1. A sensor is exposed for 0.1 seconds to a 200 Watt lamp that is 10 meters away. The sensor has an opening that is 20mm in diameter. How many photons enter the sensor, if the wavelength of the emitted light is 600 nm?

The energy content of a photon of the light is given by

\[ E = hf = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{600 \times 10^{-9}} = 3.313 \times 10^{-19} \text{ joules} \]

The total number of photons emitted per second by the light is given by

\[ \frac{\text{photons}}{\text{sec}} = \frac{P}{E} = \frac{200}{3.313 \times 10^{-19}} = 6.037 \times 10^{20} \text{ photons/sec} \]

Not all of these photons go into the sensor. We assume that the photons from the light are emitted in a spherically symmetric way. The number of photons entering the sensor is then the ratio of the sensor area to the total area of a sphere of radius 10 meters times the total number of photons per second that are emitted.

\[
\text{Area Ratio} = \frac{\text{Sensor Area}}{\text{Surface Area of Sphere}} = \frac{\pi \left(\frac{d}{2}\right)^2}{4\pi R^2} = \frac{\pi \left(\frac{20 \times 10^{-3}}{2}\right)^2}{4\pi (10)^2} = 2.5 \times 10^{-7}
\]

Number of photons/sec entering the sensor is then

\[ \frac{\text{photons}}{\text{sec}} = (2.5 \times 10^{-7}) (6.037 \times 10^{20}) = 1.51 \times 10^{14} \text{ photons/sec} \]

The number of photons in 0.1 seconds is then

\[ \text{photons} = (1.51 \times 10^{14}) (0.1) = 1.51 \times 10^{13} \]
2. Light of wavelength 550nm strikes a surface causing the ejection of photoelectrons for which the stopping potential is given by $V_{stop} = 0.19$ Volts.

(a) **What is the work function of the surface?**

This a problem in the photoelectric effect. The kinetic energy of the electron is given by

$$KE = hf - WorkFunction$$

The kinetic energy of the electron is also given by the work necessary to stop the electron by an external potential difference and this is given by

$$KE = qV$$

Equating these we then have

$$qV = hf - WorkFunction$$

$$WorkFunction = hf - qV$$

$$= \frac{hc}{\lambda} - qV$$

$$= \left(\frac{6.626 \times 10^{-34}}{550 \times 10^{-9}}\right)(3 \times 10^8) - (1.6 \times 10^{-19})(0.19)$$

$$= 3.61 \times 10^{-19} - 0.3 \times 10^{-19}$$

$$= 3.31 \times 10^{-19} \text{ joules} = 2.07 \text{ eV}$$

(b) **What is the threshold frequency for this surface?**

The threshold frequency is when the electron comes out with zero kinetic energy

$$KE = hf - WorkFunction$$

$$hf = WorkFunction$$

$$f = \frac{WorkFunction}{h} = \frac{3.31 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$$f = 5.00 \times 10^{14} \text{ Hz}$$

(c) **Now suppose radiation of wavelength 190nm is incident on the surface. What is now the necessary stopping potential for the emitted photoelectrons?**

First we need to find the frequency of the radiation

$$\lambda f = c$$

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{190 \times 10^{-9}} = 1.579 \times 10^{15} \text{ Hz}$$

Now we can find the stopping potential

$$qV = KE = hf - WorkFunction$$

$$V = \frac{q}{hf - WorkFunction}$$

$$= \left(\frac{6.626 \times 10^{-34}}{1.579 \times 10^{15}}\right) - 3.31 \times 10^{-19}$$

$$= 7.152 \times 10^{-19} \text{ joules} = 4.47 \text{ Volts}$$
3. A sample of radioactive material is initially found to have an activity of 115 decays/minute. After 4 days and 5 hours, its activity is measured to be 73.5 decays/minute.

(a) What is the half life of the material?

The activity is given by

\[ R = -\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t} \]

We are given the activity at two different times the first of which can be taken as \( t_1 = 0 \), while the second one is \( t_2 = 4 \text{ days, 5 hours} = 101 \text{ hours} \). At \( t_1 \) we then have

\[ R_1 = R_0 e^{-\lambda t_1} = R_0 \]

At \( t_2 \) we have

\[ R_2 = R_0 e^{-\lambda t_2} \]

Taking the ratio of \( R_2 \) to \( R_1 \) we have

\[ \frac{R_2}{R_1} = e^{-\lambda t_2} \]

\[ \frac{73.5}{115} = e^{-(101)\lambda} \]

\[ \ln \left( \frac{73.5}{115} \right) = -101\lambda \]

\[ -0.4476 = -101\lambda \]

\[ \lambda = 4.432 \times 10^{-3} \frac{\text{hrs}^{-1}}{} \]

The half-life in terms of the decay constant is given by

\[ t_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{4.432 \times 10^{-3}} = 156.35 \text{ hours} \]

(b) How long from the initial time will it take for the sample to reach an activity of 10.0 decays/minute?

The activity is given by

\[ R = R_0 e^{-\lambda t} \]

\( R_0 \) is just the initial activity (remember that we can start the clock at any time) which is 115 decays/minute. We then have

\[ R = R_0 e^{-\lambda t} \]

\[ \frac{10}{115} = 115 e^{-\left(4.432\times10^{-3}\right)t} \]

\[ \frac{10}{115} = 8.696 \times 10^{-2} = e^{-\left(4.432\times10^{-3}\right)t} \]

\[ \ln(8.696 \times 10^{-2}) = -\left(4.432 \times 10^{-3}\right) t \]

\[ 2.442 = \left(4.432 \times 10^{-3}\right) t \]

\[ t = 5.51 \times 10^2 \text{ hours} \]
4. An electron in a hydrogen atom undergoes deexcitation and in the process releases a photon of wavelength 487 nm. To get to the final state that the electron now finds itself in would require an excitation energy of 10.19 eV?

(a) **What is the binding energy of the initial state?**

We must first determine the energy of the emitted photon which is given by

\[
E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34}) \times (3 \times 10^8)}{487 \times 10^{-9}}
\]

\[
= 4.08 \times 10^{-19} \text{ joules} = 2.55 \text{ eV}
\]

We are told that the final state that the electron is in has an excitation energy \(E_x\) of energy 10.19 eV. That means that the energy of this level is given by

\[
E_{\text{final}} = E_1 + E_x
\]

\[
= -13.6 + 10.2
\]

\[
= -3.40 \text{ eV}
\]

The emitted photon comes from a transition from a higher state to this and the energy of this photon is also given by

\[
E_{\text{photon}} = E_{\text{initial}} - E_{\text{final}}
\]

\[
E_{\text{initial}} = E_{\text{photon}} + E_{\text{final}}
\]

\[
= 2.55 + (-3.40)
\]

\[
= -0.85 \text{ eV}
\]

The binding energy is just the energy that would be required to remove the electron from this state and therefore

\[
E_{\text{Binding}} = 0.85 \text{ eV}
\]

(b) **Between what levels is this transition between?**

The energy of any given level is given by

\[
E_n = -\frac{13.6}{n^2} \text{ eV}
\]

For the initial state we have

\[
-0.85 = -\frac{13.6}{n_{\text{init}}^2}
\]

\[
n_{\text{init}}^2 = \frac{13.6}{0.85} = 16
\]

\[
n_{\text{init}} = 4
\]

For the final state we have

\[
-3.4 = -\frac{13.6}{n_{\text{final}}^2}
\]

\[
n_{\text{final}}^2 = \frac{13.6}{3.4} = 4
\]

\[
n_{\text{final}} = 2
\]
Physics 232 Formula Sheet

Simple Harmonic Motion
\[ \ddot{F} = -k \dot{x} \]
x = A \cos(\omega t + \phi)
\[ \omega = \sqrt{\frac{k}{m}} \]
\[ \omega = 2 \pi f \]
\[ T = \frac{1}{f} \]
\[ PE = \frac{1}{2} k x^2 \]
\[ E_{Total} = PE + KE \]
\[ E_{Total} = \frac{1}{2} k A^2 \]

Doppler Effect
\[ f_L = \frac{v + v_L}{v + v_S} f_S \]

Periodic Motion
\[ y(x, t) = A \cos(k x + \omega t) \]
\[ \frac{\partial^2 y(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x, t)}{\partial t^2} \]
v = \lambda f
\[ y_{net} = \sum_i y_i \]
\[ P_{ave} = \frac{1}{2} \mu v \omega^2 A^2 \]

Waves on a String
\[ v = \sqrt{\frac{T}{\mu}} \]
\[ \lambda = \frac{2L}{n} \quad n = \text{integer} \]

Light
Reflection
\[ \theta_{\text{incident}} = \theta_{\text{reflected}} \]

Refraction
\[ n_a \sin \theta_a = n_b \sin \theta_b \]
\[ \sin \theta_c = \frac{n_a}{n_b} \quad n_a < n_b \]

Polarization
\[ I = I_0 \cos^2 \phi \]

Electromagnetic Waves
\[ E_{\text{max}} = c B_{\text{max}} \]
\[ I = S_{\text{ave}} = \frac{1}{2} \varepsilon_0 c E^2 \]
\[ \mathbf{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \]

Diffraction
\[ d \sin \theta = m \lambda \]
\[ \sin \theta_1 = 1.22 \frac{\lambda}{D} \]

Special Relativity
\[ x' = \frac{x - vt}{\sqrt{1 - v^2 / c^2}} \]
\[ y' = y \]
\[ z' = z \]
\[ t' = \frac{t - vx / c^2}{\sqrt{1 - v^2 / c^2}} \]
\[ v'_x = \frac{v_x - v}{1 - (v/c^2)v_x} \]
\[ v'_y = \frac{v_y \sqrt{1 - v^2 / c^2}}{1 - (v/c^2)v_x} \]
\[ v'_z = \frac{v_z \sqrt{1 - v^2 / c^2}}{1 - (v/c^2)v_x} \]
\[ T = \frac{T_0}{\sqrt{1 - (v/c)^2}} \]
\[ L = L_0 \sqrt{1 - (v/c)^2} \]
\[ E = \frac{m_0 c^2}{\sqrt{1 - (v/c)^2}} \]
\[ E = KE + m_0 c^2 \]
\[ E^2 = p^2 c^2 + m_0^2 c^4 \]
Refraction at a Curved Surface

\[ \frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{R} \]
\[ m = -\frac{n}{n'} \frac{s'}{s} \]

Mirrors

\[ \frac{1}{s} + \frac{1}{s'} = 1 \]
\[ f = \frac{R}{2} ; m = -\frac{s'}{s} \]

Lenses

\[ \frac{1}{s} + \frac{1}{s'} = 1 \]
\[ m = -\frac{s'}{s} \]

Blackbody Radiation

\[ \lambda_{\text{max}}T = 2.90 \times 10^{-3} m K \]
\[ I_{\text{Total}} = \sigma T^4 \]
\[ I(\lambda) = \frac{2\pi \hbar c^2}{\lambda^5 \left( e^{\frac{\hbar c}{kT}} - 1 \right)} \]
\[ E = n hf \]

Interference

\[ \delta = \delta_{\text{inh}} + \delta_{p.d.} + \delta_{\text{refl}} \]
\[ \delta_{p.d.} = 2\pi \frac{\Delta x}{\lambda} \]

Two Slit Intensity

\[ I = I_0 \cos^2 \left( \frac{\pi d}{\lambda} \sin \theta \right) \]

Reflected Intensity

\[ I_{\text{reflected}} = \left( \frac{n_a - n_b}{n_a + n_b} \right)^2 \]

Photoelectric Effect

\[ KE = hf - B.E. \]

Compton Scattering

\[ \lambda' = \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta) \]

Bohr Model

\[ L = m v r = n \frac{h}{2\pi} \]
\[ E_n = - \frac{1}{\varepsilon_0^2} \frac{m_e e^4}{8 n^2 \hbar^2} \]
\[ E_n = - \frac{13.6}{n^2} eV \]
\[ r_n = \varepsilon_0 \frac{n^2 \hbar^2}{\pi m_e e^2} \]
\[ r_n = n^2 (0.53 \text{ Angstroms}) \]

DeBroglie Hypothesis

\[ \lambda = \frac{h}{p} \]

Heisenberg Uncertainty Principal

\[ \Delta p_x \Delta x \geq \frac{h}{2\pi} \]
\[ \Delta E \Delta t \geq \frac{h}{2\pi} \]

Particle in Infinite Well

\[ E_n = \frac{n^2 \pi^2 \hbar^2}{2 m L^2} \]
\[ k_n = \frac{\sqrt{2 m E_n}}{\hbar} \]
\[ \psi_n (x) = \sqrt{\frac{2}{L}} \sin \left( \frac{n \pi x}{L} \right) \]

Hydrogen Atom

\[ E_n = - \frac{1}{\varepsilon_0^2} \frac{m_e e^4}{8 n^2 \hbar^2} \]
\[ n = 1, 2, 3, \ldots \]
\[ L = \sqrt{l(l + 1)} \hbar \]
\[ l = 0, 1, 2, \ldots, n-1 \]
\[ L_z = m \hbar \]
\[ m = 0, \pm 1, \pm 2, \ldots, \pm l \]
Physical Constants

\[ c = 3 \times 10^8 \text{ m / s} \]
\[ v_{\text{sound}} = 1100 \text{ ft / sec} \]
\[ \varepsilon_0 = 8.854 \times 10^{-12} \text{ C}^2 / \text{N} \cdot \text{m}^2 \]
\[ \mu_0 = 4\pi \times 10^{-7} \text{ T m / A} \]
\[ e = 1.6 \times 10^{-19} \text{ C} \]
\[ m_e = 9.109 \times 10^{-31} \text{ kg} \]
\[ = 0.511 \text{ MeV} / c^2 \]
\[ m_p = 1.672 \times 10^{-27} \text{ kg} \]
\[ = 938.27 \text{ MeV} / c^2 \]
\[ 1 \text{ eV} = 1.6 \times 10^{-19} \text{ joules} \]
\[ \sigma = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \text{ K}^4} \]

Useful Geometry

Circle

\[ \text{Area} = \pi r^2 \]
\[ \text{Circumference} = 2\pi r \]

Sphere

\[ \text{Surface Area} = 4\pi r^2 \]
\[ \text{Volume} = \frac{4}{3} \pi r^3 \]

Cylinder

\[ \text{Lateral Area} = 2\pi r L \]
\[ \text{Volume} = \pi r^2 L \]

Nuclear Stuff

\[ E_B = \left( Z M_H + N M_N - \frac{A}{Z} M \right) c^2 \]
\[ N = N_0 e^{-\lambda t} \]
\[ R = -\frac{dN}{dt} = R_0 e^{-\lambda t} \]
\[ t_{1/2} = \frac{0.693}{\lambda} \]
\[ \tau = \frac{1}{\lambda} \]