Time Reversal

$$\vec{r}_k{}' = \vec{r}_k = \mathcal{T}\,\vec{r}_k\,\mathcal{T}^{-1}$$

$$\vec{p}_k{}' = -\vec{p}_k, \quad \vec{s}_k{}' = -\vec{s}_k$$

$\mathcal{T}$ cannot be represented by an unitary operator. Unitary operations preserve algebraic relations between operators, while $\mathcal{T}$ changes the sign of commutation relations.

$$\left[p_x, x\right] = -i\hbar \Rightarrow \left[p_x{}', x'\right] = i\hbar$$

$$\left[s_x, s_y\right] = i s_z \Rightarrow \left[s_x{}', s_y{}'\right] = -i s_z{}'$$

In order to save the commutation relations, on has to introduce:

$$\mathcal{T} = UK$$

antiunitary

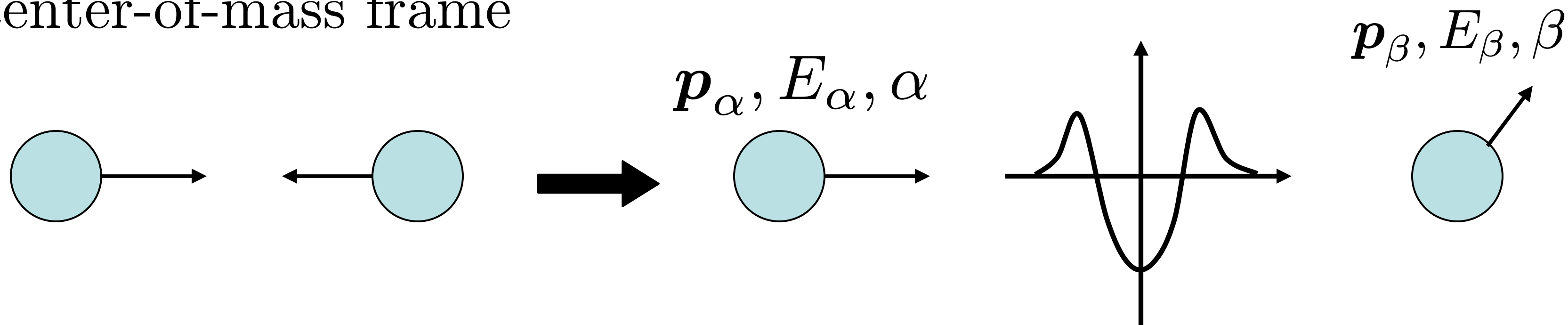
unitary

takes complex conjugate of all *c* numbers

$$\langle B | A \rangle = \langle B' | A' \rangle^*$$

Nuclear Reactions

- Nuclear reaction: $a_1 + a_2 \rightarrow b_1 + b_2$
- Reducible to a 2-body problem: scattering of a particle off a potential (elastic, inelastic or nuclear reaction) in the center-of-mass frame



- Interaction represented by Hamiltonian $\hat{H} = H_0 + \hat{V}$
- Cross-section (probability of scattering at given angle) is proportional to S-matrix element:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\alpha\rightarrow\beta} \sim |S_{\alpha\beta}|^2$$

Nuclear Reactions and S-Matrix

- Consider a wave-packet associated with one (or several) free particle for t → -∞, denoted by Ψ_α^{in}
- α stands for the quantum numbers (momentum, spin, etc.)
- After the interaction, we have a new wave-packet Ψ_β^{out}

$$S_{\alpha\beta} = \langle \Psi_\alpha^{out} | \Psi_\beta^{in} \rangle$$

- Consider now the wave-packets associated with the same free particles that do *not* interact, Φ_α and Φ_β

$$S_{\alpha\beta} = \langle \Phi_\alpha | U(+\infty, -\infty) | \Phi_\beta \rangle$$

- Evolution operator:

$$U(t, t_0) = e^{-\frac{i}{\hbar}(t-t_0)\hat{H}}$$

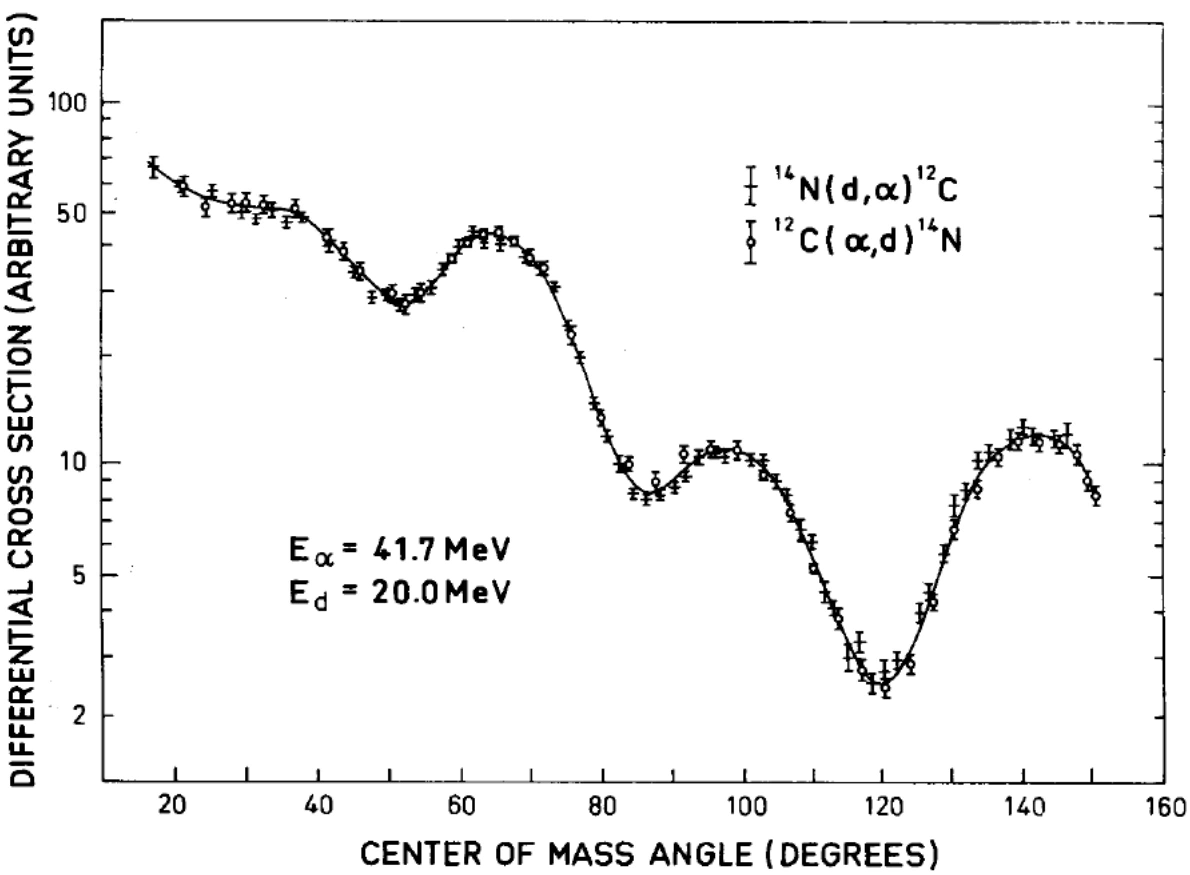
- If the Hamiltonian is time-even, the S-matrix elements are the same as for the inverse process.

Time reversal symmetry

Nuclear reactions

$$a_1 + a_2 = b_1 + b_2$$

normal and inverse kinematics!



Other Symmetries

C - interchanges particles & antiparticles (charge conjugation)

CP - violated in K^0 decay

CPT - follows from relativistic invariance

Since *CP* is violated, *T* has to be violated as well!

Remember: Measurement of the Electron Dipole moment is an attempt to find evidence of T-violati on independently of CP-violation

Isospin symmetry

Introduced 1932 by Heisenberg

- Protons and neutrons have almost identical mass: $\Delta m/m = 1.4\text{x}10^{-3}$
- Low energy np scattering and pp scattering below E=5 MeV, after correcting for Coulomb effects, is equal within a few percent in the ^1S scattering channel.
- Energy spectra of “mirror” nuclei, (N,Z) and (Z,N), are almost identical.

$$\psi_n(\vec{r},s) = \phi(\vec{r},s)\begin{pmatrix}1\\0\end{pmatrix}, \quad \psi_p(\vec{r},s) = \phi(\vec{r},s)\begin{pmatrix}0\\1\end{pmatrix}$$

wave functions

$$\tau_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}, \quad \tau_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}, \quad \tau_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

Pauli isospin matrices

$$\vec{\tau} = (\tau_x, \tau_y, \tau_z), \quad \vec{t} = \vec{\tau}/2$$

$$[t_x, t_y] = it_z, \quad [\vec{t}^{\,2}, t_i] = 0$$

SU(2) commutation roles!

$$\vec{T} = \sum_{i=1}^{A} \vec{t}_i$$

total isospin

$$[H, T_z] = 0 \Rightarrow T_z|\Psi\rangle = \frac{N-Z}{2}|\Psi\rangle$$

T_z component conserved! (charge conservation)

$$[H, T_\pm] = 0$$

charge independence

$$[H, T^2] = 0 \Rightarrow T^2|\Psi\rangle = T(T+1)|\Psi\rangle$$

T is conserved!

Isospin (Complements)

- Nucleons are eigenstates of the isospin operator (by definition)

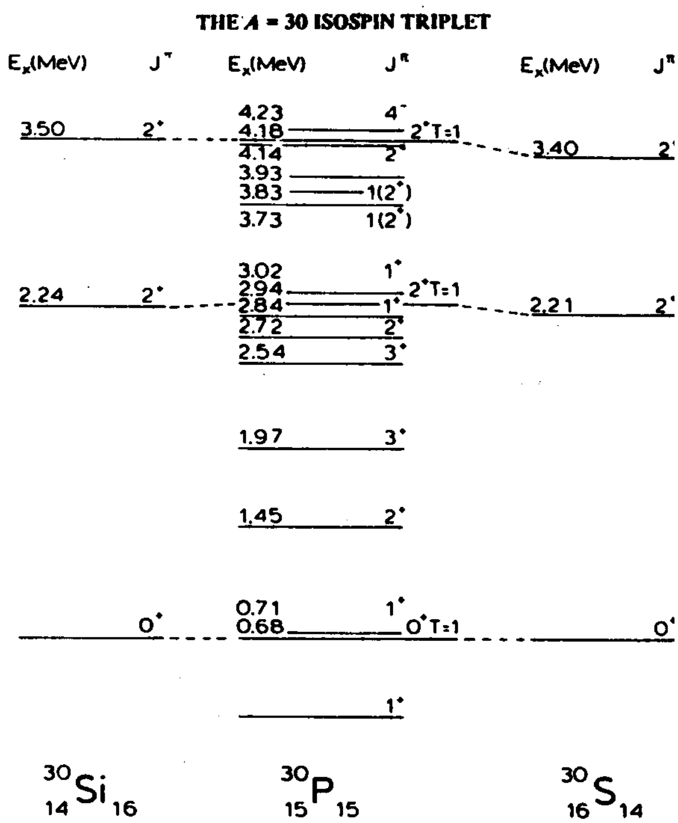
$$\begin{array}{ll} t_z = +1/2 & \text{neutron} \\ t_z = -1/2 & \text{protons} \end{array}$$

- Nucleon isospin projections (eigenvalues):
- If isospace has dimension 2, any unitary operator acting in it can be expanded on: $I, \sigma_x, \sigma_y, \sigma_z$

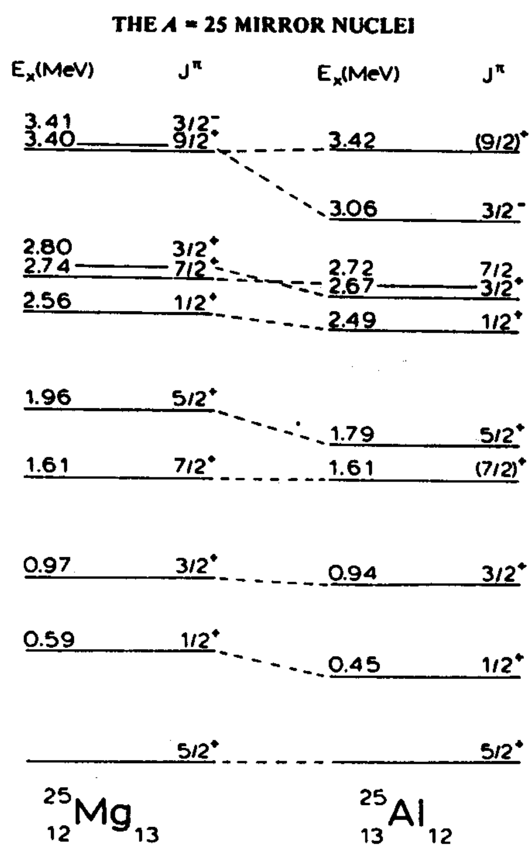
$$\Rightarrow \text{SU(2) (Lie) Algebra}$$

- t_x and t_y transform proton into neutron (and vice-versa)
- Charge independence of strong N-N interaction

Isospin Symmetry (examples)

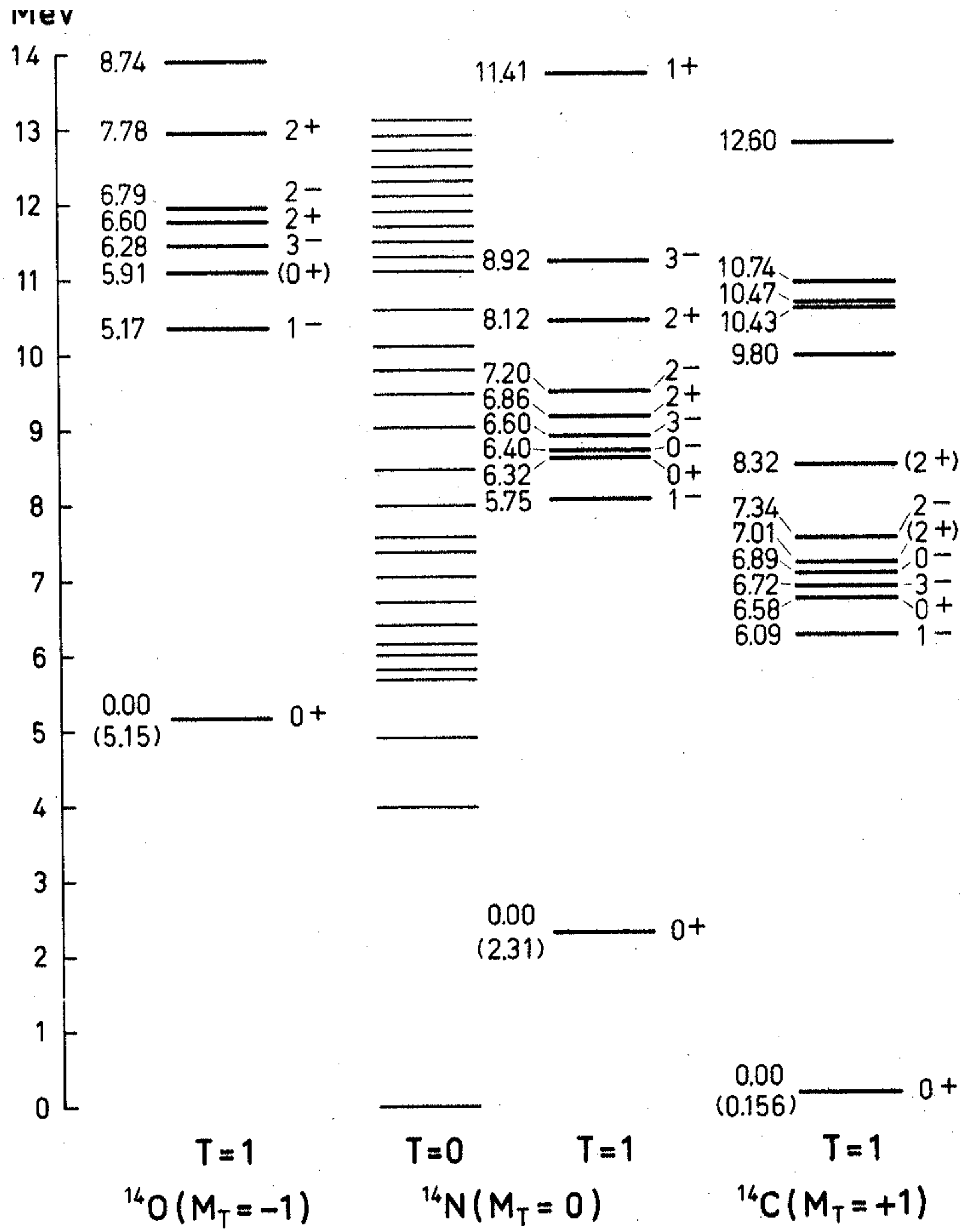


T=1 multiplet



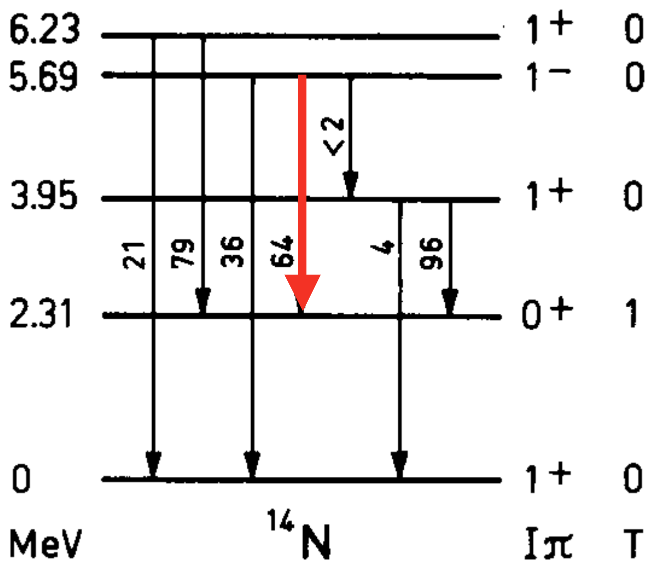
Mirror Nuclei

Isospin Symmetry (examples)



Coulomb shift, Thomas-Ehrman shift...

Isospin Symmetry
(examples; isospin selection rules)



$$B(E1) \propto \left|\langle i|D|f\rangle\right|^2$$

$$D = \sum_k e_k z_k = \frac{1}{2} e \sum_k z_k - \frac{1}{2} e \sum_k z_k \tau_z(k)$$

isoscalar **isovector**

$$|T_i - T_f| \le 1 \le T_i + T_f$$

$$\langle T010|T0\rangle = 0 \Rightarrow$$

all $T_i = T_f$ transitions in N=Z (self-conjugate) nuclei are forbidden!