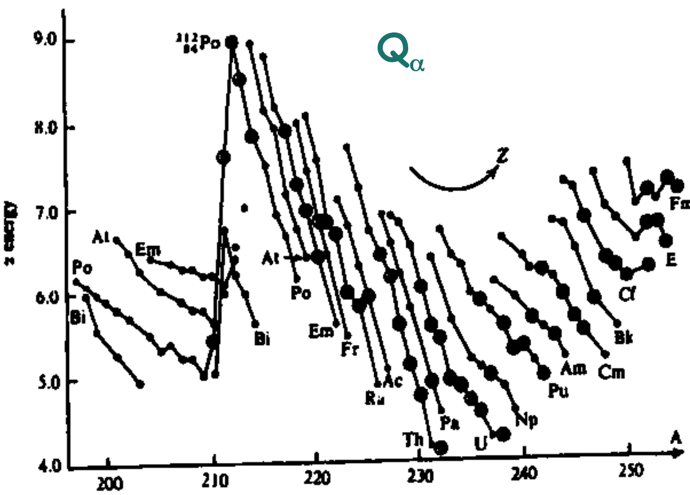
# Alpha Decay

## Energy relations

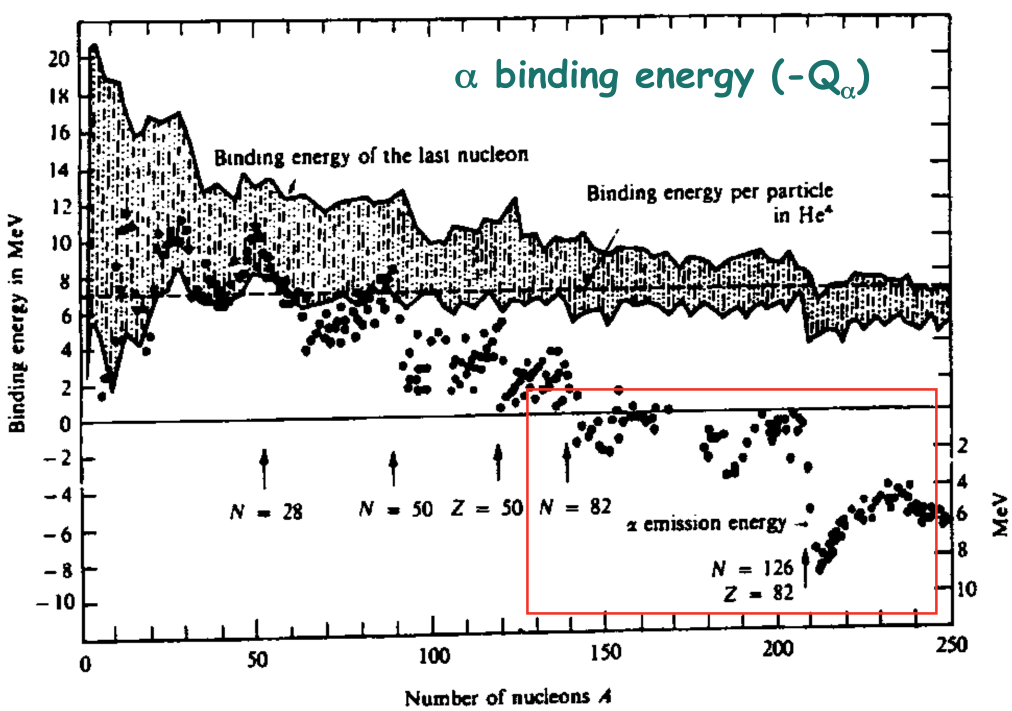
$$Q_\alpha(A,Z) = B(A-4,Z-2) + 28.3\text{MeV} - B(A,Z)$$

experimental binding energy of $^4$He




$$Q_\alpha = T_\alpha + T_d = T_\alpha\left(\frac{M_D + M_\alpha}{M_D}\right) \approx T_\alpha\left(\frac{A}{A-4}\right)$$

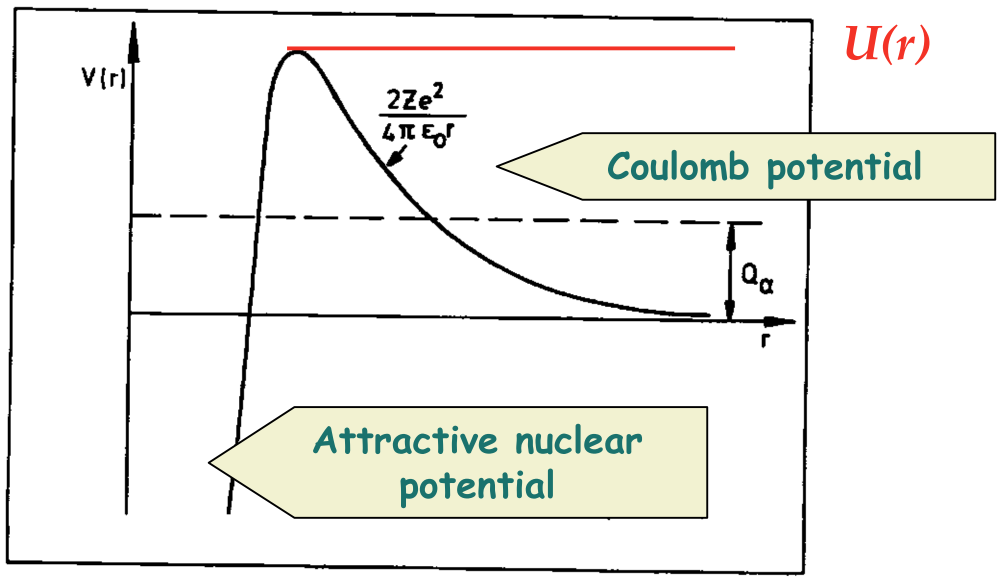
recoil term effect



# Alpha Decay

## Theory of Alpha decay

1928 Gamow




### Time-dependent approach

$$\left[-\frac{\hbar^2}{2M}\nabla^2 + V(\vec{r})\right]\psi(\vec{r},t) = i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t}$$

At t=0, alpha particle is localized inside the nucleus. It can be represented by a wave packet. At large times, the wave function is an outgoing wave. In the first step, one solves the stationary problem for the auxiliary potential U(r)

$$\left[-\frac{\hbar^2}{2M}\nabla^2 + U(\vec{r})\right]\varphi_n(\vec{r}) = E_n\varphi_n(\vec{r})$$

$$\psi(\vec{r},t) = \sum_n c_n(t)\varphi_n(\vec{r})e^{-iE_nt/\hbar}$$

Summation or integration

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$$\frac{\partial}{\partial t}\rho(\vec{r},t)+\vec{\nabla}\vec{j}(\vec{r},t)=0$$

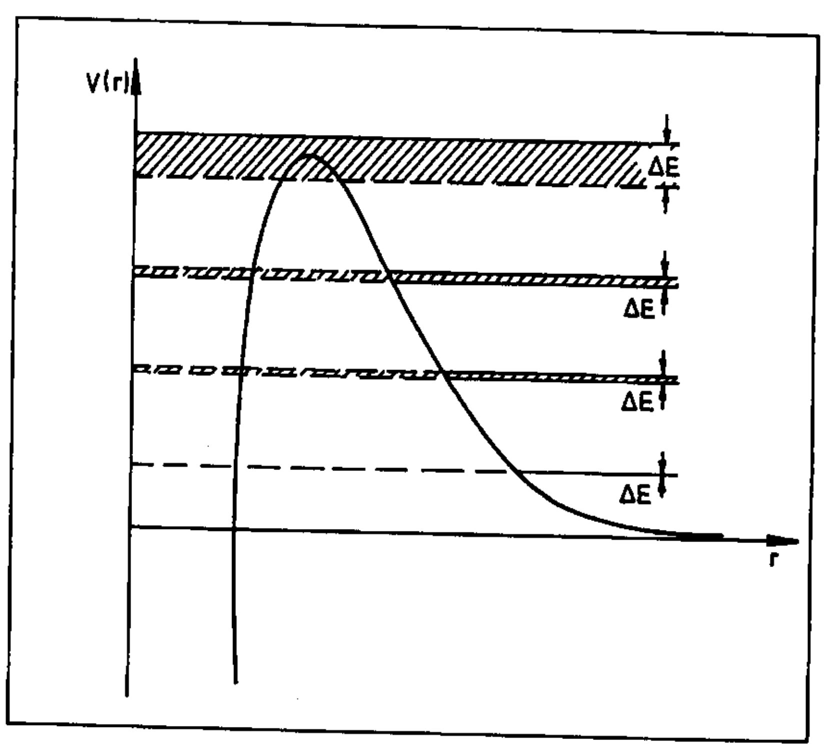
Continuity equation

Since a current of alpha-particles is leaving the nucleus, the density ρ has to be a decreasing function of time. If we want to approximate the initial wave function through a stationary state, we have to assume complex E:

$$E \Rightarrow E-\frac{i}{2}\Gamma \quad \left(\Gamma=\Delta E\right)$$

Line width

$$\rho(\vec{r},t)=\left|\psi(\vec{r},t)\right|^2=\left|\psi(\vec{r})\right|^2 e^{-\lambda t} \quad (\lambda \equiv \Gamma/\hbar)$$



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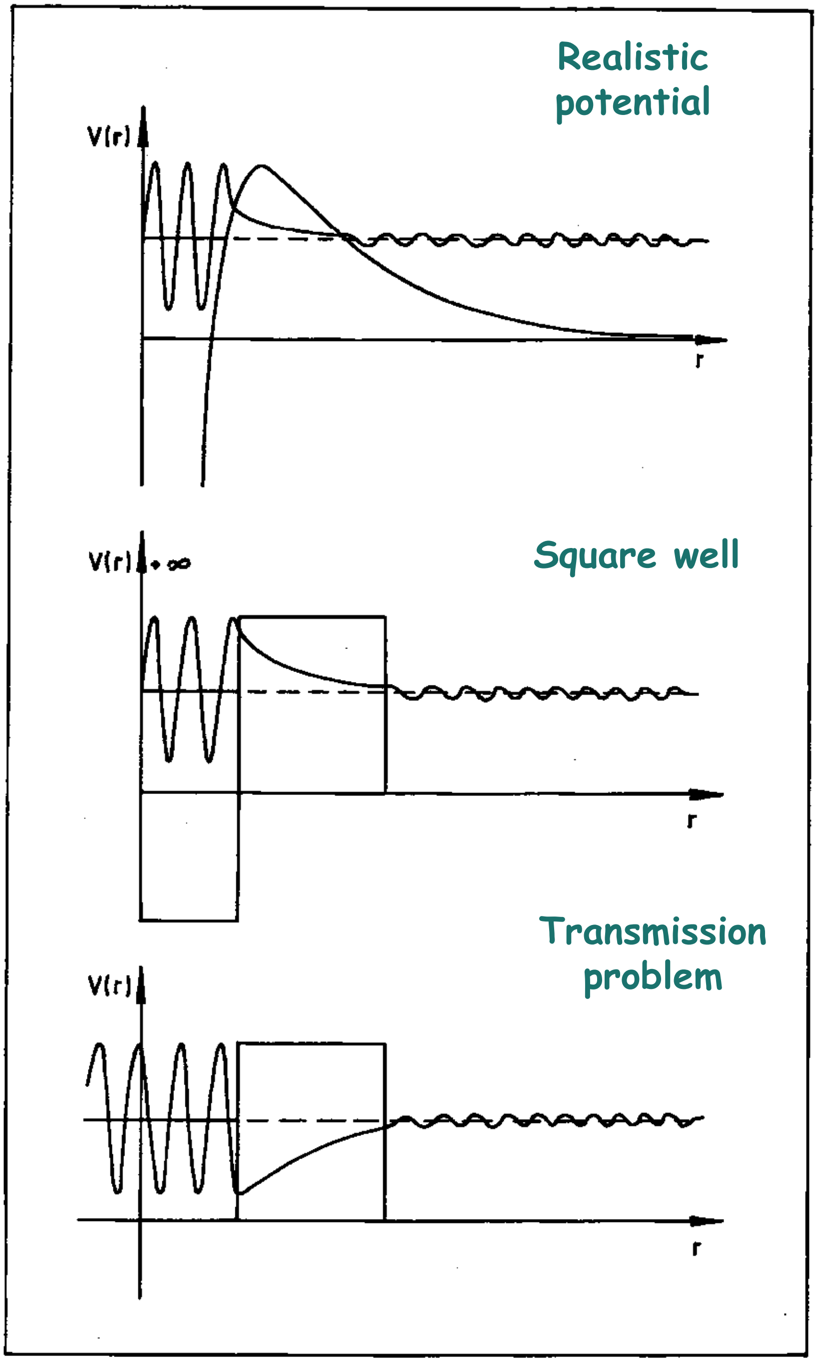
### Approximations



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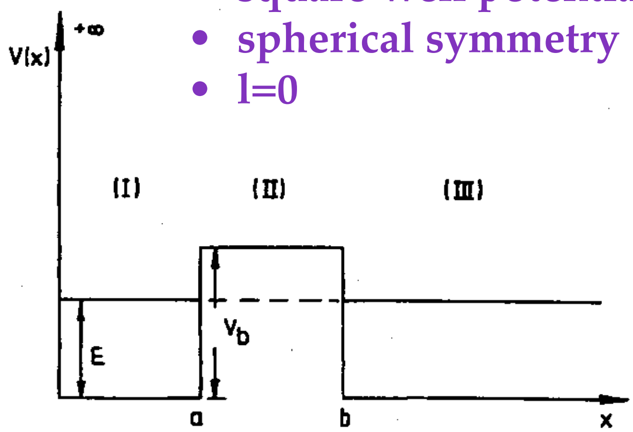


- square-well potential
- spherical symmetry
- l=0

$$\chi'' + \frac{2M}{\hbar^2}(E - V)\chi = 0 \quad (\chi = \varphi r)$$ Radial Schroedinger equation

**Region I:** $\chi_I = A \sin pr, \quad p^2 = \frac{2ME}{\hbar^2}$

**Region II:** $\chi_{II} = c_+ e^{q(r-a)} + c_- e^{-q(r-a)}, \quad q^2 = \frac{2M(V_b - E)}{\hbar^2}$

**Region III:** $\chi_{III} = c_1 e^{p(r-b)} + c_2 e^{-p(r-b)}$

$$c_\pm = i \sin pa \pm i \frac{p}{q} \cos pa, \quad G = 2q(b-a)$$

$$\frac{|\chi_{III}|^2}{|\chi_I|^2} = \frac{1}{4}\left(1 + \frac{q^2}{p^2}\right)\left(c_+^2 e^G + c_-^2 e^{-G}\right) + \frac{1}{2}\left(1 - \frac{q^2}{p^2}\right) c_+ c_-$$

In almost all cases $|\chi_{III}|$ is much larger than $|\chi_I|$. We are now interested in those situations where $|\chi_{III}|$ is as small as possible.

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$$c_+ = 0 \Rightarrow \tan(pa) = -\frac{p}{q}$$

defines "virtual" levels in region I: alpha particle is well localized; very small penetrability through the barrier