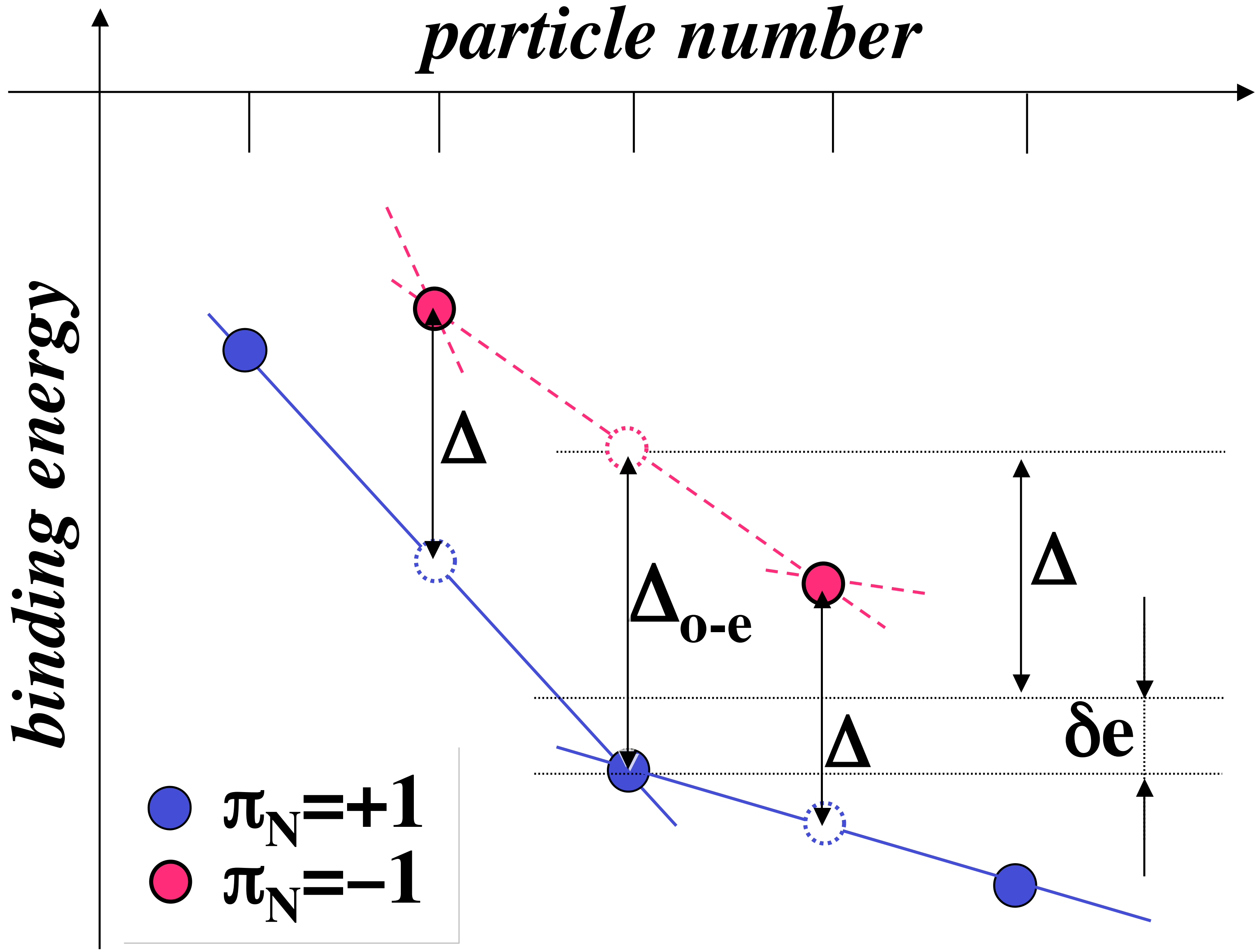
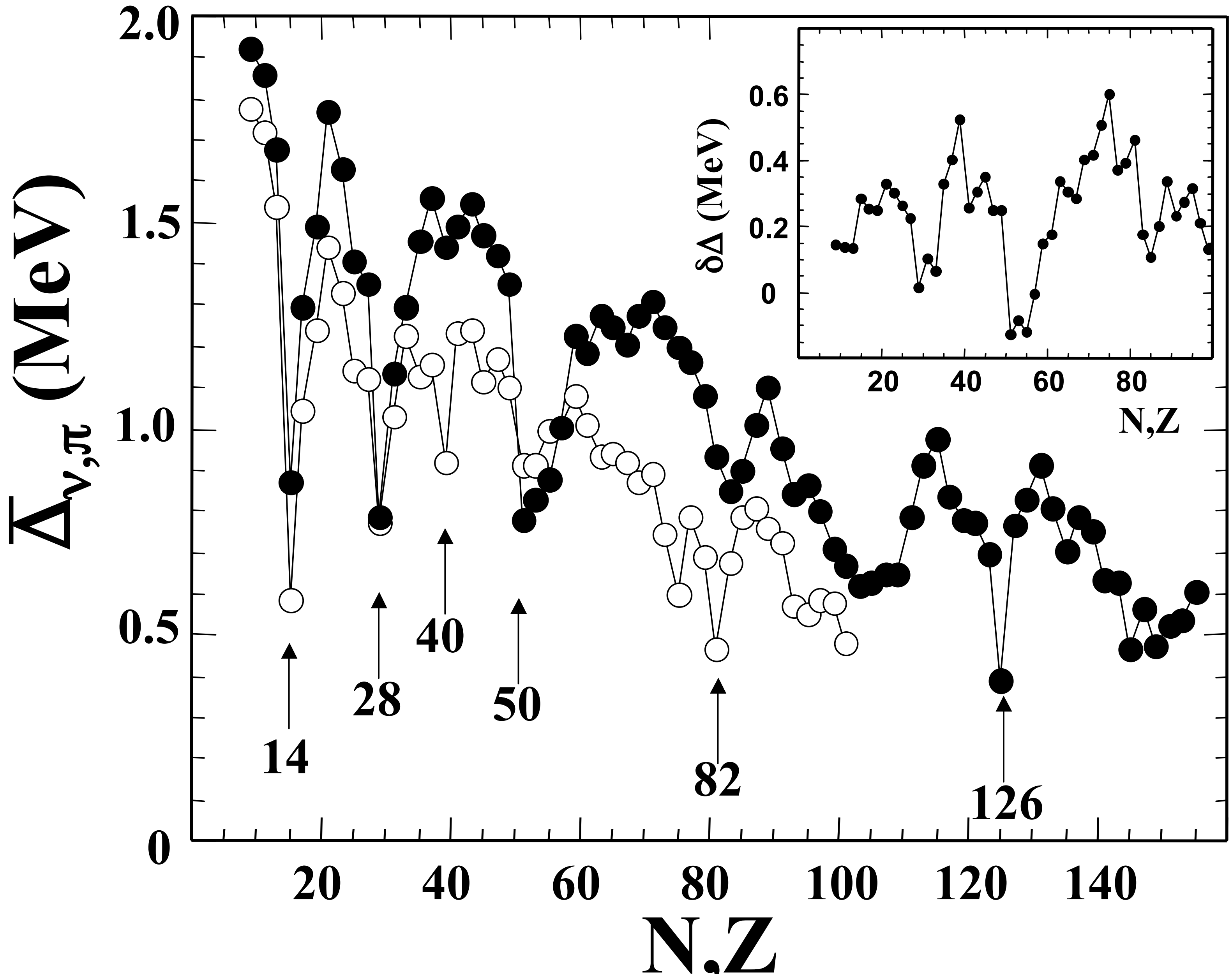


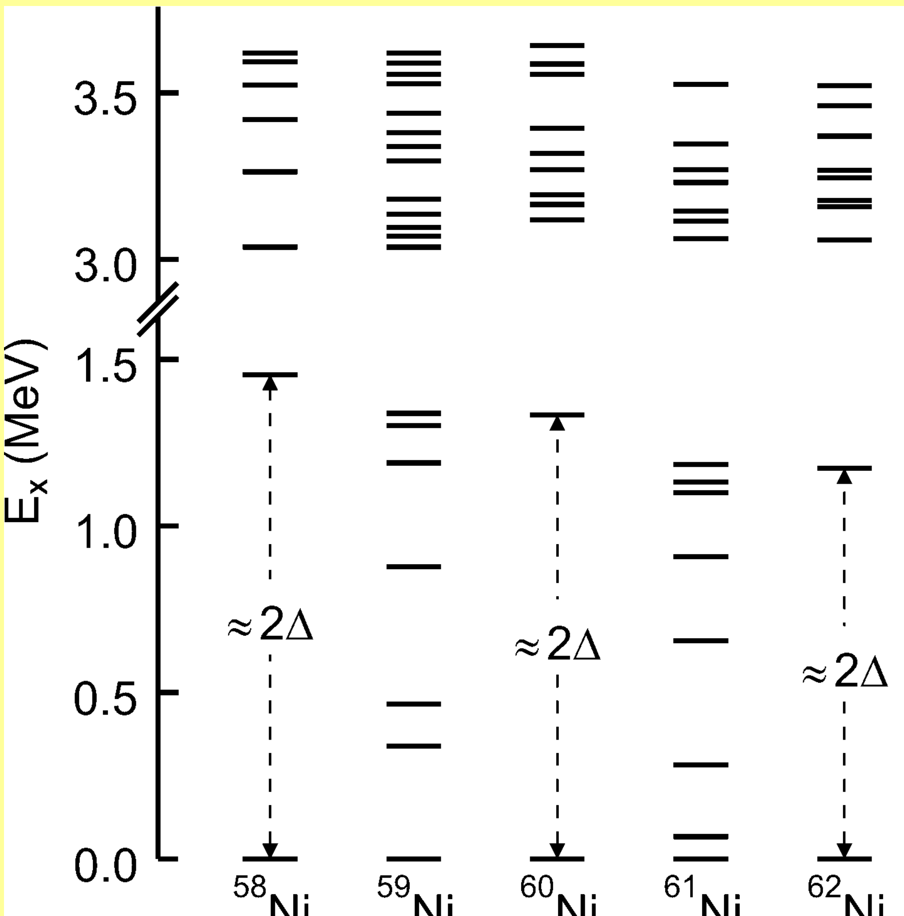
particle number
binding energy
Δ
Δ
Δ_{o-e}
Δ
δe
π_N=+1
π_N=−1



$\overline{\Delta}_{\nu,\pi}$ (MeV)
2.0
1.5
1.0
0.5
0
20
40
60
80
100
120
140
N,Z
14
28
40
50
82
126
δΔ (MeV)
0.6
0.4
0.2
0
20
40
60
80
N,Z

Evidence for pairing correlations in nuclei

(iv) Energy gap: odd-even and even-odd nuclei (especially deformed nuclei) have energy spectra different from even-even nuclei.



- e-e nuclei: only a few states at most (vibrations, rotations) appear below the pairing gap 2Δ.
- But in o-e and e-o nuclei (where the last nucleon is unpaired) many s.p. and collective states appear.
- Note: above the pair breaking energy 2Δ many excited states are possible → level density $\rho = \rho(\Delta)$

Seniority Model

The seniority model provides a simple model for pairing phenomena.
System: N fermions in a single j-shell.
Hamiltonian:

$$\begin{aligned} H &= -G \sum_{m,m'>0} \hat{a}_m^\dagger \hat{a}_{-m}^\dagger \hat{a}_{-m'} \hat{a}_{m'} \\ &= -G \hat{S}_+ \hat{S}_- \end{aligned}$$

where

$$\hat{S}_+ = \sum_{m>0} \hat{a}_m^\dagger \hat{a}_{-m}^\dagger \quad \text{and} \quad \hat{S}_- = \left(\hat{S}_+\right)^\dagger .$$

Introduce *quasi-spin operators*

$$\begin{aligned} \hat{s}_+^{(m)} &= \hat{a}_m^\dagger \hat{a}_{-m}^\dagger \\ \hat{s}_-^{(m)} &= \hat{a}_{-m} \hat{a}_m \\ \hat{s}_0^{(m)} &= \frac{1}{2}\left(\hat{a}_m^\dagger \hat{a}_m + \hat{a}_{-m}^\dagger \hat{a}_{-m} - 1\right) \end{aligned}$$

and find angular momentum commutation relations!

$$\begin{aligned} \left[\hat{s}_+^{(m)}, \hat{s}_-^{(m)}\right] &= 2\hat{s}_0^{(m)} \\ \left[\hat{s}_0^{(m)}, \hat{s}_+^{(m)}\right] &= \hat{s}_+^{(m)} \\ \left[\hat{s}_0^{(m)}, \hat{s}_-^{(m)}\right] &= -\hat{s}_-^{(m)} \end{aligned}$$

Rewrite Hamiltonian as

$$H = -G\left(\vec{S}\cdot\vec{S} - \hat{S}_0^2 + \hat{S}_0\right)$$

in terms of total quasi-spin

$$\vec{S} = \sum_{m>0} \vec{s}^{(m)}.$$

and total z-component of quasi spin

$$\hat{S}_0 = \sum_{m>0} \hat{s}_0^{(m)} = \frac{1}{2}(\hat{N} - \Omega).$$

Here, $\Omega = j + 1/2$ is the maximal number of pairs for a single j-shell. The eigenvalues S of total quasi-spin are

$$S = \frac{1}{2}|N - \Omega|, \ldots, \frac{1}{2}\Omega - 1, \frac{1}{2}\Omega.$$

Thus, the energies of the seniority model are

$$E(S, N) = -G\left[S(S+1) - \frac{1}{4}(N - \Omega)^2 + \frac{1}{2}(N - \Omega)\right].$$

Alternatively, one uses the *seniority* quantum number $s = \Omega - 2S$

$$E(s, N) = -\frac{G}{4}\left[s^2 - 2s(\Omega + 1) + 2N(\Omega + 1) - N^2\right].$$

Note:

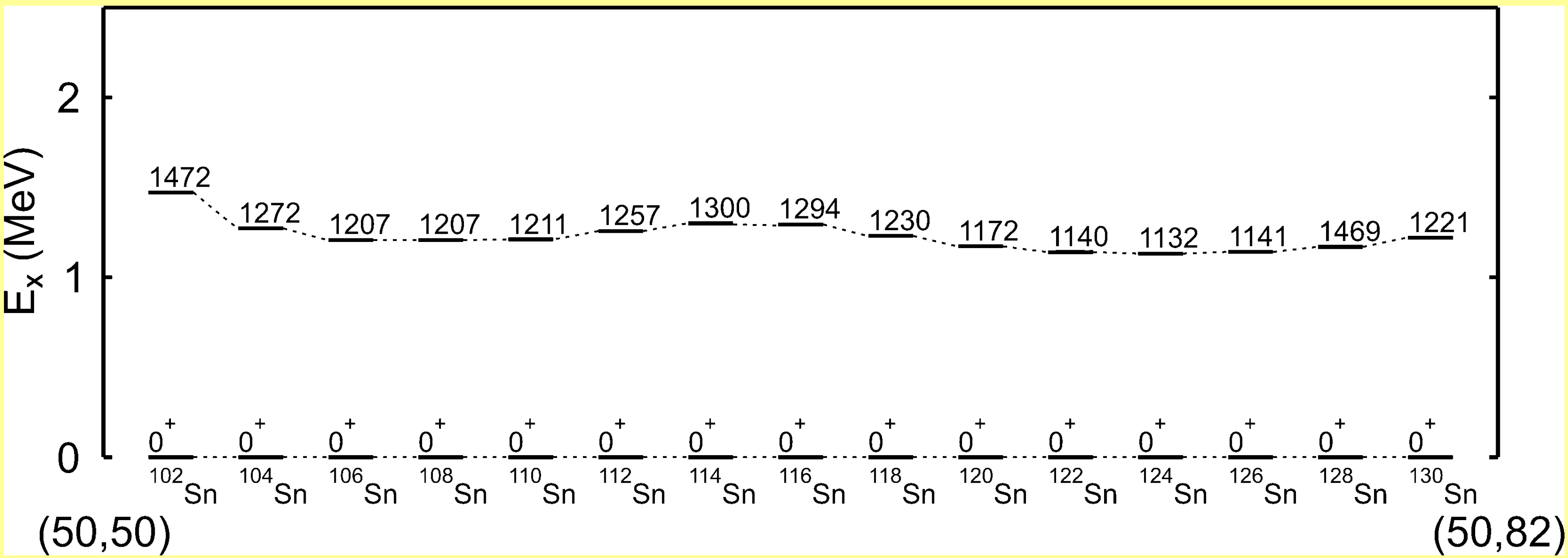
- s counts number of unpaired nucleons.
- ground state has minimal seniority $s = 0$ (or maximal quasi spin $S = \Omega/2$)
- for fixed N, excitations depend only on seniority quantum number
- $E(N, s = 2) - E(N, s = 0) = G\Omega$

Two-particle spectrum of pure pairing force: $J = 0$ ground state is separated from degenerate $J = 2, 4, 6, \ldots, 2j - 1$ levels.

Pair vibrational spectrum: $E(N, s = N) - E(N, s = 0) \approx G\Omega N/2$ for $N \ll \Omega$. (Binding increases linearly with number of pairs.)

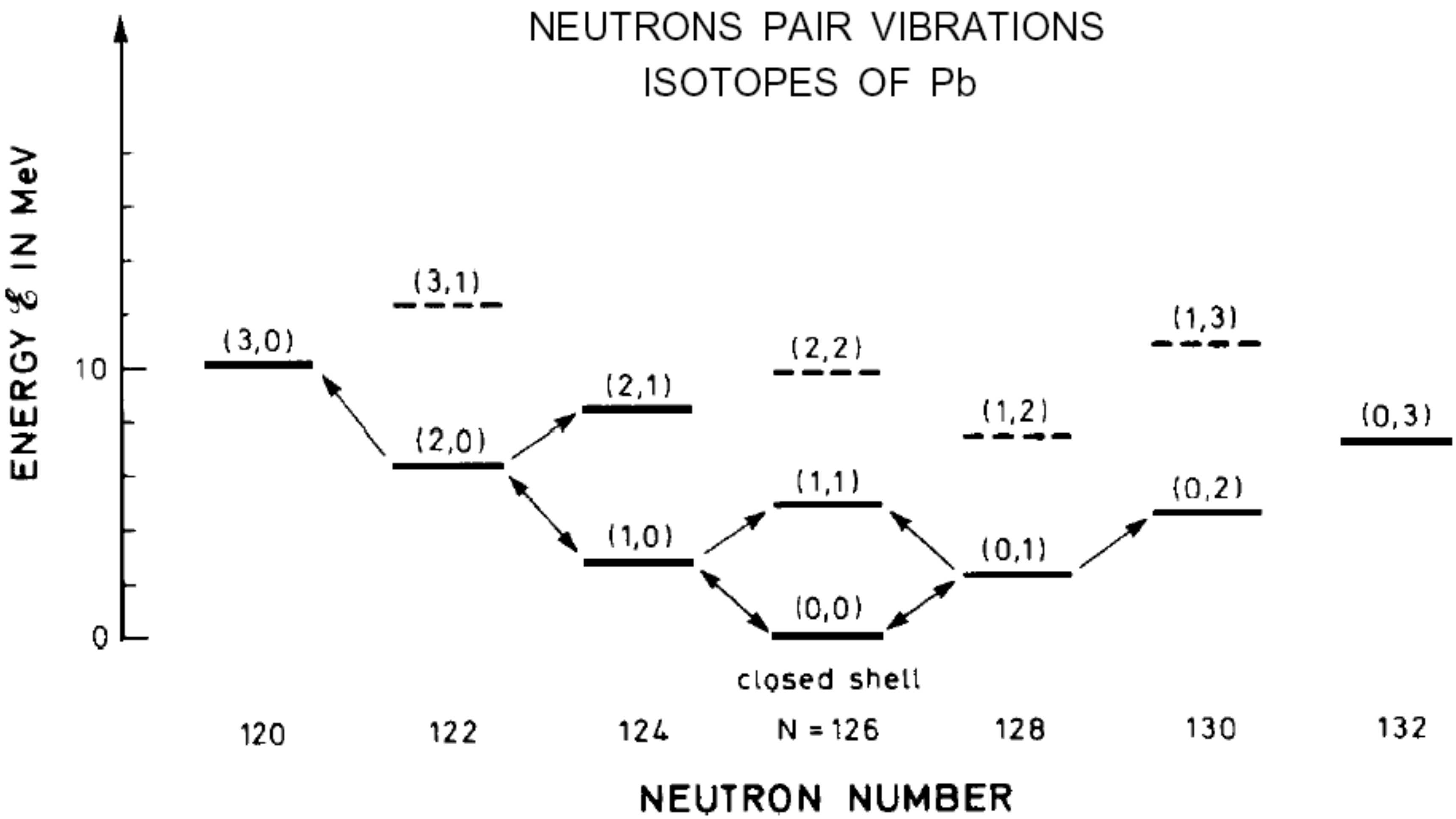
Evidence for pairing correlations in nuclei

(iii) The excitation energy of the first excited 2^+ state in nuclei remains remarkably constant over large intervals of neutron (proton) numbers.



- These 2^+ states are not rotational states but are connected to a coherent pairing condensate.

- Pair breaking energy: $2\Delta \approx 2$ MeV



NEUTRONS PAIR VIBRATIONS
ISOTOPES OF Pb
ENERGY ℰ IN MeV
10
0
(3,0)
(3,1)
(2,0)
(2,1)
(1,0)
(2,2)
(1,1)
(0,0)
closed shell
(1,2)
(0,1)
(1,3)
(0,2)
(0,3)
120
122
124
N = 126
128
130
132
NEUTRON NUMBER

Ground states

States labeled by quasi spin S and its z-component $S_0 = (N - \Omega)/2$

$$\text{vacuum:} \quad N = 0, \quad |-\rangle = |S, S_0\rangle = |\frac{\Omega}{2}, -\frac{\Omega}{2}\rangle$$
$$\text{full :} \quad N = 2\Omega, \quad |S, S_0\rangle = |\frac{\Omega}{2}, \frac{\Omega}{2}\rangle$$

Ground states

$$|S, S_0\rangle = |\frac{\Omega}{2}, \frac{N - \Omega}{2}\rangle \propto \hat{S}_+^{N/2}|-\rangle \quad \text{for even } N$$
$$|S, S_0\rangle = |\frac{\Omega}{2}, \frac{N - \Omega}{2}\rangle \propto \hat{S}_+^{(N-1)/2}\hat{a}_m^\dagger|-\rangle \quad \text{for odd } N$$

Note:

1. even-N ground state is not degenerate
2. odd-N ground state is 2Ω-fold degenerate; this explains high level density of odd systems.

Excited states

Even N: seniority $s = 2$ states. Creation operator for two particles with spin J and projection M:

$$\hat{A}^\dagger_{J,M} = \frac{1}{\sqrt{2}} \sum_{m,n} \langle jjmn|JM\rangle \hat{a}^\dagger_m \hat{a}^\dagger_n$$

$$|JM\rangle = \hat{A}^\dagger_{J,M}|-\rangle$$

Can show that (for $J \neq 0$)

$$\vec{S}^2|JM\rangle = \left(\frac{\Omega}{2} - 1\right)\frac{\Omega}{2}|JM\rangle$$

Thus, $|J, M\rangle$ has quasi spin $S = \Omega/2 - 1$ and seniority $s = 2$ (for $J \neq 0$).

N-body state with seniority $s = 2$

$$|(\Omega - 1)/2, (N - \Omega)/2\rangle \propto \hat{S}_+^{N/2-1}|JM\rangle.$$

Quadrupole deformation

$$|\langle s = 0|Q_{20}|s = 0\rangle|^2 \propto \frac{N}{2}\left(\Omega - \frac{N}{2}\right)$$

Thus, deformation is strongest for mid-shell nuclei.

Pairing rotation

Quasi spin S precesses around its z-axis. Its z-component S_0 is conserved and related to the number of particles

$$N = 2S_0 + \Omega.$$

As J_z is the infinitesimal generator of rotations around the z-axis, S_0 (and therefore N) is the infinitesimal generator of pairing rotations. This rotation of quasi spin is a *gauge transformation*, and the gauge angle is φ. The precession frequency is

$$\frac{\partial \varphi}{\partial t} = \frac{\partial E}{\partial S_0} = 2\frac{\partial E}{\partial N} = 2\mu,$$

where μ denotes the chemical potential.

Ground states of nuclei with different mass N form a *pairing rotational spectrum*