Due Date: 11-12-08

Nuclear Physics 621

Homework 8 – Shell Structure

Send your answer to:
Nicolas Schunck,
schuncknf@ornl.gov

Consider a three-dimensional anisotropic harmonic oscillator potential

$$V = \frac{1}{2}m\omega_x^2 x^2 + \frac{1}{2}m\omega_y^2 y^2 + \frac{1}{2}m\omega_z^2 z^2$$

The corresponding Hamiltonian is separable into the x-, y- and z-directions and the energy eigenvalues are easily obtained as:

$$E = \hbar\omega_x\left(n_x + \frac{1}{2}\right) + \hbar\omega_y\left(n_y + \frac{1}{2}\right) + \hbar\omega_z\left(n_z + \frac{1}{2}\right)$$

1) In the case of spherical symmetry ($\omega_x = \omega_y = \omega_z = \omega_0$), a large degeneracy is obtained. Calculate the lowest 'magic' numbers (include spin degrees of freedom).

2) In the case of axial symmetry, $\omega_x = \omega_y = \omega_\perp \neq \omega_z$. Plot the energy E (in units of $\hbar\omega_z$) as function of the ratio $r = \omega_\perp/\omega_z$, from $r = 0.1$ to 3.5. Include in your plot only the eigenvalues such that $N = n_x + n_y + n_z \leq 6$

   List the magic numbers in the two other cases with large degeneracies for r > 1 (also include spin degrees of freedom).

   Comment your results.