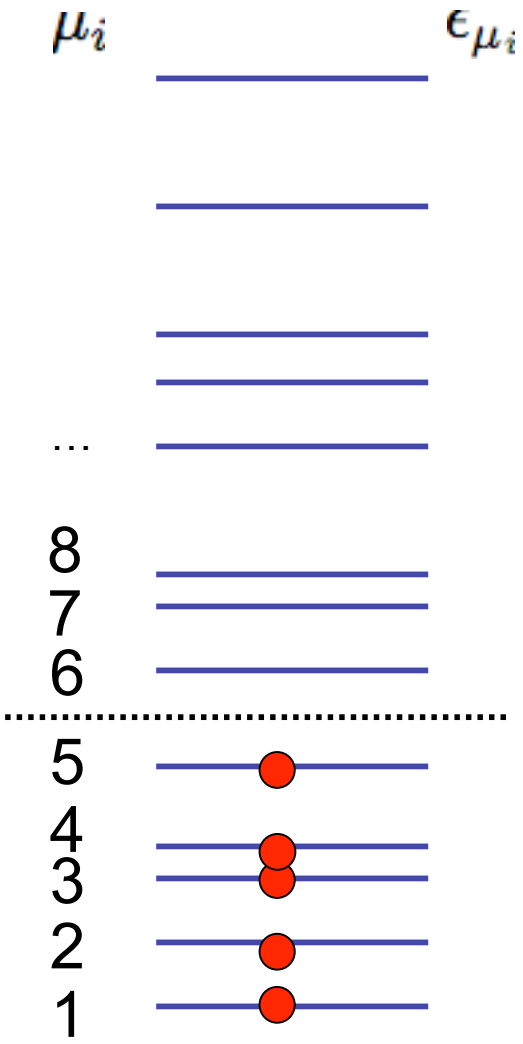
Wick's theorem for Slater determinants: examples

$$|\Phi\rangle = a^+_{\mu_1} \ldots a^+_{\mu_A} |0\rangle$$

$$\hat{A}\hat{B} = \langle\Phi|\hat{A}\hat{B}|\Phi\rangle - \langle\Phi|\hat{A}|\Phi\rangle\langle\Phi|\hat{B}|\Phi\rangle$$

Consider A=5 particles moving in non-degenerate states. The lowest energy state is the shell-model ground state:

$$|\Phi\rangle = a^+_1 a^+_2 a^+_3 a^+_4 a^+_5 |0\rangle$$



1. Find

$$a_{\mu_0},\ a_{\mu_-},\ a_{\mu_+}$$

$$a^+_{\mu_0},\ a^+_{\mu_-},\ a^+_{\mu_+}$$

$$\overbrace{a_5 a_7}$$

$$\overbrace{a_1 a^+_3} \qquad \overbrace{a_1 a^+_1} \qquad \overbrace{a^+_1 a_1}$$

$$\overbrace{a_7 a^+_8} \qquad \overbrace{a^+_7 a_7}$$

2. Find

$$\langle\Phi|a^+_2 a_1 a_2 a_3|\Phi\rangle$$

$$\langle\Phi|a^+_2 a_1 a_2 a^+_3|\Phi\rangle$$

$$\langle\Phi|a^+_2 a_3 a_2 a^+_3|\Phi\rangle \qquad \langle\Phi|a^+_2 a^+_3 a_2 a_3|\Phi\rangle$$

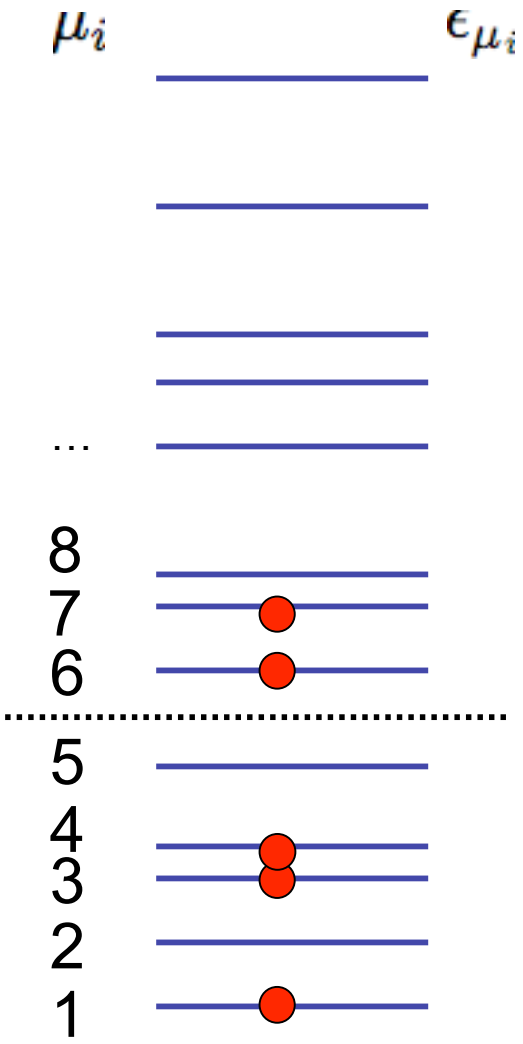
$$\langle\Phi|a^+_2 a_7 a_2 a^+_7|\Phi\rangle \qquad \langle\Phi|a_6 a_7 a^+_6 a^+_7|\Phi\rangle$$

Consider an excited state of a system:

$$|\Phi'\rangle = a_1^+ a_3^+ a_4^+ a_6^+ a_7^+ |0\rangle$$

$$\langle\Phi|\Phi'\rangle = ?$$

1. How to write $|\Phi'\rangle$ in terms of $|\Phi\rangle$?



2. Find

$$a_{\mu_0},\ a_{\mu_-},\ a_{\mu_+}$$

$$a^+_{\mu_0},\ a^+_{\mu_-},\ a^+_{\mu_+}$$

$$\overbrace{a_5 a_7}$$

$$\overbrace{a_1 a_3^+} \qquad \overbrace{a_1 a_1^+} \qquad \overbrace{a_1^+ a_1}$$

$$\overbrace{a_7 a_8^+} \qquad \overbrace{a_7^+ a_7}$$

3. Find

$$\langle\Phi'|a_2^+ a_1 a_2 a_3|\Phi'\rangle$$

$$\langle\Phi'|a_2^+ a_1 a_2 a_3^+|\Phi'\rangle$$

$$\langle\Phi'|a_2^+ a_3 a_2 a_3^+|\Phi'\rangle \qquad \langle\Phi'|a_2^+ a_3^+ a_2 a_3|\Phi'\rangle$$

$$\langle\Phi'|a_2^+ a_7 a_2 a_7^+|\Phi'\rangle \qquad \langle\Phi'|a_6 a_7 a_6^+ a_7^+|\Phi'\rangle$$

What if a Slater determinant is expressed in another basis?

$$
\begin{aligned}
a_1'^{+} &= \cos\alpha\, a_1^{+} + \sin\alpha\, a_2^{+} \\
a_2'^{+} &= -\sin\alpha\, a_1^{+} + \cos\alpha\, a_2^{+} \\
a_\mu'^{+} &= a_\mu^{+} \quad \text{for } \mu > 2
\end{aligned}
$$

$$
|\Phi''\rangle = a_1'^{+} a_2'^{+} a_3'^{+} a_4'^{+} a_5'^{+} |0\rangle
$$

Find

$$
a_{\mu_0},\ a_{\mu_-},\ a_{\mu_+}
$$

$$
a^{+}_{\mu_0},\ a^{+}_{\mu_-},\ a^{+}_{\mu_+}
$$

$$
\overbrace{a_5 a_7}
$$

$$
\overbrace{a_1 a_3^{+}} \qquad \overbrace{a_1 a_1^{+}} \qquad \overbrace{a_1^{+} a_1}
$$

$$
\overbrace{a_7 a_8^{+}} \qquad \overbrace{a_7^{+} a_7}
$$