Construction of the functional

Perlinska et al., Phys. Rev. C 69, 014316 (2004)

isoscalar (T=0) density $\left(\rho_0 = \rho_n + \rho_p\right)$

isovector (T=1) density $\left(\rho_1 = \rho_n - \rho_p\right)$

+isoscalar and isovector densities:
spin, current, spin-current tensor,
kinetic, and kinetic-spin
+ pairing densities

p-h density p-p density (pairing functional)

$$\mathcal{H}(r) = \frac{\hbar^2}{2m}\tau_0(r) + \sum_{t=0,1}\left(\chi_t(r) + \breve{\chi}_t(r)\right)$$

Most general second order expansion in densities and their derivatives

$$\begin{aligned}
\chi_0(r) &= C_0^\rho \rho_0^2 + C_0^{\Delta\rho}\rho_0\Delta\rho_0 + C_0^\tau \rho_0\tau_0 + C_0^{J0}J_0^2 + C_0^{J1}J_0^2 + C_0^{J2}\underline{J}_0^2 + C_0^{\nabla J}\rho_0\nabla\cdot J_0 \\
&+ C_0^s s_0^2 + C_0^{\Delta s} s_0\cdot\Delta s_0 + C_0^T s_0\cdot T_0 + C_0^j j_0^2 + C_0^{\nabla j} s_0\cdot(\nabla\times j_0) + C_0^{\nabla s}(\nabla\cdot s_0)^2 + C_0^F s_0\cdot F_0, \\
\chi_1(r) &= C_1^\rho \vec{\rho}^2 + C_1^{\Delta\rho}\vec{\rho}\circ\Delta\vec{\rho} + C_1^\tau\vec{\rho}\circ\vec{\tau} + C_1^{J0}\vec{J}^2 + C_1^{J1}\vec{J}^2 + C_1^{J2}\vec{\underline{J}}^2 + C_1^{\nabla J}\vec{\rho}\circ\nabla\cdot\vec{J} \\
&+ C_1^s\vec{s}^2 + C_1^{\Delta s}\vec{s}\cdot\circ\Delta\vec{s} + C_1^T\vec{s}\cdot\circ\vec{T} + C_1^j\vec{j}^2 + C_1^{\nabla j}\vec{s}\cdot\circ(\nabla\times\vec{j}) + C_1^{\nabla s}(\nabla\cdot\vec{s})^2 + C_1^F\vec{s}\cdot\circ\vec{F},
\end{aligned}$$

- Constrained by microscopic theory: ab-initio functionals provide quasi-data!
- Not all terms are equally important. Usually ~12 terms considered
- Some terms probe specific experimental data
- Pairing functional poorly determined. Usually 1-2 terms active.
- Becomes very simple in limiting cases (e.g., unitary limit)

Pairing correlations: Introduction

This Lecture closely follows chapter 6 in Ring & Schuck *The nuclear many-body problem*.

Idea of pairing correlations due to

1. *energy gap:* Even-even nuclei have few levels up to 1–2 MeV. Odd-even nuclei have several levels in this range.
2. *level density:* For a few nucleons in single j-shell, calculated level density exceeds experimental level density.
3. *odd-even staggering:* Odd-mass nuclei are less bound than their even-mass neighbors.
4. *moments of inertia:* Moments of inertia from single-particle picture too large when compared to data.
5. *deformation:* Nuclei close to closed shell are spherical, deformation sets in suddenly when further away from shell closure.

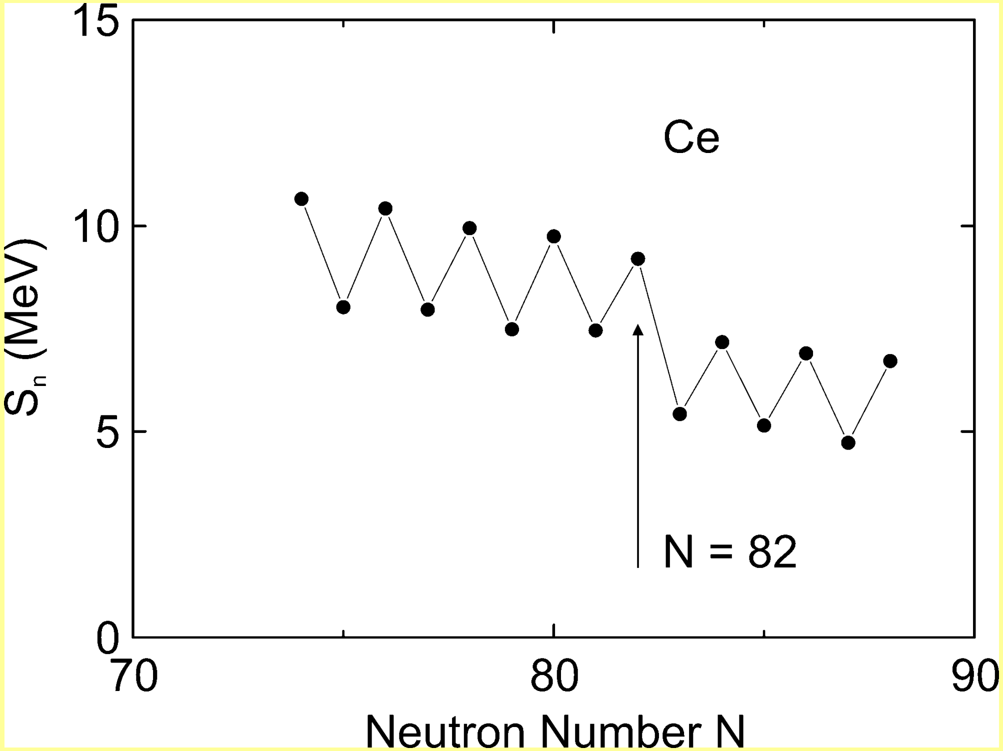
Even-even nuclei have $J = 0$ ground states. Göppert-Mayer (1950) showed that the $J = 0$ coupling of two nucleons in a single j-shell is energetically favored for a short-ranged, attractive nucleon-nucleon force. This is due to the large overlap of single-particle orbitals with $\pm m$ angular momentum projection.

$$|J = 0, M = 0\rangle = \frac{1}{\sqrt{2}} \sum_m \langle j, j, m, -m|00\rangle a^\dagger_m a^\dagger_{-m} |-\rangle.$$

The observations 1.–5. listed above can be explained when one assumes that (i) spin-0 pairs are an important component of the wave function, (ii) the breaking of a pair costs about 1–2 MeV, (iii) pair condensation yields superfluid components.

Evidence for pairing correlations in nuclei

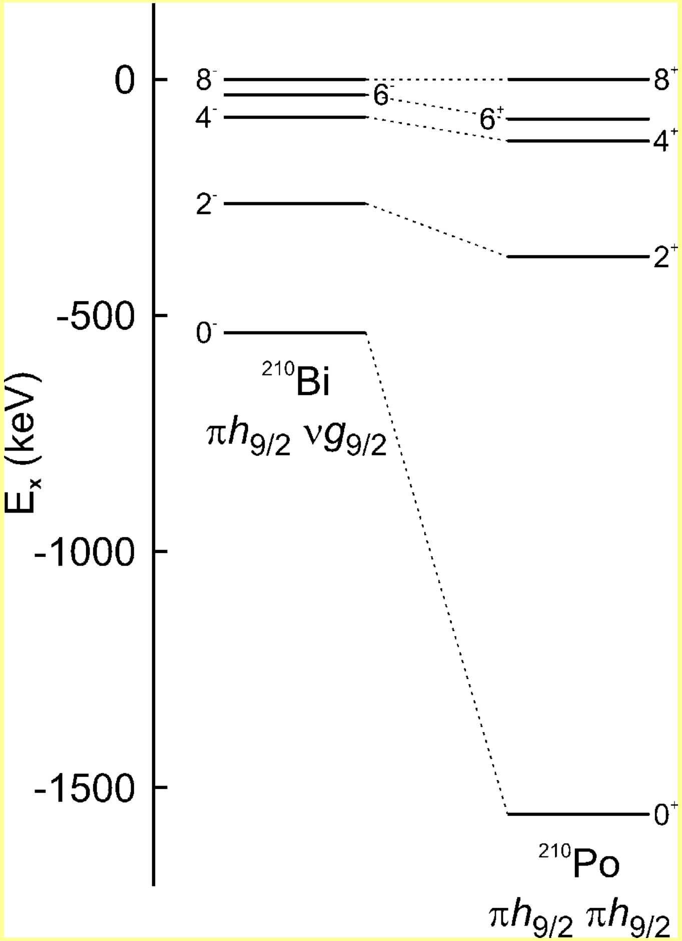
(i) Odd-even effect: mass of an odd-even nucleus is larger than the mean of adjacent two even-even nuclear masses → shows up in S_n and S_p for all nuclei.



- Behavior points towards pair formation of nucleons.

Evidence for pairing correlations in nuclei

(ii) Energy spectra for nuclei near closed shells (A ± 2, A ± 4) show a clear gap for the 0^+ g.s.



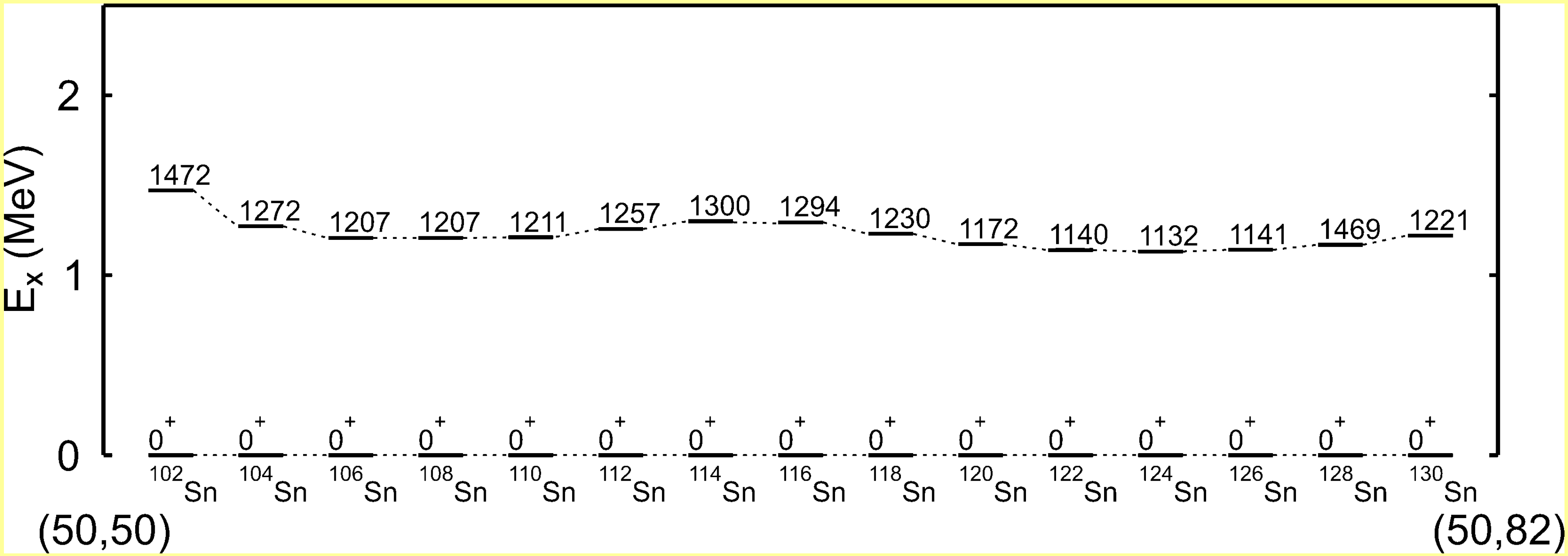
- In ^{210}Po the configuration outside the doubly-closed shell core of ^{208}Pb is $(1h_{9/2})^2$. If there were no interaction between the two π's constituting the pair, i.e. if they behaved like independent particles, the various $(1h_{9/2})^2$ spin couplings, which reflect the orbital alignments, would lead to states degenerate in energy.

  → correlated pair of two π's

- Pairing effect ≈ 2Δ

Evidence for pairing correlations in nuclei

(iii) The excitation energy of the first excited 2^+ state in nuclei remains remarkably constant over large intervals of neutron (proton) numbers.



- These 2^+ states are not rotational states but are connected to a coherent pairing condensate.
- Pair breaking energy: $2\Delta \approx 2$ MeV

Pairing gaps in even-even nuclei, reduced moments of inertia, etc.

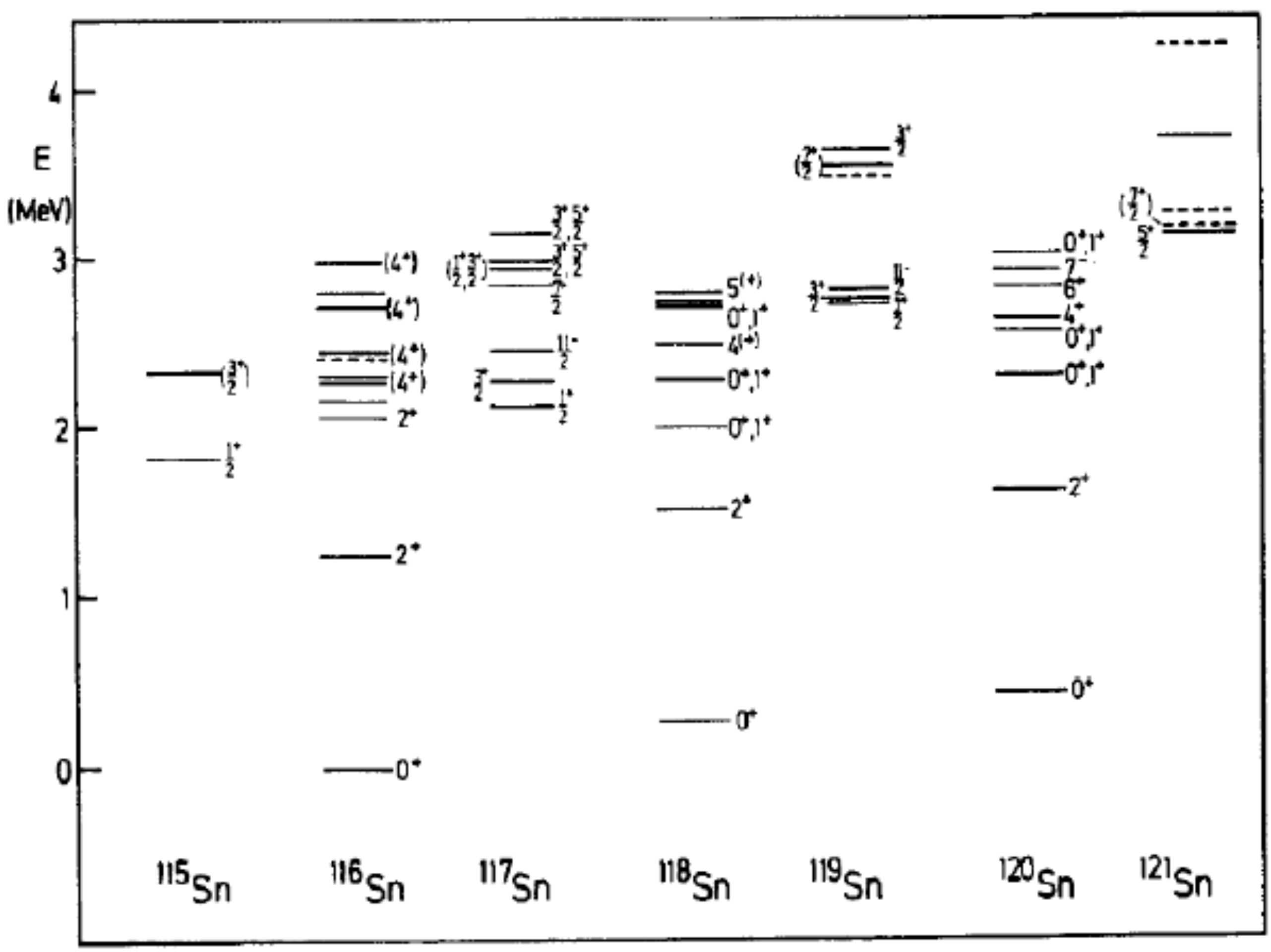
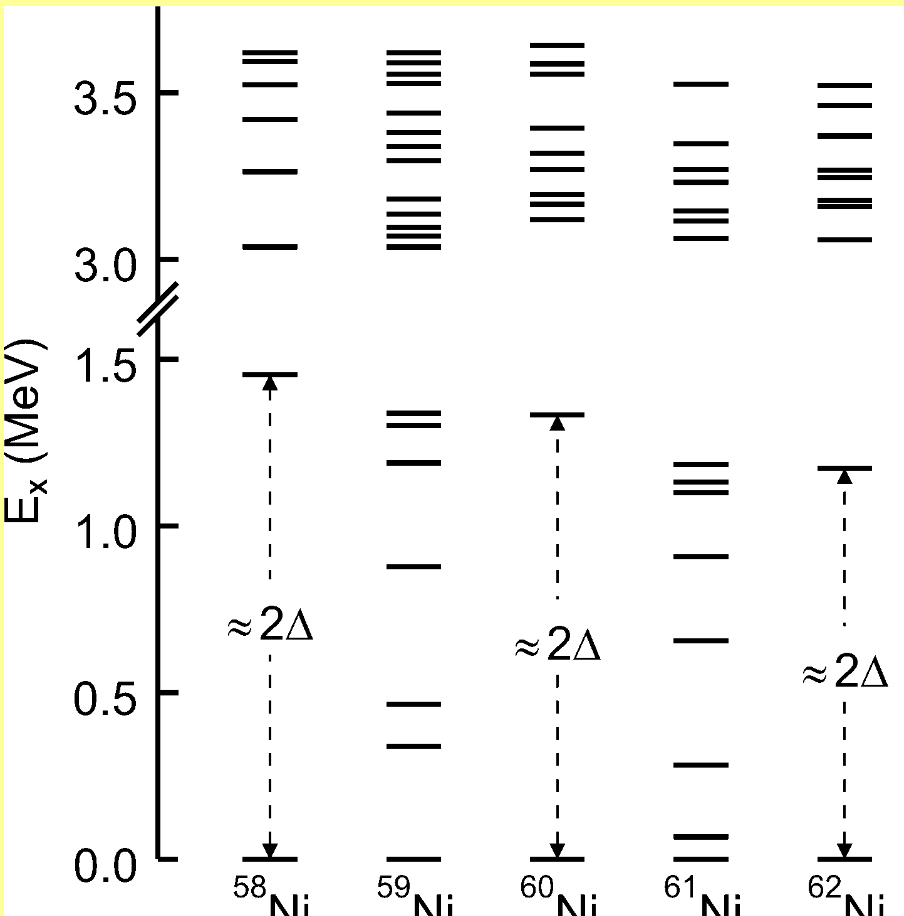


Figure 6.1. Excitation spectra of the $_{50}$Sn isotopes.

Evidence for pairing correlations in nuclei

(iv) Energy gap: odd-even and even-odd nuclei (especially deformed nuclei) have energy spectra different from even-even nuclei.



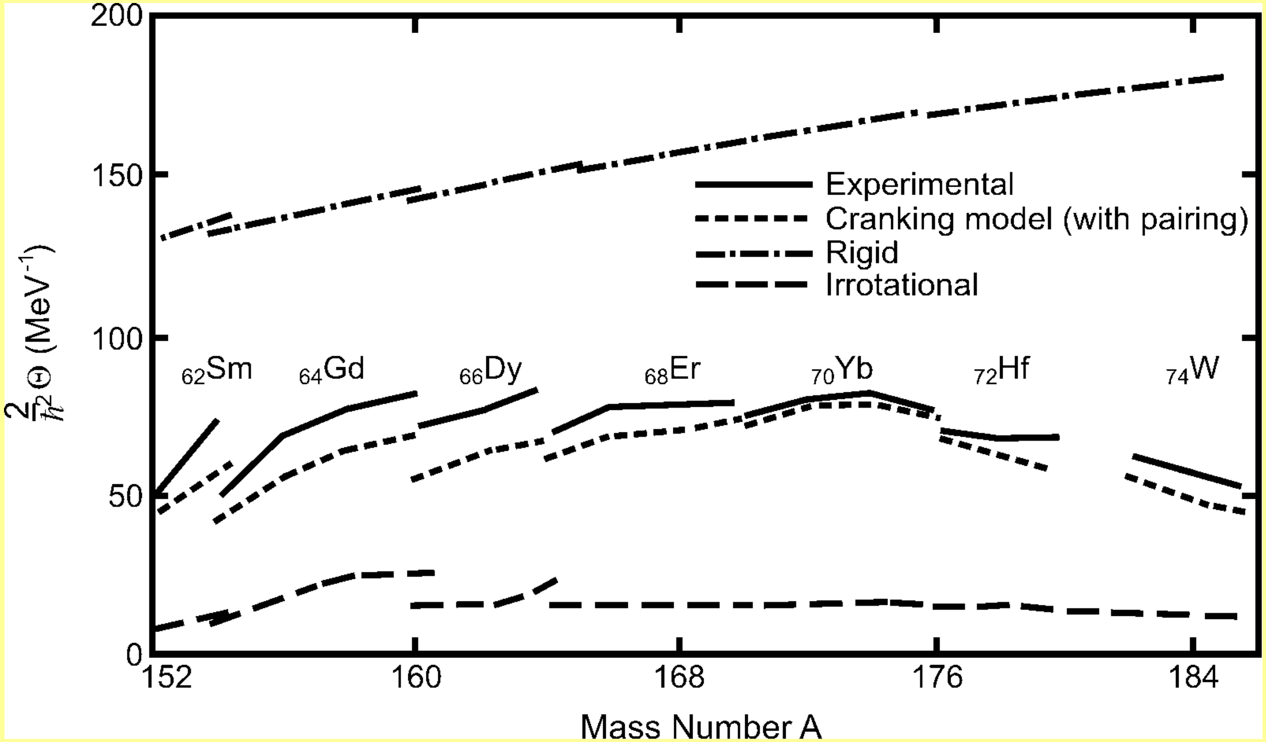
- e-e nuclei: only a few states at most (vibrations, rotations) appear below the pairing gap 2Δ.
- But in o-e and e-o nuclei (where the last nucleon is unpaired) many s.p. and collective states appear.
- Note: above the pair breaking energy 2Δ many excited states are possible → level density $\rho = \rho(\Delta)$

Evidence for pairing correlations in nuclei

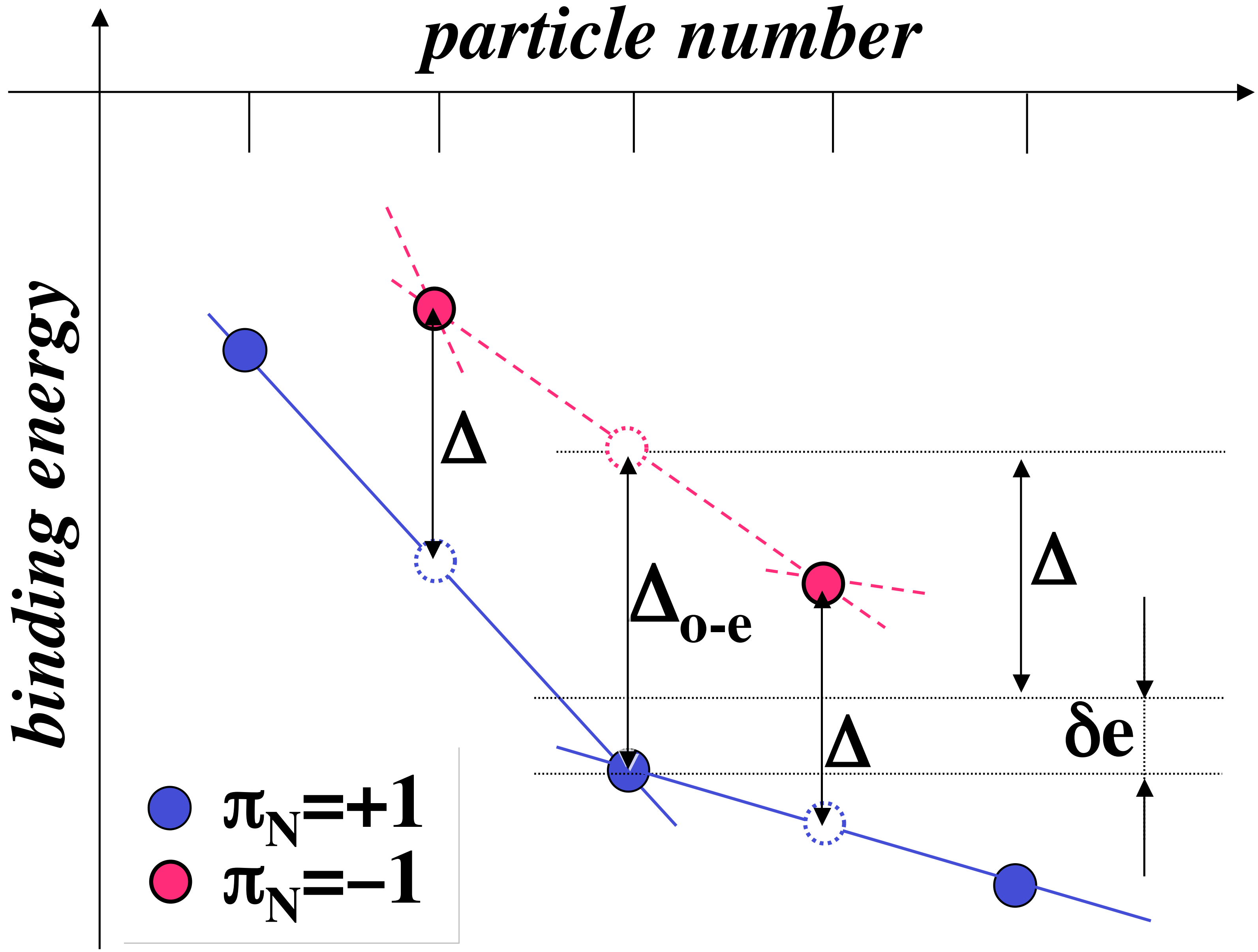
(v) Moment of inertia: extracted from level spacing in rotational bands

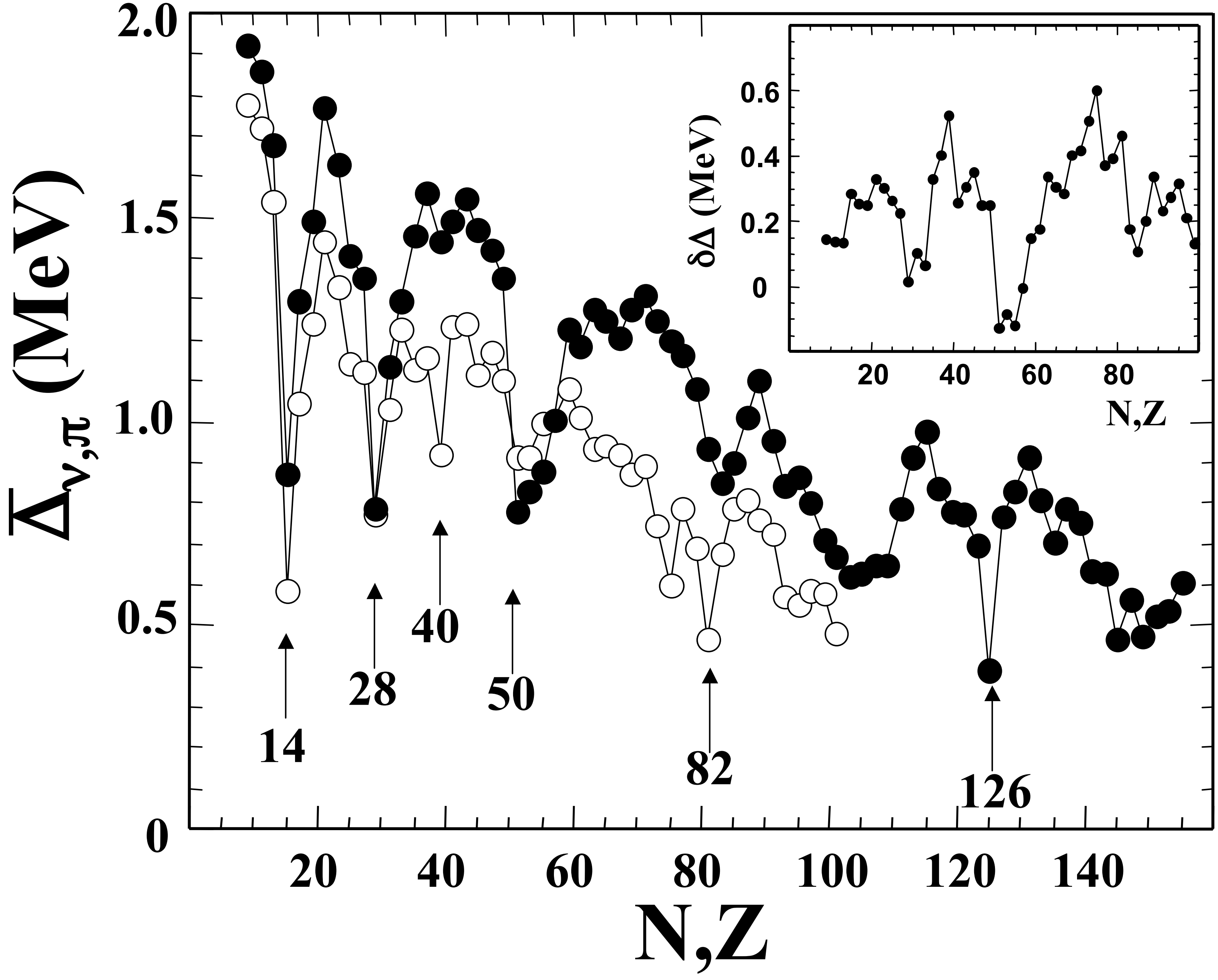
$$E = \frac{\hbar^2}{2\Theta} J(J+1)$$

deviates about a factor of two from the rigid rotor values.



- $\Theta_{irrot} < \Theta_{exp} < \Theta_{rigid}$
- Pairing correlations have a dramatic influence on collective modes.


particle number
binding energy
Δ
Δ
Δ
Δ_{o-e}
δe
π_N=+1
π_N=−1



$\overline{\Delta}_{\nu,\pi}$ (MeV)
2.0
1.5
1.0
0.5
0
20
40
60
80
100
120
140
N,Z
14
28
40
50
82
126
δΔ (MeV)
0.6
0.4
0.2
0
20
40
60
80
N,Z