelength are related by

\[ v = \lambda f \]
\[ \lambda = \frac{v}{f} = \frac{100}{10} = 10 \text{ m} \]

d. **If the maximum displacement is 1 mm, what is the equation of the wave?**

The wave function for a wave traveling to the left is

\[ y(x, t) = y_{\text{max}} \cos (kx + \omega t) \]

We need to determine \( k \) and \( \omega \).

\[ k = \frac{2\pi}{\lambda} \quad \text{and} \quad \omega = 2\pi f \]
\[ k = \frac{6.283}{10} = 0.6283 \]
\[ \omega = 6.283 (10) = 62.83 \]

So the wave function is then

\[ y(x, t) = 0.001 \cos (0.6283 x + 62.83 t) \]
3. You are driving west on I-40 at 100 ft/sec. You notice in your rear view mirror a state trooper who is traveling at 120 ft/sec. The state trooper sounds his siren at a frequency of 300 Hz.

This is a problem involving the Doppler effect. The appropriate equation is

$$f_L = \frac{v + v_s}{v + v_s} f_S$$

with the sign convention for velocities being - from the listener to the source is positive.

a. **What is the frequency that you hear?**

Here both velocities are in the opposite sense to the positive direction for velocities. So $v_S = -120$ ft/s and the velocity of the listener is $v_L = -100$ ft/s. We then have

$$f_L = \frac{1100 - 100}{1100 - 120} \cdot 300 = \frac{1000}{980} \cdot 300 = 306.122 \text{ Hz}$$

b. **You breath a sigh of relief when the trooper passes you. What is the frequency that you now hear?**

Now both velocities are in the same sense as going from the listener to the source. So $v_S = 120$ ft/s and the velocity of the listener is $v_L = 100$ ft/s. We then have

$$f_L = \frac{1100 + 100}{1100 + 120} \cdot 300 = \frac{1200}{1220} \cdot 300 = 295.1 \text{ Hz}$$
4. A ray of light is incident on a slab of glass. The ray of light makes an angle of $45^\circ$ with respect to the normal to the surface.

a. It is found that the ray of light makes an angle of $30^\circ$ with respect to the normal in the glass. What is the index of refraction of the glass?

This a problem involving Snell’s Law

$$n_a \sin \theta_a = n_b \sin \theta_b$$

with the medium $a$ being the air and medium $b$ being the glass

$$\frac{1 \sin 45^\circ}{n_b \sin 30^\circ} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{0.707}{0.5} = 1.414$$

b. What is the speed of light in the glass?

The speed of light in a medium is reduced from that in a vacuum by the factor of the medium.

$$v = \frac{c}{n_b} = \frac{3 \times 10^8}{1.414} = 2.122 \times 10^8 \text{ m/s}$$

c. The ray of light now stars out in the glass and heads towards the air-glass boundary. What is the maximum angle the ray of light can make with respect to the normal before total internal reflection takes place?

Now medium $a$ is the glass and medium $b$ is the air. The critical angle is when the outgoing angle is $90^\circ$

$$1.414 \sin \theta_{crit} = 1 \ (1)$$

$$\sin \theta_{crit} = \frac{1}{1.414} = 0.707$$

$$\theta_{crit} = 45^\circ$$
Maybe Useful formulae and constants:

\[ \vec{F} = -k \vec{x}; \quad \vec{x} = a \cos (\omega t + \phi); \quad U = \frac{1}{2}kx^2 \]

\[ \omega = \sqrt{\frac{k}{m}}; \quad y(x, t) = y_{\text{max}} \cos (kx \pm \omega t); \quad f_L = \frac{v + v_l}{v + v_s} f_s \]

\[ v = \sqrt{\frac{\mu}{T}}; \quad \omega = 2\pi f; \quad T = \frac{1}{f} \]

\[ I = I_0 \cos^2 \phi; \quad I = S_{\text{ave}} = \epsilon_0 c E_{\text{max}}^2; \quad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}} \]

\[ n_a \sin \theta_a = n_b \sin \theta_b; \quad n = \frac{c}{v}; \quad n = \frac{c}{v}; \quad \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \]

\[ E = cB; \quad \rho_{\text{rad}} = \frac{I}{c}; \quad \rho_{\text{rad}} = \frac{2I}{c} \]

\[ c = 3 \times 10^8 \text{ m/sec}; \quad \mu_0 = 4\pi \times 10^{-7} \text{T} \cdot \text{m/A} \]

\[ \epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 \quad \text{v}_{\text{sound}} = 1100 \text{ ft/sec} = 340 \text{ m/sec} \]